Oddness of the number of Nash equilibria: the case of polynomial payoff functions

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Abstract: The main question of this talk is: does there exist "generically" an odd number of Nash equilibria \((x_1, ..., x_N) \in X_1 \times ... \times X_N\), i.e. solutions of

\[ u_i(x_1, ..., x_{i-1}, d_i, x_{i+1}, ..., x_N) \leq u_i(x_1, ..., x_{i-1}, x_i, x_{i+1}, ..., x_N) \quad \forall i = 1, ..., N, \forall d_i \in X_i \]

for polynomial payoffs \(u_i\), where each \(X_i\) is some nonempty, convex and compact semi-algebraic subset of some Euclidean space, and each \(u_i\) is assumed to be concave with respect to \(x_i\). In 1971, Robert Wilson (SIAM Journal on Applied Mathematics) proved the so-called "oddness theorem" when \(u_i\) are the payoffs of mixed extensions of finite games (in that case, \(u_i\) are affine and \(X_i\) are simplices). In this paper, we prove the oddness theorem for large classes of polynomial payoff functions and semi-algebraic sets of strategies, and we provide some applications to recent models.

References:


