Convergence analysis of SDDP like algorithms

- Mael Forcier (CERMICS, Ecole des Ponts)
- **Vincent Leclère** (CERMICS, Ecole des Ponts)

Keywords: Stochastic Optimization, SDDP.

Abstract: Multistage stochastic programming problems are notoriously difficult to solve. Let $(\boldsymbol{\xi}_t)_{t\in[T]}$ be a sequence of independent random variables with known distribution affecting the system with a known distribution. Leveraging the Markovian property of the system, a number of Dynamic Programming inspired, time-decomposition algorithm derived from the the Stochastic Dual Dynamic Programming (SDDP) [1] algorithm have been developed in the last thirty years. These various algorithm iteratively compute and refine approximation of the value functions based on specific property of the problem (convexity, binary variables...) Noticeably, almost all rely at some point on the value function's Lipschitz regularity and compactness of the state set, which are the two assumptions we keep.

In this talk we present a flexible framework using inner and outer approximations of the Bellman's value function that cover numerous known algorithms (like SDDiP [3], SLDP [2], GAPM [4]...) and derive convergence and complexity results.

In addition, our analysis do not rely on finitely supported $\boldsymbol{\xi}$ contrary to former results in this area, paving the way to SDDP like algorithm for non-finitely supported $\boldsymbol{\xi}$ which are generally thought as out of reach without a first discretization step.

Finally, we detail, in the multistage stochastic linear setting with generic cost distribution, how to obtain exact oracle and adapt the SDDP algorithm.

References:

[1] M. Pereira, L. Pinto. Multi-stage stochastic optimization applied to energy planning. *Mathematical programming*, 1991.

[2] S. Ahmed, F. Cabral and B. da Costa Stochastic Lipschitz dynamic programming. *Mathematical Programming*, 2020.

[3] J. Zou, S. Ahmed, A. Sun Stochastic dual dynamic integer programming. *Mathematical Programming*, 2019.

[4] M. Forcier, V. Leclère Generalized adaptive partition-based method for two-stage stochastic linear programs: convergence and generalization *Arxiv*, https://arxiv.org/abs/2109.04818, 2022.