

# On the Optimal Impulse Control of Stochastic Portfolio

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**Abstract:** We consider stochastic portfolio optimization problem in finite-time horizon with general utility function and general term transaction costs. We adopt an optimal impulse control theory approach to propose a new portfolio value process, and to characterize an optimal investment strategy maximizing the finite-time horizon expected discounted utility of the investor's wealth. Indeed, we derive the dynamic programming equation (DPE) associated to the impulse control problem, then we show that the value function of the stochastic portfolio is the unique viscosity solution to the given equation. Therefore, by proving a verification theorem, we obtain an optimal investment strategy. The considered portfolio optimization problem turns to a stochastic impulse control problem with general in term of the form and cost of impulses, a problem that enjoys a wide range of applications in various fields.

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