Multilevel Physics Informed Neural Networks

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Abstract: The approximation of the solution of partial differential equations (PDEs) by artificial neural networks (ANNs) dates back to the 90s [1], but it is only in these last years that this topic fully emerged and gave rise to an active field of research.

In this field, the most famous network architecture is the one of PINNs (physics informed neural networks), introduced for the first time in [2]. Since their introduction, the PINNs architecture has encountered a growing interest and the good performance observed in practice have been later supported by theoretical results, cf. for instance [3, 4].

Despite their good performance, the training of such networks may still represent a challenge in case of difficult problems, such as highly nonlinear problems. In this case really large networks may be needed to correctly represent the sought solution, leading to the need of solving a large-scale optimization problem, for which standard training methods may show a slow convergence, and it may be difficult to properly tune the learning rate.

When dealing with linear PDEs, multigrid (MG) methods are by far the most effective methods for the solution of large scale problems [5]. The improved performance of MG methods derives from the fact that alternating relaxations among fine and coarse grids allows us to more efficiently reduce all the components of the error, smooth and oscillatory ones.

In this work we propose a multilevel PINN approach (MPINN), based on writing the solution of the PDE as a sum of two terms, a fine and a coarse one. Each term is a PINN depending on a different number of parameters and trained on a different training set, which are optimized independently the one from the other, in an alternate fashion. As in classical MG indeed, the method proceeds by alternating relaxations on the two levels, which in this case are epochs of training of each PINN. We show that interestingly this approach allows us to reproduce the acceleration typically observed in classical MG methods, in the context of the training of PINNs.

References:

[1] I. E. Lagaris, A. Likas, D. I. Fotiadis. Artificial neural networks for solving ordinary and partial differential equations. *IEEE Transactions on Neural Networks* , 9(5):987–1000, 1998.

[2] M. Raissi, P. Perdikaris, G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations *Journal of Computational Physics*, 378:686–707, 2019.

[3] Y. Shin, J. Darbon, G. E. Karniadakis. On the convergence of physics informed neural networks for linear second-order elliptic and parabolic type PDEs. *ArXiv preprint 2004.01806*, 2020.

[4] S. Mishra, R. Molinaro. Estimates on the generalization error of physics informed neural networks (PINNs) for approximating PDEs. *ArXiv preprint 2006.16144*, 2020.

[5] U. Trottenberg, C. W. Oosterlee, A. Schuller. Multigrid. *Elsevier, New-York*, 2000.