Approximation of an optimal control problem posed on a network with penalized trajectories

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Abstract: This work is connected to Hamilton-Jacobi equations theory and more specifically to Hamilton-Jacobi equations posed on networks (see [1, 2, 5]). We consider the particular network $C := 0_{\mathbb{R}^2} \cup (\mathbb{R}_+^*(0,1)) \cup (\mathbb{R}_+^*(-1,0)) \cup (\mathbb{R}_+^*(0,-1)) \cup (\mathbb{R}_+^*(1,0))$, and for $\varepsilon > 0$ the following system:

$$\left\{ \begin{array}{lcl} {X'}^{x,\alpha,\varepsilon}(t) & = & f(X^{x,\alpha,\varepsilon}(t), \alpha(t)) - \dfrac{\nabla d(X^{x,\alpha,\varepsilon}(t))}{\varepsilon}, \quad t > 0 \\ {X'}^{x,\alpha,\varepsilon}(0) & = & x \end{array} \right. , \tag{S_ε}$$

where $f : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is bounded and uniformly lipschitz w.r.t its first variable, $\alpha \in L^\infty\left(\mathbb{R}_+, \mathbb{R}^2\right)$, and $d(x, y) = x^2y^2$, $(x, y) \in \mathbb{R}^2$. Properties of d will constrain trajectories to evolve nearby the network C and to remain on it at the limit. So we have to consider both the convergence of the sequences $(X^{x,\alpha,\varepsilon})_{\varepsilon>0}$ and that of $(V^\varepsilon)_{\varepsilon>0}$ (optimal value functions associated with (S_ε)). We propose some conditions which allow to characterize the limit V of $(V^\varepsilon)_{\varepsilon>0}$ as the value function of an optimal control problem posed on C in the sense of the literature, and that the limit $X^{x,\alpha}$ of $(X^{x,\alpha,\varepsilon})_{\varepsilon>0}$ is an admissible trajectory for it. The authors in [3] obtained some similar but different results. Our approach is quite different: we constrain the trajectories using the function d, keeping a large set of admissible controls. We took inspiration from the study of the Skorokhod problem [4]. We meet several stability issues that we will present. We will also explain the choice of the network's structure and propose some ideas to extend it to some other situations.

References:

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