

Nested replicator dynamics and nested logit choice

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Keywords: Nested replicator dynamics; nested logit choice; learning; independence of irrelevant alternatives; blue bus / red bus paradox.

Abstract: We consider population dynamics for games whose action sets are endowed with a nested similarity structure. Each agent concurrently runs multiple updating procedures; in each of these procedures, the agent considers imitating an opponent whose strategy is sufficiently similar to their own, switching to the opponent’s strategy with probability proportional to the difference between the strategies’ payoffs. The resulting aggregate behavior process, which we call the *nested replicator dynamics*, does not satisfy standard monotonicity conditions for imitative game dynamics. Nevertheless, we show that it retains all of the main stability and convergence properties of the classic replicator dynamics. In the context of online learning in normal form games – where the agents’ choice probabilities are determined by the strategies’ cumulative payoffs – the nested replicator dynamics describe the evolution of mixed strategies when choice probabilities are based on the nested logit choice rule of [1] and [2]. This result generalizes an existing relation between the classic replicator dynamics and the exponential weights (EW) algorithm in online learning, and provides an additional layer of interpretation to our convergence analysis and results.

References:

- [1] Ben-Akiva, M. *Structure of Passenger Travel Demand Models*. PhD thesis, MIT, 1973.
- [2] McFadden, D. Modelling the choice of residential location. In Karlqvist, A., Lundqvist, L., Snickars, F., and Weibull, J. W. (eds.), *Spatial Interaction Theory and Planning Models*. North Holland, Amsterdam, 1978.