Oddness of the number of Nash equilibria: the case of polynomial payoff functions

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Abstract: The main question of this talk is: does there exist "generically" an odd number of Nash equilibria $(x_1, ..., x_N) \in X_1 \times ... \times X_N$, i.e. solutions of

$$u_i(x_1, ..., x_{i-1}, d_i, x_{i+1}, ..., x_N) \leq u_i(x_1, ..., x_{i-1}, x_i, x_{i+1}, ..., x_N) \ \forall i = 1, \dots, N, \forall d_i \in X_i$$

for polynomial payoffs u_i, where each X_i is some nonempty, convex and compact semi-algebraic subset of some Euclidean space, and each u_i is assumed to be concave with respect to x_i. In 1971, Robert Wilson (SIAM Journal on Applied Mathematics) proved the so-called "oddness theorem" when u_i are the payoffs of mixed extensions of finite games (in that case, u_i are affine and X_i are simplices). In this paper, we prove the oddness theorem for large classes of polynomial payoff functions and semi-algebraic sets of strategies, and we provide some applications to recent models.

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