# Hidden convexity for the uniform optimal quantization problem


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**Abstract:** Several issues in machine learning and inverse problems require to generate discrete data as if sampled from a model probability distribution $\rho$ on a compact domain $\Omega$ of $\mathbb{R}^d$. A natural way to do so relies on the construction of a uniform probability distribution over a set of $N$ points which minimizes the Wasserstein distance to the model distribution:

$$\min_{y_1,\ldots,y_N\in\mathbb{R}^N} F(y_1,\ldots,y_N) := \mathrm{W}_2\left(\frac{1}{N}\sum_{1\leq i\leq N}\delta_{y_i},\rho\right).$$

This minimization problem, where the unknowns are the positions of the atoms, is non-convex. Yet, in most cases, a suitably adjusted version of Lloyd's algorithm — in which Voronoi cells are replaced by Power cells — seems to lead to configurations with small Wasserstein error. We explain quantitatively this behaviour, by showing in particular that if the points $y_1^0,\ldots,y_N^0\in\Omega$ are not too close to each other, i.e. $\|y_i-y_j\|\gtrsim N^{-1/d}$, then a *single* step of gradient descent,

$$y_i^1 = y_i^0 - N\nabla_{y_i}F(y_1^0,\ldots,y_N^0),$$

is sufficient to get a configuration $(y_1^1,\ldots,y_N^1)$ which is Wasserstein-close to $\rho$:

$$\mathrm{W}_2^2\left(\frac{1}{N}\sum_{1\leq i\leq N}\delta_{y_i^1},\rho\right)\lesssim N^{-1/d}.$$

We will also discuss a more recent result, showing in dimension $d=2$ that the quantization energy of *stable* critical points is actually commensurable to the energy of the minimizer. This is based on joint works with A. Figalli, F. Santambrogio and C. Sarrazin.


**References:**


[1] Q. Mérigot, F. Santambrogio, C. Sarrazin Non-asymptotic convergence bounds for Wasserstein approximation using point clouds. *Proceedings of Advances in Neural Information Processing Systems 34* (NeurIPS 2021).