Fictitious Play for Identical Interest and Zero-Sum Stochastic Games

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Abstract: Game theory is a mathematical framework to model systems where several agents take decisions and get a payoff that depends on all agents' action. The most classical concept to study these systems is the so-called Nash equilibrium defined as an action profile where no players have any incentive to deviate. This immediately raises the question of how such an equilibrium is reached. A line of work is to study a repeated version of the game where players can choose their action using the past rounds of the game, following strategies that enable them to learn an action profile. This dynamical study of games can be used to justify the static equilibrium concepts such as Nash ones, or as a solution concept in itself. The oldest and probably most well known procedures is Fictitious Play (FP) [2, 4] where players play a best-response to the average empirical actions of other players.

We propose an extension of the decentralized, discrete-time, fictitious play learning procedure to a dynamic model called discounted stochastic game. A stochastic game (in contrast to a static game) is a game with a state variable into which players typically want to maximize their instantaneous payoff but also maximize their future reward by moving to a future profitable state. Our family of discrete-time FP procedures is proven to converge to the set of stationary Nash equilibria in identical interest discounted stochastic games. This extends similar convergence results of FP for static games.

We then analyze the continuous-time counterpart of our FP procedures, which include as a particular case the best-response dynamic introduced and studied by Leslie et al. [3] in the context of zero-sum stochastic games. This is a differential inclusion and we prove the convergence of this dynamics to stationary Nash equilibria in identical-interest and zero-sum discounted stochastic games. Thanks to the stochastic approximation framework [1] (that we extend for asynchronous systems), we can infer from the continuous-time convergence some discrete time results such as the convergence to stationary equilibria in zero sum-and team stochastic games.

References:

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