Newton-type inertial algorithms for solving monotone equations governed by sums of potential and nonpotential operators

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Keywords: proximal-gradient algorithm, inertial method, Hessian-driven damping, nonpotential operator, cocoercive operator, structured monotone equation.

Abstract: Proximal-gradient algorithms are powerful tools for solving optimization problems with an additively separable and smooth plus nonsmooth structure. Howerver, many situations coming from physics, biology, human sciences involve equations containing both potential and nonpotential terms. To describe such situations, precisely, in a Hilbert space setting we focus on solving the additively structured monotone problem

$$\text{Find } x \in \mathcal{H} \; : \; \nabla f(x) + B(x) = 0,$$

where ∇f is the gradient of a convex continuously differentiable function $f : \mathcal{H} \to \mathbb{R}$ (that is the potential part), and $B : \mathcal{H} \to \mathcal{H}$ is an operator which is supposed to be monotone and cocoercive (that is the nonpotential part). This work is mainly devoted to the study of a class of first-order algorithms which aim to solve such structured monotone equations involving the sum of potential and nonpotential operators that is based on the inertial autonomous dynamic

$$\ddot{x}(t) + \gamma\dot{x}(t) + \nabla f(x(t)) + B(x(t)) + \beta_f \nabla^2 f(x(t))\dot{x}(t) + \beta_b B'(x(t))\dot{x}(t) = 0, \; t \geq 0. \quad \text{(DINAM)}$$

These dynamics involve dampings controlled respectively by the Hessian of f, and by a Newton-type correction term attached to B (see [1]). These geometric dampings attenuate the oscillations which occur with the inertial methods with viscous damping. Temporal discretization of this dynamic provides fully splitted proximal-gradient algorithms. Then the authors showed the weak convergence of the sequences generated by algorithms towards the zeros of $\nabla f + B$. Their convergence properties are claimed using Lyapunov analysis under certain conditions on parameters (see [2]). These results open the door to the design of first-order accelerated algorithms in numerical optimization taking into account the specific properties of potential and nonpotential terms.

References:

[1] S. Adly, H. Attouch, V.N. Vo. Asymptotic behavior of Newton-like inertial dynamics involving the sum of potential and nonpotential terms. *Fixed Point Theory and Algorithms for Sciences and Engineering*, 2021:17 (2021).

[2] S. Adly, H. Attouch, V.N. Vo. Newton-type inertial algorithms for solving monotone equations governed by sums of potential and nonpotential operators. *Accepted for publication in Applied Mathematics and Optimization.* (2022)