

# Sufficient conditions for a “simple” Second Welfare Theorem with consumption externalities

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**Abstract:** Besides convexity assumptions, the Second Welfare Theorem relies on another crucial assumption, namely that individuals are self-interested, meaning that their preferences are independent of the actions of others. It is well known that dropping self-interest, the Second Welfare Theorem may fail. We consider a pure exchange economy with consumption externalities in preferences. We use the notion of competitive equilibrium that combines Arrow–Debreu with Nash. We point out that a simple condition for restoring the Second Welfare Theorem is that the set of Pareto optimal allocations is included in that of *internal* Pareto optimal allocations. In this paper, we provide two assumptions to ensure such inclusion.

The first one is called Social Redistribution. We show that our assumption is weaker than other relevant assumptions introduced in [1] and [4]. Another important concept that has been used to obtain the Second Welfare Theorem in the presence of externalities was introduced in [6]. This is known as Non-malevolence. While in Non-malevolence there is no redistribution, we prove that Non-malevolence is stronger than Social Redistribution as well. Furthermore, we provide several examples showing that Social Redistribution is not equivalent to any of the previous assumptions.

We then move towards a differentiable approach (for instance, see [2] and [3]), since it is a natural way to study the behavior of individuals through their marginal utilities. We introduce our second assumption, that we call Directional Social Redistribution. This is the differential counterpart of Social Redistribution in terms of first order effects of the externalities on utility functions.<sup>1</sup>This assumption entails a crucial consequence of Directional Social Redistribution, namely Theorem 16. That is, if the positive weighted sum of the supporting prices for externalities is the same for everybody, then it can be used to implement the Pareto optimal allocation as a competitive equilibrium. Besides, we show that Directional Social Redistribution is weaker than Local non-malevolence introduced in [5].

Finally, we show that, for Bergson-Samuelson utility functions, Directional Social Redistribution is ensured by a specific property of the Jacobian matrix. This is relevant because it leads us to sufficient conditions on the entries of that matrix, which deliver significant information on the nature of the externalities.

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<sup>1</sup>By “first order effects”, we mean directional derivatives.