

# Quantitative stability of barycenters in the Wasserstein space

- Guillaume Carlier (Université Paris Dauphine-PSL)
- **Alex Delalande** (Université Paris-Saclay)
- Quentin Mérigot (Université Paris-Saclay)

**Keywords:** Optimal transport

**Abstract:** Wasserstein barycenters [1] define averages of probability measures in a geometrically meaningful way. Their use is increasingly popular in applied fields (see for instance [2, 3, 4] for successful application in image/geometry/language processing), which calls for the study of their statistical properties. A natural question in this spirit is the question of the stability of Wasserstein barycenters: given a set of probability measures, how does a perturbation of these measures affect the corresponding barycenters?

We answer this question by showing that the Wasserstein barycenter depends in a Hölder-continuous way on the data, under relatively mild assumptions. Our proof relies on recent quantitative stability for optimal transport maps and a new result quantifying the modulus of continuity of the push-forward operation under a (not necessarily smooth) optimal transport map.

## References:

- [1] Agueh, Martial and Carlier, Guillaume. Barycenters in the Wasserstein space *SIAM Journal on Mathematical Analysis, Society for Industrial and Applied Mathematics*, 43:2:904-924, 2011.
- [2] Rabin, Julien and Peyré, Gabriel and Delon, Julie and Marc, Bernot. Wasserstein Barycenter and its Application to Texture Mixing *SSVM'11*, 2011.
- [3] Solomon, Justin and de Goes, Fernando and Peyré, Gabriel and Cuturi, Marco and Butscher, Adrian and Nguyen, Andy and Du, Tao and Guibas, Leonidas. Convolutional Wasserstein Distances: Efficient Optimal Transportation on Geometric Domains *ACM Trans. Graph.*, 34:4, 2015.
- [4] Colombo, Pierre and Staerman, Guillaume and Piantanida, Pablo and Clavel, Chloé. Automatic Text Evaluation through the Lens of Wasserstein Barycenters *EMNLP 2021*.