

# Workshop: Tissue growth and movement

Week 10-14th January 2022  
(IHP, Paris)

## POSTERS

Titles and abstracts

**Giorgia Ciavolella** (University of Rome and Sorbonne Université)

**Title: Membrane problem and application to cell invasion**

**Abstract:** Studying tumour evolution, a crucial and challenging scenario is represented by cancer cells invasion through thin membranes. In particular, one of the most difficult barriers for cells to cross is the basement membrane. The drastically different scales and mobility rates between the membrane and the adjacent tissues lead to consider the limit as the thickness of the membrane approaches zero and, consequently, membrane limit conditions, called Kedem-Katchalsky conditions.

In this poster, I will present the main tools to recover the rigorous limit problem for a porous-medium type equation, called effective problem, and the transmission conditions on the limiting zero-thickness membrane, formally derived by Chaplain et al. (2019). Moreover, I will show an extension of the Turing theory in the case of a reaction-diffusion system with membrane conditions. Finally, I will illustrate an applied biological membrane model, that we are studying in collaboration with a group of biologists.

**Noemi David** ( University of Bologna and Sorbonne Université)

**Title: On the incompressible limit for tumor growth models**

**Abstract:** Both compressible and incompressible porous medium models are used in the literature to describe the mechanical properties of living tissues. These two classes of models can be related using a stiff pressure law. In the incompressible limit, density-based models generate free boundary problems of Hele-Shaw type, where saturation holds in the moving domain. In this poster, I will present a model including the effect of a nutrient (or possibly an external drift). Then, a badly coupled system of equations describes the cell population density and the nutrient concentration. For this reason, the derivation of the free boundary limit was an open problem, in particular, the main difficulty is to establish the so-called complementarity relation which allows recovering the pressure using an elliptic equation. To this end, we prove the strong compactness of the pressure gradient, blending two new techniques: an extension of the usual Aronson-Bénilan estimate in an  $L^2$ -setting, and a sharp  $L^4$ -uniform bound of the pressure gradient.

**Kathrin Hellmuth** (University of Wuerzburg)

**Title: An inverse problem for bacterial movement: determining the chemotactic tumbling coefficient from macroscopic measurements**

**Abstract:** Bacteria move by running a straight line until they stop and choose a new direction. When the movement is directed, e.g. induced by a gradient in the concentration of an attracting chemical substance, this phenomenon is called chemotaxis. On the population level, this "run-and-tumble" process is often summarized by a kinetic chemotaxis partial differential equation (PDE) in the mesoscopic phase space. In the scaling limit, these models are typically linked to macroscopic PDEs, e.g. the Keller-Segel equation, describing the bacteria density in space and time. Similar models appear in angiogenesis which plays a role e.g. in growth of some tumours. On the kinetic level, the movement is determined by the law of changing direction which is encoded in the tumbling coefficient of the chemotaxis equation. It is thus of great interest to biologists and practitioners to determine this model parameter and thus fit the model to reality. To do so, experiments have to be conducted where bacteria density is measured. Since velocity dependent measurements are expensive, we model the measurements to be of macroscopic type, i.e. the velocity averaged bacteria density is measured. Inferring the tumbling kernel from these data constitutes the inverse problem we are considering. We shall report on work in progress on this topic. This is joint work with Christian Klingenberg (Wuerzburg, Germany), Qin Li (Madison, Wisc., USA) and Min Tang (Shanghai, China).

**Christian Klingenberg** (Wuerzburg University)

**Title: Computer tomography for body tissue in motion**

**Abstract:** Tomographic image reconstruction is well understood if the specimen being studied is stationary during data acquisition. However, if this specimen changes its position during the measuring process, standard reconstruction techniques can lead to severe motion artifacts in the computed images. Solving a dynamic reconstruction problem therefore requires to model and incorporate suitable information on the dynamics in the reconstruction step to compensate for the motion.

Many dynamic processes can be described by partial differential equations which thus could serve as additional information for the purpose of motion compensation. In this poster, we consider the Navier-Cauchy equation which characterizes small elastic deformations and serves, for instance, as a simplified model for respiratory motion. Our goal is to provide a proof-of-concept that by incorporating the deformation fields provided by this PDE, one can reduce the respective motion artifacts in the reconstructed image. To this end, we solve the Navier-Cauchy equation prior to the image reconstruction step using suitable initial and boundary data. Then, the thus computed deformation fields are incorporated into an analytic dynamic reconstruction method to compute an image of the unknown interior structure. The feasibility is illustrated with numerical examples from computerized tomography.

This is joint work with Bernadette Hahn and Sandra Warnecke.

**Florian Kreten** ( University of Bonn)

**Title: Traveling waves of an FKPP-type model for self-organized growth**

**Abstract:** We consider a reaction-diffusion system of densities of two types of particles, introduced by E. Hannezo et al. in the context of branching morphogenesis. It is a simple model for a growth process: active, branching particles form the growing boundary layer of an otherwise static tissue, represented by inactive particles. The active particles diffuse, branch and become irreversibly inactive upon collision with a particle of arbitrary type. In absence of active particles, this system is in a steady state, without any a priori restriction on the amount of remaining inactive particles. Thus, while related to the well-studied FKPP-equation, this system features a game-changing continuum of steady state solutions, where each corresponds to a possible outcome of the growth process. However, simulations indicate that this system self-organizes: traveling fronts with fixed shape arise under a wide range of initial data. In the present work, we describe all positive and bounded traveling wave solutions, and obtain necessary and sufficient conditions for their existence. We find a surprisingly simple symmetry in the pairs of steady states which are joined via heteroclinic wave orbits. Our approach is constructive: we first prove the existence of almost constant solutions and then extend our results via a continuity argument along the continuum of limiting points.

**Léo Meyer** (Université d'Orléans)

**Title: Modelling adipose cell distribution**

**Abstract:** We are interested in modelling the particular bimodal distribution in size of adipose cell. We introduce two model based on the Lifshitz-Slyozov equations and the Becker-Doring equations. Following classical results, we prove convergence from one model to the other and we investigate the addition of a diffusive term to reflect biological results. We also present some numerical result using a finite volume scheme

**Josephine Solowiej-Wedderburn** (University of Surrey)

**Title: Pulling on springs: exploring feedback mechanisms in cellular mechanosensation**

**Abstract:** Cells are active systems that respond to their environment. In particular, it is becoming increasingly apparent that physical force and the mechanical properties of their microenvironment play a crucial role in determining cell behaviour. Gaining a better appreciation of how these systems function may have significant implications for both tissue engineering applications and in understanding how mechanical factors affect the development and progression of a broad range of diseases such as cancer, osteoporosis, or cardiomyopathies. Here, we explore cellular contractility as a mechanism for mechanosensation. We take a theoretical continuum-mechanics approach, modelling cellular contractility as an active stress. The mathematical model is analysed and solved using both analytical approaches (exploiting approximations and symmetry arguments) and Finite Element Methods. We use this model to investigate energy constraints as a potential feedback mechanism, guiding how the cell may regulate its contractility in response to different environmental cues. We focus our model in the context of the most common biophysical experiments for mechanosensation where cells adhere to flat substrates with known mechanical properties. In such experiments the shape, size and pattern of adhesions between the cell and substrate, and distribution of contractility throughout the cell have significant implications for cellular mechanosensation. Hence, we also investigate the implications of these factors on cellular contractility, subject to our energy constraints.

**Jakub Skrzeczkowski** (University of Warsaw)

**Title: Fast reaction limit with nonmonotone reaction function**

**Abstract:** We consider a mass-conserving reaction-diffusion system with nonmonotone reaction function  $F$  and one non-diffusing component. As the speed of reaction tends to infinity, the concentration of the non-diffusing component exhibits fast oscillations. We identify precisely its Young measure which, as a by-product, proves strong convergence of the diffusing component, a result that is not obvious at all from a priori estimates. Our work is based on the analysis of regularization for forward-backward parabolic equations by Plotnikov. We rewrite his ideas in terms of kinetic functions which clarifies the method, brings new insights, relaxes assumptions on model functions, and provides a weak formulation for the evolution of the Young measure. We also refine the method of Plotnikov by application of classical Radon-Nikodym theorem.