Optimal bacterial resource allocation

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Abstract: The study of living microorganisms through resource allocation models has increasingly become relevant for its capacity to elucidate natural behaviors of microbia through very simple dynamical models. The core idea is to represent the distribution of cellular resources through optimal control strategies, based on the assumption that evolutionary processes have tuned these endogenous allocation strategies to attain nearly-optimal levels. Numerous problems arise in this context, one of them being the optimal production of metabolites regulated by an external control capable of arresting bacterial growth. To this end, a resource allocation approach can help understand how to modify the naturally-evolved allocation strategies so as to efficiently produce such chemical compounds. These biosynthetic strategies have been studied in different industrial frameworks. The simplest case can be modelled by omitting the dynamics of the substrate in the medium, representing the case where exponential growth can be attained. Another relevant, more complex case is in continuous bioreactors, where an inflow of fresh medium rich in substrate and an outflow of the culture at the same volumetric flow rate produce a constant volume of the culture in the device. In that case, a steady state is attained, and studies are mostly oriented to reach such state in a cost-effective way. In fed-batch fermentation, the process starts with an initial culture, and the bioreactor is progressively filled through an inflow of rich medium, increasing the volume of the culture until it reaches maximum volume. Once the maximum volume is attained, the culture evolves as a closed process, known in the field as batch processing. As no mass comes in or out of the device, the remainder of the nutrients in the medium are progressively consumed until the mass becomes entirely final products.

The latter is the subject of this talk, as we tackle the problem of batch processing from a resource allocation perspective. A simple coarse-grained self-replicator model is introduced, based on previous works, with minimal biological assumptions and including a heterologous pathway for the production of a value-added chemical compound. Using mass conservation laws related to the closedness of the bio-process, it is possible to analyse the asymptotic behavior and stability of the dynamical system, showing that, for every possible allocation strategy, all component of the system are transformed either into proteins or into metabolites, a condition later defined as *full depletion*. Then, two cases are consedered: biomass maximisation, representing the natural objective of wild-type microbial cultures; and metabolite maximisation, using the full bacterial model that includes the pathway for metabolite synthesis for industrial purposes. Both problems are analysed in infinite time and in finite time, the latter being studied thanks to Pontrjagin maximum principle.

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