One-to-many Matching Games

- **Felipe Garrido-Lucero** (LAMSADE, Université Paris Dauphine-PSL)
- Rida Laraki (LAMSADE, Université Paris Dauphine-PSL; Computer sciences department, University of Liverpool)

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Abstract: In this ongoing work, we present a matching model in which doctors are assigned to hospitals and, simultaneously, play a strategic game that depends on their identity (so different coalitions of doctors and hospitals have potentially different games). Doctors can be assigned to at most one hospital while hospitals have the capacity of receiving many doctors up to a quota. This model seeks to generalize two articles from the literature: (1) Hatfield and Milgrom's matching with contracts model [7], where doctors and hospitals get matched and choose contracts from a set of finitely many options, model that does not capture possible strategic behaviours from the agents, and (2) our own model of one-to-one matching games [5], where agents within a couple play a two-person strategic game. Due to the presence of games, besides of the classical pairwise stability of Gale and Shapley [4], a second notion of stability comes from players' incentives to deviate in strategies. The mix of these two notions of stability refines the solutions on many well-known articles in the literature.

Besides of generalizing the two articles mentioned above, the model of one-to-many matching games is rich enough for including many of the most important models in the literature of one-to-one matching markets [2, 4, 9], as well as problems in one-sided markets e.g. roommates [8], roommates with transferable utilities [1], and hedonic games [3]. An interesting application of this model could be a symmetric case whose stable solutions segregate the agents.

Once presented the model we extend the results in [5, 6] to the separable one-to-many case in which hospitals' payoff functions are obtained as the sum over the utilities obtained with each of their doctors. Then, we focus on a one-sided matching market in which doctors only care about the people matched with them and not the assigned hospital. In the case in which doctors can be matched only in couples, as in a roommate setting, the work of Alkan and Tuncay gives insights on how to compute allocations that are externally and internally stable (as defined in [5]) or expressing the infeasibility of the problem in case of non-existence.

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