A two-stage robust approach for minimizing the weighted number of tardy jobs with profit uncertainty

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Abstract: We investigate a stochastic variant of the well-know $1|r_j|\sum w_jU_j$ problem, in which the jobs are subject to unexpected failure which leads to additional costs. The decision maker is then allowed to take recourse actions such as outsourcing or spending more time on the jobs to fix them. We are interested in worst-case optimization, with polyhedral uncertainty set affecting the objective function.

An instance of our problem consists of a set of jobs $\mathcal{J}$, each of which is characterized by a release date r_j, a due date d_j, and a nominal processing time p_j. A weight w_j can be interpreted as the cost for executing the job tardy, or the opposite of the profit of processing the job on time. At the first stage, *here-and-now* decisions are to select a subset of jobs $\mathcal{J}^* \subseteq \mathcal{J}$ to process. After that, a subset of the jobs can be affected by unexpected failures, those being governed by the uncertainty set $\Xi = \left\{ \xi \in \mathbb{R}_+^{|J|} \,\middle|\, \xi_j \leq 1, \forall J_j \in \mathcal{J} \text{ and } \sum_{j|J_j\in\mathcal{J}} \xi_j \leq \Gamma \right\}$. The realization of alea $\xi \in \Xi$ determines a profit degradation for each job $J_j \in \mathcal{J}$ defined as $\delta_j(\xi) = \bar{\delta}_j\xi_j$, where $\bar{\delta}_j$ is the maximum additional cost linked to the job's failure. Input parameter Γ is the largest number of jobs that can incur their maximum degradation. At the second stage *recourse* actions have to be taken. For each $j \in \mathcal{J}^*$, one can choose *(i)* to keep the revealed profit ; *(ii)* to repair the job, adding τ_j time units to its processing time to recover its initial profit ; or *(iii)* to reject the job, and pay a fixed outsourcing cost f_j. Finally, jobs in $\mathcal{J}^*$ that are not rejected must be scheduled so that they meet their time windows. The objective is to select a subset of jobs as well as the recourse actions that minimize the worst-case overall cost (equivalently, maximizes the overall worst-case profit).

The contribution of this talk is to propose an exact method to solve this problem. It is first formulated using a non-linear mixed integer model, based on the model proposed in [1] for the deterministic case. This model is reformulated using the approach described in [2], to obtain a Mixed Integer Linear Program with an exponentially large number of variables. It is solved using a branch-and-price approach.

Our approach is compared against the $K-$adaptability method of [3], which results in a heuristic formulation. We solve this model directly using a general purpose commercial solver. We compare both approaches on a set of 3200 randomly generated instances. Our branch-and-price algorithm solves to optimality all 20 job-instances of our test bed within one hour, and 85% of the 25 job-instances. Our method provides as by-product, for each solved instance, the number K^* of recourse solutions required to achieve optimality. When using K^* as the parameter of the finite adaptability model, it fails at solving some 10 job-instances. It solves less than 17% of the instances for which $K^* \geq 2$ and $|I| = 25$.

References:

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