Min-sup-min robust combinatorial optimization with few recourse solutions

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Abstract: In this work, we consider a variant of adaptive robust combinatorial optimization problems where the decision maker can prepare K solutions and choose the best among them upon knowledge of the true data realizations. The problem formalizes as:

$$v(X_0, \Xi) := \min_{x \in X_0^K} \sup_{\xi \in \Xi} \min_{k \in [K]: x^k \in X_1(\xi)} f(\xi, x^k), \tag{1}$$

where $[K] = \{1, \ldots, K\}$, $\Xi \subseteq \mathbb{R}^n$ is a convex set, $f : X \times \Xi \to \mathbb{R}$ is a function concave in $\xi \in \Xi$, $X_0 \subseteq \mathbb{Z}^n$ is a combinatorial set, and $X_1(\xi)$ is defined by L constraints

$$g_\ell(\xi, x) \leq b_\ell, \quad \ell \in [L], \tag{2}$$

where $g_\ell : \Xi \times X \to \mathbb{R} : (\xi, x) \mapsto g_l(\xi, x)$ is the function characterizing the ℓ-th constraint.

While several studies (e.g., [5, 6]) have illustrated the practical relevance of problem (1), exact solution algorithms have stayed behind. Two general algorithms have been proposed: [5] reformulates the problem through a Mixed-Integer Linear Programming (MILP) formulation involving big-M, and [6] introduces an ad-hoc branch-and-bound algorithm based on generating a relevant subset of scenarios $\Xi' \subseteq \Xi$ and enumerating over their assignment to the K solutions. Unfortunately, these two approaches can hardly solve the shortest path instances proposed by [5] with more than 25 nodes. The approach proposed in [3] had more success with these instances, solving all of them to optimality (up to 50 nodes) in the special case $K = 2$. Yet this latter approach requires f to be linear, Ξ to have a special structure and does not scale up with K.

We propose a new exact algorithm for solving these problems when the feasible set of the nominal optimization problem does not contain too many good solutions. Our algorithm enumerates these good solutions, generates dynamically a set of scenarios from the uncertainty set, and assigns the solutions to the generated scenarios using a vertex p-center formulation, solved by a binary search algorithm. Our numerical results on adaptive shortest path and knapsack with conflicts problems show that our algorithm compares favorably with the methods proposed in the literature. We additionally propose a heuristic extension of our method to handle problems where it is prohibitive to enumerate all good solutions. This heuristic is shown to provide good solutions within a reasonable solution time limit on the adaptive knapsack with conflicts problem.

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