A simplex method for two stage linear stochastic problems with general cost distribution

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Keywords: Stochastic Programming, Simplex, Polyhedron, Normal fan, Chamber complex.

Abstract: Let $\boldsymbol{q}$ be an integrable random variable and define the following 2-stage stochastic linear problem (see e.g. [3])

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & c^\top x + \mathbb{E}\begin{bmatrix} \min\limits_{y\in\mathbb{R}^m} & \boldsymbol{q}^\top y \\ \text{s.t.} & Tx + Wy \leq h \end{bmatrix} \\ \text{s.t.} \quad & Ax \leq b \end{aligned} \tag{2SLP}$$

The structure of this 2SLP can be enlightened through polyhedral geometry. We first recall the links between linear programming and polyhedral geometry by comparing the geometric interpretation of the simplex algorithm, as a walk on the vertices of the polyhedron of admissible points, with its combinatorial interpretation, as a pivot from a basis to another one. After presenting the notions of faces, normal fan and active constraints, we explain how they are related through a one-to-one correspondence. This polyhedral and combinatorial frameworks allow us to derive analytical results for 2SLP. In particular, we show that the cost distribution can be replaced by a discrete cost distribution as shown in [1], taking the expected value of the cost at each cone of a normal fan. We define the chamber complex of a polyhedron P along a projection π as the polyhedral complex obtained by intersecting the projections $\pi(F)$ for F describing the faces of P. Finally, we present an algorithm to solve 2SLP. Geometrically, this algorithm is a walk on the vertices of a chamber complex where we reduce the value of the function step by step. Combinatorially, the pivot can be interpreted as a flip from one collection of active constraint sets (understood as a regular subdivision as defined in [4]) to another one. Thus, this new algorithm can be seen as a generalization of the simplex method for 2SLP.

References:

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