Workshop on Mathematical Modeling and Statistical Analysis in Neuroscience, January 31 - February 4, 2022.

Organizers: Susanne Ditlevsen, Olivier Faugeras, Antonio Galves, Patricia Reynaud-Bouret, Delphine Salort and Shigeru Shinimoto.

NB : Lunches are not provided but it is easy to find restaurants and snacks in the neighborhood of IHP.

NB2 : Cocktail is cancelled due to the sanitary restrictions, but coffee dispenser …

P= Present R = Remote

Monday January 31th

Opening

9h30-10h30 Welcoming of participants

10h30-11h30 Guilherme Ost and Claudia Vargas (Federal University of Rio de Janeiro) P/R
Retrieving the structure of probabilistic sequences of auditory stimuli from electroencephalographic (EEG) signals

11h30-12h Shigeru Shinomoto (Kyoto University) R
Inferring monosynaptic connections from a cross-correlogram of neuronal spike trains

Statistics and Artificial Intelligence Afternoon

14h30-15h30 Ingrid Bethus and Alexandre Muzy (Université Côte d'Azur) P
Linking individually varying behavioral learning to neuronal code using granular computational cognitive models

15h30-16h00 Uri Eden (Boston University) R
Marked Point Process Modeling and Estimation Problems in Neural Data Analysis

Break

16h30-17h00 Thibault Taillefumier (University of Texas) R
Neural networks in the replica-mean field limits

17h00-17h30 Adeline Samson (Université Grenoble Alpes) R
TBA

17h30-18h00 Robert E. Kass (Carnegie Mellon University) R
Statistical Models of Interaction Across Brain Regions from Multi-Electrode Recordings

Tuesday February 1st

Statistics and Artificial Intelligence: the morning session animated by Ingrid Bethus and Alexandre Muzy (Université Côte d'Azur)

10h00-10h20 Flavio Rusch (Sao Paulo) P?
Self-organized criticality in hierarchical modular networks of Galves-Löcherbach neurons
10h20-10h40 Tien Cuong Phi (Université Côte d'Azur) P
A new method to simulate point processes
10h40 -11h00 Antonio Carlos Costa (Amsterdam) P?
Maximally predictive ensemble dynamics from data

11h00 -12h00 Discussion

Algebra and Geometry Afternoon

14h30-15h00 Claire Guerrier (Université Côte d'Azur) R
Multi-scale modeling of vesicular release at neuronal synapses
15h00-15h30 Carey E. Priebe (Johns Hopkins University) R
Learning 1-Dimensional Submanifolds for Subsequent Inference on Random Dot Product Graphs
15h30-16h00 Carina Curto (Pennsylvania State University) P?
Graph rules and topological insights for inhibitory network dynamics

Break

16h30-17h30 Lida Kanari and Kathryn Hess (EPFL) R
Topological insights on neuronal morphologies

Wednesday February 2nd

Algebra and Geometry: the morning session animated by Lida Kanari and Kathryn Hess (EPFL)

10h00-10h20 Yuri Rodrigues (Université Côte d'Azur) P
A new synaptic rule to unify experimental heterogeneity
10h20 -10h40 Sunil Modhara (Nottingham) P?
Neural fields with rebound currents: novel routes to patterning
10h40-11h00 Alejandro Ramos Lora (Granada) P?
Beyond Blow-Up for Nonlinear Noisy Leaky Integrate and Fire neuronal models: numerical approach to the "plateau" state

11h00-12h00 Discussion

Probability Afternoon
14h30-15h30 Eva Löcherbach (Université Panthéon Sorbonne) and Christophe Pouzat (Université de Strasbourg) P

System of interacting neurons with short term synaptic facilitation

15h30-16h00 Etienne Tanré (Inria Sophia-Antipolis) P

Spontaneous oscillations in a pure excitatory mean field networks of neurons

Break

16h30-17h00 Wilhem Stannat (TU Berlin) R

Fluctuation limits for mean-field interacting nonlinear Hawkes processes

17h00-17h30 Romain Veltz (Inria Sophia Antipolis) P

Some recent results on a mean field of a network of 2d spiking neurons.

Thursday February 3rd

Probability: the morning session animated by Eva Löcherbach (Université Panthéon Sorbonne) and Christophe Pouzat (Université de Strasbourg)

10h00-10h20 Morgan André (UniCamp and NeuroMat) P

Time averages of a metastable system of spiking neurons

10h20 -10h40 Luyan Yu (University of Texas) R

Metastable spiking networks in the replica-mean-field limit

10h40-11h00 Michel Davydov (ENS Paris – Inria) P

Propagation of Chaos and Poisson Hypothesis for Replica Mean-Field Models

11h00-12h00 Discussion

PDE and Dynamical systems Afternoon
14h30-15h30 Afia Ali (UCL) and Mathieu Desroches (Inria Sophia-Antipolis) P/R

Understanding Synaptic Mechanisms: Why a Multi-disciplinary Approach is Important ?

15h30-16h00 Daniele Avitabile (Vrije Universiteit Amsterdam) R

Projection methods for spatially-extended neurobiological models

Break

16h30-17h00 Grégory Faye (Université Paul Sabatier Toulouse) P ?

Front initiation in continuous neural fields

17h00-17h30 José A. Carillo (Oxford) P ?

Noise-driven bifurcations in a neural field system modelling networks of grid cells

17h30-18h00 Bruno Cessac (Inria Sophia-Antipolis) P

The non linear dynamics of retinal waves

Friday February 4th

PDE and Dynamical systems: the morning session animated by Afia Ali (UCL) and Mathieu Desroches (Inria Sophia-Antipolis)

10h00-10h20 Zhennan Zhou (Peking University) R
Towards understanding the time periodic solutions in a kinetic model for neuron networks
10h20 -10h40 Kota Ikeda (Meiji University) R
Theoretical study of the emergence of periodic solutions for the inhibitory NNLIF neuron model with synaptic delay
10h40-11h00 Nicolas Torres (Sorbonne Université) P
A multiple time renewal equation for neural assemblies with elapsed time model

11h00-12h00 Discussion

Closure

14h30 Reward ceremony (MNA) P
14h40-15h40 Stephen Coombes and Peter Liddle (University of Nottingham) R
A neural mass model for abnormal beta-rebound in schizophrenia

ABSTRACTS

NB : the other abstracts are merged at the end

Afia B. Ali (UCL School of Pharmacy, London, UK) and Mathieu Desroches (Inria - Méditerranée Research Centre, Sophia Antipolis, France)

Understanding Synaptic Mechanisms: Why a Multi-disciplinary Approach is Important

A multi-disciplinary approach in understanding the functions mediated by synapses will reveal information at a multi-scale level from sub-cellular events to large scale network activity. This will result in better holistic understanding of the principles by which synapses orchestrate information flow in the brain, both in health and in pathological states. This is one of the greatest challenges in neuroscience which involves overcoming two key aspects: on the one hand, the heterogeneity of the brain, for example, the different receptor types, their morphological structure and on the other hand overcoming the temporal and spatial scales. Our study attempts to take these challenges to the forefront, by combining a number of techniques, including:
i) Intracellular whole-cell recording, a specialised electrophysiological technique which involves simultaneously recording two neurons that are synaptically connected, thus allowing the investigation and manipulation of the chemical or electrical communication between neurons
ii) Biocytin and dual immunofluorescence labelling that enables the identification of the neurons recorded
iii) Incorporation of our current understanding of the biomolecular machinery, which regulates exocytosis, the process enabling neurotransmitters to be released
iv) Mathematical multi-scale modelling that bridges the gap between protein and electrical activity enabling the synthesis of the aforementioned empirical observations

This multi-disciplinary approach sheds new light onto how the synaptic molecular machinery fine tunes cellular processes of neurotransmitter release. For example, we have derived a novel invariant mathematical principle, which we coined *Activity Induced Transcritical Canards, which we* hypothesize will lead to a better understanding of pathological cellular mechanisms of Alzheimer's disease.

This is joint work with Serafim Rodrigues (Ikerbasque, BCAM, Bilbao, Spain).

Morgan André (UniCamp and NeuroMat)

Time averages of a metastable system of spiking neurons

We study a stochastic system of spiking neurons in which the spikes of the neurons are represented by a family of interacting point processes on the positive real line. The model depends on a parameter gamma, representing the intensity of the natural leakage of the neurons. This model has already been proven to exhibit several interesting behaviors. Firstly it undergoes phase transition with respect to the parameter gamma [1]. Moreover the time of extinction of finite versions of the system have been proven to be asymptotically memory-less for small gamma [2][3], a characteristic property of metastable systems. Here we show that this last result actually holds in the whole subcritical region and that previous to extinction the finite versions of the system are in a regime which in some sense resemble stationarity. This is the second characteristic property of metastable dynamics. The main idea is to use a bypass through the theory of "Interacting particles systems".

Bibliography:

[1] P.A. Ferrari, A. Galves, I. Grigorescu and E. Löcherbach (2018). "Phase Transition for Infinite Systems of Spiking Neurons". Journal of Statistical Physics, Vol.172, pp 1564-1575.

[2] M. Andre (2019). "A Result of Metastability for an Infinite System of Spiking Neurons". Journal of Statistical Physics, Vol. 177, pp. 984-1008.

[3] M. Andre and L. Planche (2021). "The Effect of Graph Connectivity on Metastability in a Stochastic System of Spiking Neurons". Stochastic Processes and their Applications, Vol. 131, pp. 292-310.

Daniele Avitabile (Vrije Universiteit Amsterdam)

Projection methods for spatially-extended neurobiological models

We will discuss recent progress on the numerical analysis of neural fields and, more generally, on nonlocal, continuum, spatially-extended neurobiological networks. Such systems are often simulated heuristically and, in spite of their popularity in mathematical neuroscience, their numerical analysis is not yet fully established. We introduce generic projection methods for neural fields, and derive a-priori error bounds for these schemes. We extend an existing framework for stationary integral equations to the time-dependent case, which is relevant for neuroscience applications. We find that the convergence rate of a projection scheme for a neural field is determined to a great extent by the convergence rate of the projection operator. This abstract analysis, which unifies the treatment of collocation and Galerkin schemes, is carried out in operator form, without resorting to quadrature rules for the integral term, which are introduced only at a later stage, and whose choice is enslaved by the choice of the projector. Using an elementary timestepper as an example, we demonstrate that the error in a time stepper has two separate contributions: one from the projector, and one from the time discretisation. We give examples of concrete projection methods: two collocation schemes (piecewise-linear and spectral collocation) and two Galerkin schemes (finite elements and spectral Galerkin); for each of them we derive error bounds from the general theory, introduce several discrete variants, provide implementation details, and present reproducible convergence tests.

Ingrid Bethus and Alexandre Muzy (Université Côte d’Azur)

Linking individually varying behavioral learning to neuronal code using granular computational cognitive models

In animal neuroscience, neurobiologists usually try to correlate neuronal activity with a specific behavior. However, when learning to discover a task, it is unclear which behaviors the animal remembers, perceives and predicts to take its decisions.

Indeed, in the navigational learning task we are interested here, rodents may found their decisions considering last (next) actions, food locations (or events), etc. Also, this learning process may differ between individuals. Therefore, it is not possible
to know when an action starts and ends, which events are relevant,..., and so to correlate to it the neuronal activity. To solve this problem, we propose to automatically generate different candidates of computational cognitive models. Each of them refers to a different cognitive representation of the segmentation of the action to be learned (or granularity). Then, model selection is applied to decide which granularity choice (turns and/or paths in our experiment) a given individual is adopting. This gives a direct access to the inter individual variability of learning strategies and environment/self perception. Once the more plausible cognitive model is identified for each animal, neuronal code analysis can be performed with respect to the behaviors, events and locations relevant for the animal. To apply this approach we decided to use a continuous alternative T maze where rats freely navigate during 20 min without gates and have to alternate successively to the right and the left to receive a food pellet on each side. Using computational cognitive models we are able to determine which granularity of actions is relevant for the rats to learn the task. Finally, for each rat, using the environmental cues and actions identified by these models, we have preliminary results showing that particular neurons are encoding the prediction of these actions and events: their firing rate increases significantly when the rat will take a right or a left turn leading to the pellet reward.

José A. Carillo (Oxford)

Noise-driven bifurcations in a neural field system modelling networks of grid cells

The activity generated by an ensemble of neurons is affected by various noise sources. It is a well-recognised challenge to understand the effects of noise on the stability of such networks. We demonstrate that the patterns of activity generated by networks of grid cells emerge from the instability of homogeneous activity for small levels of noise. This is carried out by upscaling a noisy grid cell model to a system of partial differential equations in order to analyse the robustness of network activity patterns with respect to noise. This is rigorously achieved by mean-field type arguments. Inhomogeneous network patterns are numerically understood as branches bifurcating from unstable homogeneous states for small noise levels. We prove that there is a phase transition occurring as the level of noise decreases. Our numerical study also indicates the presence of hysteresis phenomena close to the precise critical noise value. This talk is a summary of two works in collaboration with A. Clini, H. Holden and S. Solem.

Bruno Cessac (Université Côte d’Azur, Inria)

The non linear dynamics of retinal waves

We investigate the dynamics of stage II retinal waves via a dynamical system model, grounded on biophysics, and analysed with bifurcation theory. We show how the nonlinear cells coupling and bifurcation structure explain how waves start, propagate, interact and stop. Especially, we analyse how the existence of a small region in the parameters space close to bifurcations, where dynamics returns in a recurrent way, give rise to a very rich dynamics.

Stephen Coombes and Peter Liddle (University of Nottingham)

A neural mass model for abnormal beta-rebound in schizophrenia

Many patients with psychotic illnesses including schizophrenia, suffer persisting disability despite treatment of delusions and hallucinations with antipsychotic medication. There is substantial evidence that disorganization of mental activity makes major contribution to persisting disability, by disrupting thought, emotion and behaviour. Growing evidence suggests that disorganization of mental activity reflects impairment of predictive coding. Specifically, there is well replicated evidence that disorganization is associated with a reduction in transient bursts of beta oscillations that occur post-movement (known as Post Movement Beta Rebound, PMBR). Evidence indicates that in healthy people, PMBR reflects confirmation of the internally generated forward models that guide our perceptions, intentions and actions. Therefore, understanding the mechanisms underlying PMBR and the way in which these are impaired in psychotic illness, is a potentially fruitful approach to understanding the causes of persisting disability, and could help guide development of better treatment. We will present a new neural mass model of PMBR and demonstrate that plausible pathological changes in the parameters of that model might account for reduced PMBR schizophrenia. We propose that PMBR is modulated by processes occurring in spatially remote brain regions. We will also outline the way in which our model might be incorporated into an account of the brain as a distributed network of interacting regions that serve to optimise our navigation of the world.

<u>Related references</u>

PM Briley, EB Liddle EB, M Simmonite, M Jansen, TP White, V Balain, L. Palaniyappan, R. Bowtell, KJ Mullinger, PF Liddle 2021 Regional Brain Correlates of Beta Bursts in Health and Psychosis: A Concurrent Electroencephalography and Functional Magnetic Resonance Imaging Study. Biological Psychiatry: Cognitive Neuroscience and Neuroimaging. Vol 6, 1145-1156.

Á Byrne, R O' Dea, M Forrester, J Ross and S Coombes 2020 Next generation neural mass and field modelling, Journal of Neurophysiology, Vol 123, 726–742

Á Byrne, S Coombes and P F Liddle 2019 A neural mass model for abnormal beta-rebound in schizophrenia, Handbook of Multi-scale Models of Brain Disorders, Ed. V Cutsuridis, Springer.

Carina Curto (Pennsylvania State University)

Graph rules and topological insights for inhibitory network dynamics

Many networks in the nervous system possess an abundance of inhibition, which serves to shape and stabilize neural dynamics. The neurons in such networks exhibit intricate patterns of connectivity, whose structure controls the allowed patterns of neural activity. In this talk, we will focus on inhibitory threshold-linear networks whose dynamics are dictated by an underlying directed graph. We'll introduce a set of parameter-independent graph rules that enable us to predict features of the dynamics from properties of the graph. Graph rules also lead us to consider some natural topological structures, such as nerves and sheaves, stemming from various graph covers. Our results provide a direct link between the structure and function of inhibitory networks, and yield new insights into how connectivity may shape dynamics in real neural circuits. We will illustrate this with some applications to central pattern generator circuits and related examples of neural computation.

Uri Eden (Boston University)

Marked Point Process Modeling and Estimation Problems in Neural Data Analysis

Point process modeling and estimation methods have become pervasive in the analysis of spiking data from neural populations. Recent technological developments have led to a massive increase in the size and dimensionality of neural datasets and focused researchers on models that can capture the structure of activity from populations of simultaneously recorded neurons. This has spurred the development of new modeling and estimation methods based on the theory of marked point processes.
In this talk, I will present a case study focused on understanding how mental exploration drives learning, which highlights a number of statistical problems that can be addressed under a marked point process modeling framework. I will discuss adaptive models that can capture nonstationarities in the data, goodness-of-fit methods that allow for model assessment and refinement, and state-space estimation methods that allow us to decode signals directly from the observed neural activity.

Claire Guerrier (Université Côte d'Azur)

Multi-scale modeling of vesicular release at neuronal synapses

Cells specific geometries and organizations are most of the time the main actors shaping their function. In the same time, taking into account such complex geometries in stochastic or PDE models usually leads to heavy simulations and limited analysis.
In the context of vesicular release at neuronal synapses, we used conformal mapping and asymptotic analysis to compute the narrow escape time at a cusp-like geometry. We used this result to build a simple discrete-continuum model that takes into account both a stochastic regime governed by rare events and a continuous regime in the bulk. With this model, we then investigated the mechanisms inducing paired-pulse facilitation and depression at neuronal synapses.

Rob Kass (Department of Statistics & Data Science, Machine Learning Department, and Neuroscience Institute, Carnegie Mellon University)

Statistical Models of Interaction Across Brain Regions from Multi-Electrode Recordings

Statistical and machine learning methods, which form the foundation for artificial intelligence, have much in common. Approaches identified with statistics are centered on statistical models, which are, in spirit, similar to mathematical models used in neuroscience: they aim not only to perform well in reproducing the variation seen in data, but also to provide insight into the nature of the variation. Statistical models decompose data variation into a component to be understood (the signal) and a component to be ignored (the noise).
I will illustrate the value of careful statistical modeling using three recent examples, all having to do with the identification of interactions among neural populations across brain areas.

One high-level conclusion from these studies is that it is important to define "population" and "interaction" carefully. A second is that it can be useful to consider neural population activity as arising from unobserved, latent drivers. From this perspective, communication across neural populations corresponds to interactions among latent variables, with neural activity across cortical and sub-cortical brain areas forming statistical hierarchies.

Eva Löcherbach (Université Panthéon Sorbonne) and Christophe Pouzat (Université de Strasbourg)

System of interacting neurons with short term synaptic facilitation

After reviewing the behavioral studies of working memory and of its cellular substrate, we argue that metastable states constitute candidates for the type of transient information storage required by working memory. We then present a simple neural network model made of stochastic units whose synapses exhibit short-term facilitation. This model is specifically designed to be analytically tractable, simple to simulate numerically and to exhibit metastability.
(Joint work with Errico Presutti and Antonio Galves)

Guilherme Ost & Claudia D. Vargas (Federal University of Rio de Janeiro)

Retrieving the structure of probabilistic sequences of auditory stimuli from electroencephalographic (EEG) signals

It has long been conjectured that the brain learns statistical regularities from sequences of stimuli. The ability to extract and memorize these regularities over time plays a crucial role in perception, motor control and decision-making. Using a new probabilistic approach we model the relationship between sequences of auditory stimuli generated by stochastic chains and EEG signals acquired while participants are exposed to those sequences of stimuli. The structure of the chains generating the stimuli are characterized by rooted and labeled trees whose leaves, also called contexts, represent the sequences of past stimuli governing the choice of the next stimulus. If the brain assigns probabilistic models to samples of stimuli then the context tree generating the sequence of stimuli should be encoded in the brain activity. In this talk we will present and discuss an innovative procedure allowing us to retrieve these context trees from EEG signals.

<u>For further reading</u>

Duarte et al., Mathematics 2019, 7(5), 42. https://doi.org/10.3390/math7050427

Hernandez et al., Scientific Reports 2021, 11, 3520. https://doi.org/10.1038/s41598-021-83119-x

Tien Cuong Phi (LJAD, NeuroMod, Université Côte d'Azur)

A new method to simulate Hawkes processes

In Neuroscience, Hawkes process is one of the most popular model to capture the activity of neural networks. In this talk, we are interested in simulating Hawkes processes with a huge number of neurons. By using Kalikow decomposition, we modify the classical Ogata algorithm to obtain a new algorithm, that is more tractable in practice. We present some interesting mathematical and simulated results. This based on recent works with Patricia Reynaud-Bouret (LJAD), Alexandre Muzy (I3S, Inria) and Eva Locherbach (Paris 1). This is a joint work with Paul Gresland (NeuroMod)

Carey E. Priebe (Johns Hopkins University)

Learning 1-Dimensional Submanifolds for Subsequent Inference on Random Dot Product Graphs

A random dot product graph (RDPG) is a generative model for networks in which vertices correspond to positions in a latent Euclidean space and edge probabilities are determined by the dot products of the latent positions. We consider RDPGs for which the latent positions are randomly sampled from an unknown 1-dimensional submanifold of the latent space. In principle, restricted inference, i.e., procedures that exploit the structure of the submanifold, should be more effective than unrestricted inference; however, it is not clear how to conduct restricted inference when the submanifold is unknown. We submit that techniques for manifold learning can be used to learn the unknown submanifold well enough to realize benefit from restricted inference. To illustrate, we test 1- and 2-sample hypotheses about the Fréchet means of small communities of vertices, using the complete set of vertices to infer latent structure. We propose test statistics that deploy the Isomap procedure for manifold learning, using shortest path distances on neighborhood graphs constructed from estimated latent positions to estimate arc lengths on the unknown 1-dimensional submanifold. Unlike conventional applications of Isomap, the estimated latent positions do not lie on the submanifold of interest. We extend existing convergence results for Isomap to this setting and use them to demonstrate that, as the number of auxiliary vertices increases, the power of our test converges to the power of the corresponding test when the submanifold is known. Finally, we apply our methods to an inference problem that arises in studying the connectome of the Drosophila larval mushroom body. The univariate learnt manifold test rejects ($p < 0.05$), while the multivariate ambient space test does not ($p >> 0.05$), illustrating the value of identifying and exploiting low-dimensional structure for subsequent inference.

Shigeru Shinomoto (ATR and Kyoto University)

Inferring monosynaptic connections from a cross-correlogram of neuronal spike trains

It was more than 50 years ago that Perkel, Gerstein, and Moore suggested a paradigm for detecting monosynaptic interactions from a cross-correlogram. While the original method may theoretically give plausible inferences, it sometimes suggests spurious connections in practice. This is because cross-correlograms are often accompanied by large fluctuations caused by common inputs to pairs of neurons. There have been many attempts to purge misinterpretations, by shuffling spike trains, by jittering spike times, or by taking fluctuating inputs into account. Here we developed a state-space method to detect the signature of monosynaptic connections or direct interaction buried in large fluctuations in a cross-correlogram. We evaluated the performance of the estimation accuracy by fitting the model to a large network of model neurons and then applied the model to real data.

Thibault Taillefumier (University of Texas)

Neural networks in the replica-mean field limits

In this talk, we propose to decipher the activity of neural networks via a "multiply and conquer" approach. This approach considers limit networks made of infinitely many replicas with the same basic neural structure. The key point is that these so-called replica-mean-field networks are in fact simplified, tractable versions of neural networks that retain important features of the finite network structure of interest. The finite size of neuronal populations and synaptic interactions is a core determinant of neural activity, being responsible for non-zero correlation in the spiking activity and for finite transition rates between metastable neural states. Mathematically, we develop our replica framework by expanding on ideas from the theory of communication networks to rigorously establish Poissonian mean-field limits for spiking networks. Computationally, we leverage this replica approach to characterize the stationary spiking activity of certain networks of the Galves Löcherbach type via reduction to tractable functional equations. We conclude by discussing perspectives about how to use our replica framework to probe nontrivial regimes of spiking correlations and transition rates between metastable neural states.

Romain Veltz (Université Côte d'Azur, Inria)

Some recent results on a mean field of a network of 2d spiking neurons.

In this talk, I will present some results regarding the dynamics of a network of stochastic spiking neurons akin to the "generalized linear model" connected with a mean-field coupling. This network is an elaboration of the one introduced in [De Masi et al. 2014] by generalizing the dynamics of the individual neurons. This allows to capture most of the known intrinsic neuronal spiking, like bursting for example, and thus to study the effect of the intrinsic neuron dynamics on the macroscopic one.

I will study the property of the mean-field limit. In effect, it is a nonlinear Piecewise Deterministic Markov process with explosive flow and unbounded total rate function. I will first present some theoretical results regarding the solution of the linearized SDE which is shown to be regular and has mean firing rate characterized by an integral equation. I will then show that the solution is ergodic and give many quantitative properties of its invariant distribution. I will then extend some of these results (well posedness, stationary distributions) to the nonlinear case.

Luyan Yu (University of Texas)

Metastable spiking networks in the replica-mean-field limit

Characterizing metastable neural dynamics in finite-size spiking networks remains a daunting challenge. We propose to address this challenge in the recently introduced replica-mean-field (RMF) limit. In this limit, networks are made of infinitely many replicas of the finite network of interest, but with randomized interactions across replica. Such randomization renders certain excitatory networks fully tractable at the cost of neglecting activity correlations, but with explicit dependence on the finite size of the neural constituents.

However, metastable dynamics typically unfold in networks with mixed inhibition and excitation. Here, we extend the RMF computational framework to point-process-based neural network models with exponential stochastic intensities, allowing for mixed excitation and inhibition. Within this setting, we show that metastable finite-size networks admit multistable RMF limits, which are fully characterized by stationary firing rates. Technically, these stationary rates are determined as solutions to a set of delayed differential equations under certain regularity conditions that any physical solutions shall satisfy. We solve this original problem by combining the resolvent formalism and singular- perturbation theory. Importantly, we find that these rates specify probabilistic pseudo-equilibria which accurately capture the neural variability observed in the original finite-size network. We also discuss the emergence of metastability as a stochastic bifurcation, which can also be interpreted as a static phase transition in the RMF limits. In turn, we expect to leverage the static picture of RMF limits to infer purely dynamical features of metastable finite-size networks, such as the transition rates between pseudo-equilibria.

Zhennan Zhou (Peking University)

Towards understanding the time periodic solutions in a kinetic model for neuron networks

In this talk, we are concerned with a kinetic model for neuron networks, where individual neurons are characterized by their voltage and conductance. The dynamics of the voltage is influenced by the conductance and when the voltage is reaching a threshold, it is immediately reset to a lower value. By exploring a series of simplified models, we aim to identify the cause of the emergence of time-periodic solutions in such Fokker-Planck equations.

Maximally predictive ensemble dynamics from data

Antonio C. Costa[a], Tosif Ahamed[b,c], David Jordan[d], Greg J. Stephens[a,c]
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[c] *Biological Physics Theory Unit, OIST Graduate University, Okinawa 904-0495, Japan*
[d] *Gurdon Institute, University of Cambridge, United Kingdom*

We leverage the interplay between microscopic variability and macroscopic order to connect physical descriptions across scales directly from data, without underlying equations. We reconstruct a state space by concatenating measurements in time, building a maximum entropy partition of the resulting sequences, and choosing the sequence length to maximize predictive information. Trading non-linear trajectories for linear, ensemble evolution, we analyze reconstructed dynamics through transfer operators. The evolution is parameterized by a transition time τ: capturing the source entropy rate at small τ and revealing timescale separation with collective, coherent states through the operator spectrum at larger τ. Applicable to both deterministic and stochastic systems, we illustrate our approach through the Langevin dynamics of a particle in a double-well potential and the Lorenz system. Applied to the behavior of the nematode worm *C. elegans*, we derive a "run-and-pirouette" navigation strategy directly from posture dynamics which is orders of magnitude faster. We demonstrate how sequences simulated from the ensemble evolution recover effective diffusion in the worm's centroid trajectories and introduce a top-down, operator-based clustering which reveals subtle subdivisions of the "run" behavior.

arXiv preprint: `https://arxiv.org/abs/2105.12811`

Propagation of Chaos and Poisson Hypothesis for Replica Mean-Field Models

Michel Davydov (École Normale Supérieure & INRIA Paris)

In order to model neural computations resulting from myriads of neuronal interactions, intensity-based spiking neural networks are commonly used. Unfortunately, most relevant dynamics involve complex graphs of interactions for which an exact computational treatment is impossible. To circumvent this difficulty, the replica-mean-field approach focuses on randomly interacting replicas of the networks of interest. In contrast with classical thermodynamic mean-fields, interaction couplings do not scale with the number of neurons and preserve both the geometry of the network and the finite-size correlations. In the limit of an infinite number of replicas, these networks become analytically tractable under the so-called Poisson Hypothesis, which postulates that replicas become asymptotically independent and arrivals to a given neuron become Poisson distributed. This hypothesis is often conjectured or numerically validated but not proven. We show the validity of the Poisson Hypothesis for large classes of processes that include for example Galves-Löcherbach models.

Front initiation in continuous neural fields

Grégory Faye
CNRS, Institut de Mathématiques de Toulouse

In this talk, I shall present some recent and ongoing results regarding the question of front initiation in continuous scalar neural fields. The problem is to characterize, given a localized initial condition, the long-time dynamic of the solution of the associated Cauchy problem. When the underlying kinetics of the homogenous problem is of bistable type with two stables states, say $u = 0$ and $u = 1$, with $u = 1$ more favorable than $u = 0$, one expects that *small* (in some appropriate sense) localized initial condition will lead to uniform convergence of the solution towards $u = 0$ (this is referred to as *extinction*) and *large* localized initial condition will lead to convergence on compact sets towards $u = 1$ (this is referred to as *propagation*). The identification of a threshold between extinction/propagation within a given one parameter family of localized initial condition is a well-known difficult problem, even for local bistable reaction-diffusion equations, and I will explain some progress made in that direction for two types of continuous neural fields: the Amari-Wilson-Cowan equation and the nonlocal Nagumo equation.

Topological insights on neuronal morphologies

Lida Kanari and Kathryn Hess

Abstract

The morphological diversity of neurons supports the complex information-processing capabilities of biological neuronal networks. A major challenge in neuroscience has been to reliably describe neuronal shapes with universal morphometrics that generalize across cell types and species. Inspired by algebraic topology, we have developed a topological descriptor of trees that couples the topology of their complex arborization with their geometric structure, retaining more information than traditional morphometrics.

The topological morphology descriptor (TMD) has proved to be very powerful in separating neurons into well-defined groups on morphological grounds. The TMD algorithm led to the discovery of two distinct morphological classes of pyramidal cells in the human cortex that also have distinct functional roles, suggesting the existence of a direct link between the anatomy and the function of neurons. The TMD-based classification also led to the objective and robust morphological clustering of rodent cortical neurons.

The TMD of neuronal morphologies is also essential for the computational generation (i.e., synthesis) of dendritic morphologies. Our results demonstrate that a topology-based synthesis algorithm can reproduce both morphological and electrical properties of reconstructed biological rodent cortical dendrites. Since the topology-based synthesis can be generalized to a wide variety of different dendritic shapes, it is suitable for the generation of unique neuronal morphologies to populate the digital reconstruction of large-scale, physiologically realistic networks.

Theoretical study of the emergence of periodic solutions for the inhibitory NNLIF neuron model with synaptic delay

Kota Ikeda*, Pierre Roux†, Delphine Salort‡ Didier Smets†

In neural networks, fast global oscillations was observed in [1] and are named *gamma oscillation.* Among other models aimed at understanding the self-sustained oscillations, the NNLIF model with synaptic delay and weakly firing inhibitory neurons was developed two decades ago [2]. Periodic solutions have been numerically observed in this model, but despite intensive study of this model in several researches, there was up-to-date no analytical result on this topic. In this talk, we propose to approximate formally these solutions by a Gaussian wave whose periodic movement is described by an associate difference-differential equation. We prove the existence of a periodic solution for the position in time of the centre of the Gaussian wave and we give a rigorous asymptotic result on these solutions when the connectivity parameter b goes to $-\infty$. Finally we provide heuristic and numerical evidence of the validity of our approximation.

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Beyond Blow-Up for Nonlinear Noisy Leaky Integrate and Fire neuronal models: numerical approach to the "plateau" state

María J. Cáceres Alejandro Ramos-Lora

The Nonlinear Noisy Leaky Integrate and Fire neuronal models are mathematical models that describe the activity of neural networks. These models have been studied at a microscopic level, using Stochastic Differential Equations, and at a mesoscopic/macroscopic level, through the mean field limits using Fokker-Planck type equations. To advance in the understanding of the NNLIF models, we have analyzed in depth the behaviour of the classical and physical solutions of the Stochastic Differential Equations and we compare it with what is already known about the Fokker-Planck equation, using a numerical study of their particle systems. This allows us to understand what happens in the neural network when an explosion occurs in finite time, which is one of the most important open problems about this kind of models. This allows us to go beyond the mesoscopic/macroscopic description. We answer one of the most important open questions about these models[1]: what happens after all the neurons in the network fire at the same time? We find that the neural network converges towards its unique steady state, if the system is weakly connected. Otherwise, its behaviour is more complex, tending towards a stationary state or a "plateau" distribution (membrane potentials are uniformly distributed between reset and threshold values). To our knowledge, these distributions have not been described before for these nonlinear models.

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Title and abstract for “Mathematical modelling and statistical analysis in neuroscience” workshop

Institut Henri Poincaré, Paris

January-February 2022

Sunil Modhara

Title

Neural fields with rebound currents: novel routes to patterning

Abstract

The understanding of how spatio-temporal patterns of neural activity may arise in the cortex of the brain has advanced with the development and analysis of neural field models. To replicate this success for sub-cortical tissues, such as the thalamus, requires an extended approach which includes relevant ionic currents that are able to further shape firing response. Here we advocate for one such approach that can accommodate slow currents. By way of illustration we focus on incorporating a T-type calcium current into the standard neural field framework. Direct numerical simulations are used to show that the resulting tissue model has many of the properties seen in more biophysically detailed model studies, and most importantly the generation of oscillations, waves, and patterns that arise from rebound firing. To explore the emergence of such solutions we focus on one- and two-dimensional spatial models and show that exact solutions describing homogeneous oscillations can be constructed in the limit that the firing rate nonlinearity is a Heaviside function. A linear stability analysis, using techniques from non-smooth dynamical systems, is used to determine the points at which bifurcations from synchrony can occur. Furthermore, we construct periodic travelling waves and investigate their stability with the use of an appropriate Evans function. The stable branches of the dispersion curve for periodic travelling waves are found to be in excellent agreement with simulations initiated from an unstable branch of the synchronous solution.

A new synaptic rule to unify experimental heterogeneity

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ABSTRACT

How memory and learning shape, our brain has been a long-standing question in neuroscience. Synaptic plasticity has been credited as a path to answer this since the demonstration of synaptic efficacy persistence. In the 70s, methods to induce potentiation and depression of synaptic plasticity were for the first time described. After that, the diversification of techniques allowed researchers to hypothesise on the nature of synaptic rules. However, the heterogeneity of experimental conditions shows that different outcomes can be achieved with the same stimulation pattern. Despite the relevance of diversity, this hampered the formalisation of how plasticity works, implicating reproducibility and replicability issues. Here, a computational model is developed to explain how heterogeneous experimental conditions affect plasticity outcomes. The model provides a new plasticity rule unifying developmental aspects, aCSF and temperature influence on the synaptic mechanisms. This is achieved by analysing the combined dynamics of synaptic enzymes and applying a new geometrical readout. In this way, this model can cover a broader range of experimental results than previous models for plasticity.

Self-organized criticality in hierarchical modular networks of Galves-Löcherbach neurons

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Abstract

One of the observed features of the neocortex is the existence of activity avalanches characterized by power-law size and duration distributions, putatively suggesting that the neocortex operates at a critical state. Several models have been proposed to explain how neocortical networks can reach a critical state through self-organizing mechanisms. These self-organized criticality (SOC) models have explored networks with different topologies, e.g. fully connected and Erdős-Rényi. Here we study SOC in a hierarchical modular network using stochastic neurons of the Galves-Löcherbach type. The system has two mechanisms that make the critical region an attractor of the SOC dynamics: (i) dynamical gains, which produces adaptation in the neuronal firing rates, and (ii) dynamical synapses, which represent homeostatic mechanisms. We characterize the size and duration avalanches displayed by the model and study the emergence of synchronized activity across the network modules.

Keywords: self-organized criticality; neuronal avalanche; hierarchical modular network; stochastic neuron

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Fluctuation limits for mean-field interacting nonlinear Hawkes processes

We investigate the asymptotic behavior of networks of interacting nonlinear Hawkes processes modelling a homogeneous population of neurons in the large population limit. In particular, we prove a functional central limit theorem for the mean spike-activity, thereby characterizing the asymptotic fluctuations in terms of a stochastic Volterra integral equation. Our approach differs from the usual approach via tightness of the associate martingale problem. Instead, we make use of the resolvent of the associated Volterra integral equation in order to represent fluctuations as Skorokhod continuous mappings of weakly converging martingales. Since the Lipschitz properties of the resolvent are explicit, our analysis in principle also allows to derive approximation errors in terms of driving martingales. We also discuss extensions of our results to multi-class systems.

The talk is based on joint work with S. Heesen.

S. Heesen, W. Stannat: Fluctuation limits for mean-field interacting nonlinear Hawkes processes, Stochastic Process. Appl. 139 (2021), 280–297.

Spontaneous oscillations in a pure excitatory mean field networks of neurons

Etienne Tanré - Inria, UCA - France

We consider a model of network of interacting neurons based on jump processes. Briefly, the membrane potential V_t^i of each individual neuron evolves according to a one-dimensional ODE. Neuron i spikes at rate which only depends on its membrane potential, $f(V_t^i)$. After a spike, V_t^i is reset to a fixed value V^{rest}. Simultaneously, the membrane potentials of any (post-synaptic) neuron j connected to the neuron i receives a *kick* of value $J^{i,j}$.

We study the limit (mean-field) equation obtained where the number of neurons goes to infinity. In this talk, we describe the long time behaviour of the solution. Depending on the intensity of the interactions, we observe convergence of the distribution to a unique invariant measure (small interactions) or we characterize the occurrence of spontaneous oscillations for interactions in the neighbourhood of critical values.

The talk is based on joint works with Quentin Cormier (Princeton) and Romain Veltz (Inria)

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A multiple time renewal equation for neural assemblies with elapsed time model

Nicolas Torres

Abstract

We introduce and study an extension of the classical elapsed time equation in the context of neuron populations that are described by the elapsed time since the last discharge, i.e., the refractory period. In this extension we incorporate the elapsed since the penultimate discharge and we obtain a more complex system of integro-differential equations. For this new system we prove convergence to stationary state by means of Doeblin's theory in the case of weak non-linearities in an appropriate functional setting, inspired by the case of the classical elapsed time equation. Moreover, we present some numerical simulations to observe how different firing rates can give different types of behaviors and to contrast them with theoretical results of both classical and extended models.

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