

Existence of optimal shapes with reach condition for geometric functionals

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Abstract: In this talk we will discuss the existence of shape minimizers for a large class of shape optimization problems defined on the boundary of a full-dimensional domain and whose cost involve the geometry of the hypersurface. We consider as admissible shapes orientable hypersurfaces satisfying a so-called reach condition, also known as uniform ball property, which ensures a $\mathcal{C}^{1,1}$ regularity of the hypersurface. Our main contribution is to provide a simpler framework and more concise proofs for the results in [4], [2], and [1]. Indeed using the signed distance introduced by Delfour and Zolesio (see e.g. [3]), we avoid the intensive and technical use of local charts by Dalphin or Guo and Yang. This framework, originally developed to tackle an existence problem in [5], can be easily extended to costs involving different mathematical objects associated with the domain, such as solutions of elliptic equations on the hypersurface.

References:

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