First-order methods for nonconvex quartic optimization

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Abstract: We study optimization problems of the form

$$\min_{x \in \mathcal{K}} \rho(x) - g(x) \tag{P}$$

where $\mathcal{K}$ is a finite-dimensional convex cone, and ρ, g are two convex functions on $\mathcal{K}$. We assume that ρ is a quartic polynomial induced by a symmetric 4-linear form. Such structure arises in low-rank optimization, Euclidean distance matrix recovery and phase retrieval.

We propose a bilevel scheme for finding a stationary point of Problem (P), relying on successive linear approximations of the concave function $-g$. The inner problems are convex and can be solved efficiently with fast gradient methods by leveraging the *quartic conditioning* of function ρ. This property can be seen as the fourth-degree counterpart of the standard condition number encountered in quadratic optimization.