Input Similarity from the Neural Network Perspective

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Overview of this talk

- **I - Remote sensing image segmentation and registration**
- **II - Dataset self-denoising**
- **III - Notion of similarity from the neural network viewpoint**
- **IV - Back to dataset self-denoising**

Work in collaboration with Nicolas Girard, Loris Felardos & Yuliya Tarabalka

$\hookrightarrow$ TAU team, at INRIA Saclay & Titane team, at INRIA Sophia-Antipolis

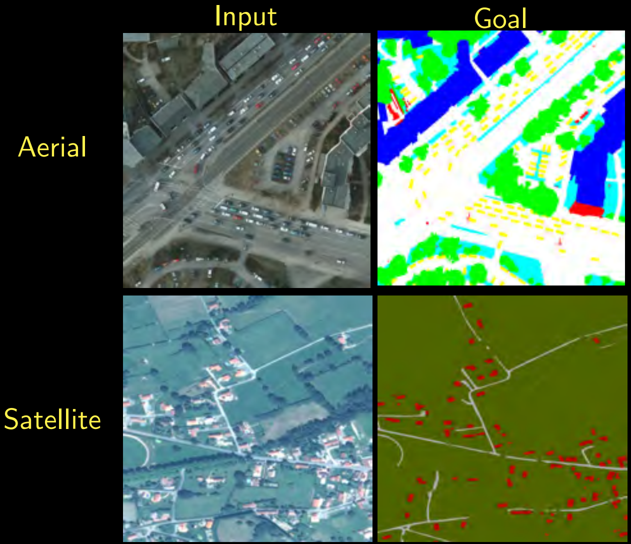
Part I

Remote sensing image segmentation and registration

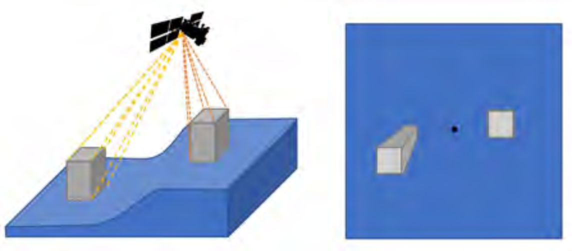
Task 1: Remote sensing image segmentation

- **Goal:** semantic segmentation of satellite images
  i.e.: each pixel $\mapsto$ *class* $\in$ {building, road, ...}
- **Tool:** neural networks with varied architectures
- **Obstacles:** no reliable dataset

Goal: semantic segmentation



Issue: misalignment



Deformations inherent to aerial or satellite photography

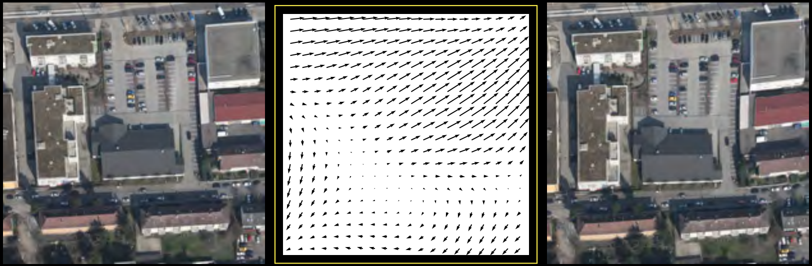
Issue: misalignment

- Registration: cadaster map (cyan) vs. photo RGB

Automatic realignment?


Multimodal pair of images: aerial RGB image / binary vector-format cadastral image (buildings in white)

Task 2: Multimodal Registration



Example of deformation. Image I ; a deformation ϕ, i.e. a $\mathbb{R}^2$ vector field ; associated deformed image $I \circ \phi$.

Approach

Optimization criterion: Euclidean norm of the prediction error

$$C(\mathrm{w}) = \mathop{\mathbb{E}}_{(I_1,I_2,\phi_{\mathrm{GT}})\in\mathcal{D}} \left[\sum_{\mathrm{x}\in\Omega(I_2)} \left\| \widehat{\phi}_{(\mathrm{w})(I_1,I_2)}(\mathrm{x}) - \phi_{\mathrm{GT}}(\mathrm{x}) \right\|_2^2 \right]$$

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- Issue: the network doesn't learn
- for prediction: each pixel $\mapsto$ deformation ± 25 px is too hard
- Idea: deformation ± 1 px is easy

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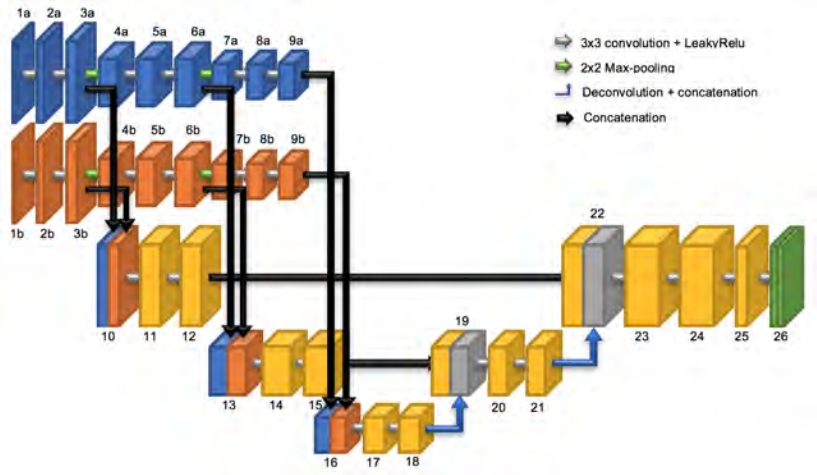
Task at scale s: Solve the alignment problem for the image pair (I_1, I_2), with a precision required of $\pm 2^s$ pixels, under the assumption that the amplitude of the registration to be found is not larger than 2^{s+1} pixels.

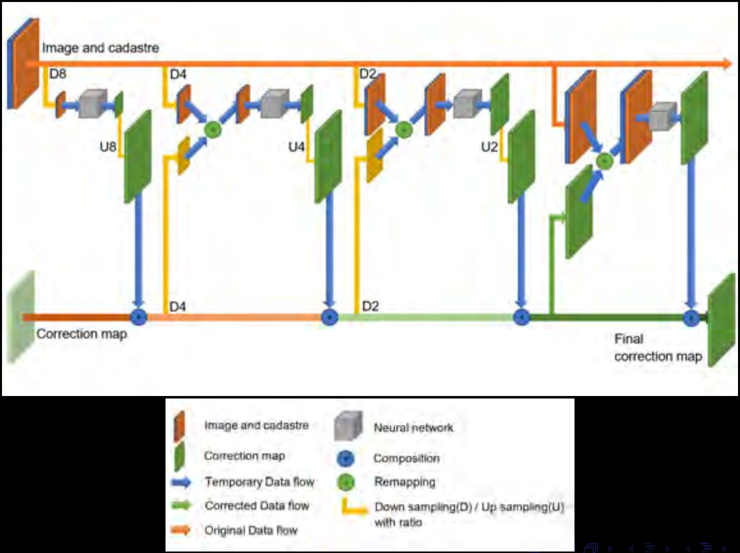
Solution for task at scale s: Downsample the images by a factor 2^s; solve the alignment task at scale 0 for these reduced images, and upsample the result with the same factor.

Full alignment algorithm: Given an image pair (I_1, I_2) of width w, iteratively solve the alignment task at scale s, from $s = \log_2 w$ until $s = 0$.

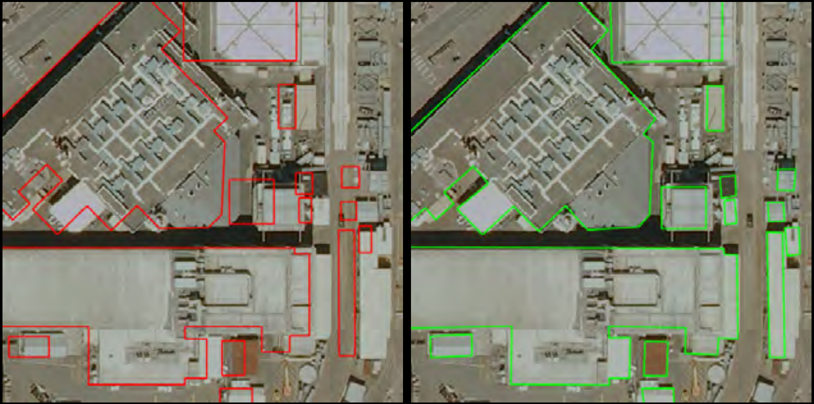
Network to process a specific scale



Global network: chain of scale-specific networks $\simeq$ compositional ResNet



Results



Example of image alignment. Original image and OpenStreetMap (OSM) map / Alignment result.

Part II

Dataset self-denoising

Training set for the alignment task

- pick locations where RGB image I and cadaster map M look not too badly aligned (or align manually)
  $\Longrightarrow$ training sample $\big((I, M), \mathrm{Id}\big)$
- generate random smooth deformations ϕ, and add $\big((I, M \circ \phi), \phi\big)$ to the training set

$\Longrightarrow$ sensitivity of the training w.r.t. alignment quality between original I and M?

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Dealing with noisy training data

- Dataset of examples $(x, y + \varepsilon)$ with noisy labels
- Is it possible to train and get accuracy higher than the noise variance?

Dataset self-denoising

Red: given ground truth
Green: the real but unavailable one

Nicolas's idea: update the dataset iteratively

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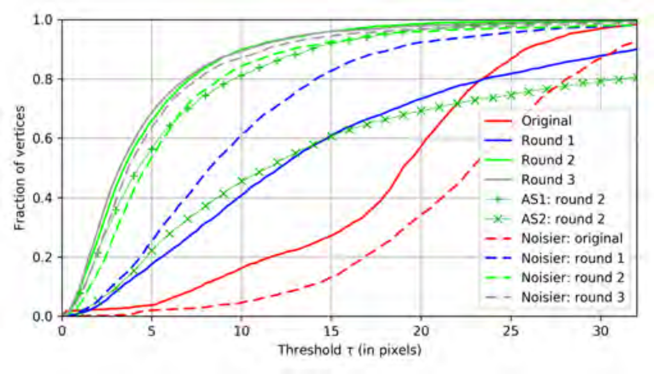
- ...

Dataset self-denoising

Quantitative results:



Dashed lines: control experiment with added noise on ground truth

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  - presented n times with noisy labels $y_i = y + \varepsilon_i$
  - assumption: i.i.d. noise ε, centered.
  - L^2 loss:

$$\inf_{\hat{y}} \sum_i \|\hat{y} - y_i\|^2$$

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$$\hat{y} \simeq y \pm \frac{1}{\sqrt{n}}$$

[Noise2Noise: Learning Image Restoration without Clean Data; Lehtinen et al., 2018]

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- Number of similar examples?
- Quantify: input similarity?

Part III

Input similarity
from the network's point of view

Notions of similarity

- Predefined metric (e.g., pixelwise L^2)
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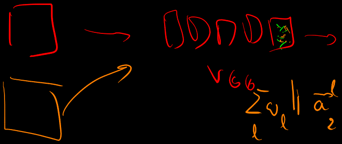
  [Johnson, Alahi and Li Fei-Fei: Perceptual losses for real-time style transfer and super-resolution, ECCV 2016]
  - to evaluate auto-encoder reconstruction error
  - compare VGG activities $\Longrightarrow$ more semantic
  - in practice: arbitrary choices (pick one layer)



$$\sum_l \omega_l \| a^l_2 - a^l_1 \|$$

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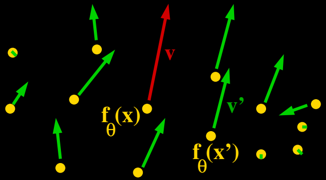
- Principled way?

Defining similarity by undissociability

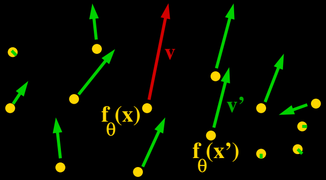
- Given a trained neural network f_θ
- and two input points x and x′
- how similar are x and x′ **for the network**?

Output space:



Quantify the influence of a data point x over another one x′ by how much the tuning of parameters θ to obtain a desired output change v for $f_\theta(x)$ will affect $f_\theta(x')$ as well.

Output space:



Influence of x over x′ = how much the tuning of parameters θ to obtain a desired output change v for $f_\theta(x)$ will affect $f_\theta(x')$ as well.

Derivation in 1-dim output case

- To change $f_\theta(x)$ by a small quantity ε, update θ by $\delta\theta = \varepsilon \frac{\nabla_\theta f_\theta(x)}{\|\nabla_\theta f_\theta(x)\|^2}$.
- Indeed, after parameter update, new value at x:

$$f_{\theta+\delta\theta}(x) \quad = \quad f_\theta(x) + \nabla_\theta f_\theta(x) \cdot \delta\theta + O(\|\delta\theta\|^2) \quad = \quad f_\theta(x) + \varepsilon + O(\varepsilon^2).$$

- This parameter change induces a value change at any other point x′ :

$$f_{\theta+\delta\theta}(x') \; = \; f_\theta(x') + \nabla_\theta f_\theta(x') \cdot \delta\theta + O(\|\delta\theta\|^2) \; = \; f_\theta(x') + \varepsilon \frac{\nabla_\theta f_\theta(x') \cdot \nabla_\theta f_\theta(x)}{\|\nabla_\theta f_\theta(x)\|^2} + O(\varepsilon^2).$$

Symmetric similarity

$$k_\theta(x, x') = \frac{\nabla_\theta f_\theta(x)}{\|\nabla_\theta f_\theta(x)\|} \cdot \frac{\nabla_\theta f_\theta(x')}{\|\nabla_\theta f_\theta(x')\|}$$

- kernel, valued in $[-1, 1]$
- Neural Tangent Kernel!

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Properties for vanilla neural networks:

Theorem 1

For any real-valued neural network f_θ whose last layer is a linear layer (without any parameter sharing) or a standard activation function thereof (sigmoid, tanh, ReLU...), and for any inputs x and x',

$$k_\theta(x, x') = 1 \implies \nabla_\theta f_\theta(x) = \nabla_\theta f_\theta(x') \implies f_\theta(x) = f_\theta(x')$$

Symmetric similarity

$$k_\theta(\mathrm{x}, \mathrm{x}') = \frac{\nabla_\theta f_\theta(\mathrm{x})}{\|\nabla_\theta f_\theta(\mathrm{x})\|} \cdot \frac{\nabla_\theta f_\theta(\mathrm{x}')}{\|\nabla_\theta f_\theta(\mathrm{x}')\|}$$

- kernel, valued in $[-1, 1]$
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Properties for vanilla neural networks:

Theorem 2

For any real-valued neural network f_θ without parameter sharing, if $k_\theta(\mathrm{x}, \mathrm{x}') = 1$ for two inputs x, x', then all useful activities computed when processing x are equal to the ones obtained when processing x'.

Link with the *perceptual loss*

- Perceptual loss:

$$\sum_{\text{activities } i \neq 0} \lambda_{\text{layer}(i)} \; a_i(\mathrm{x}) \; a_i(\mathrm{x}')$$

- Our similarity measure for vanilla networks:

$$k_\theta(\mathrm{x}, \mathrm{x}') = \sum_{\text{activities } i} \lambda_i(\mathrm{x}, \mathrm{x}') \; a_i(\mathrm{x}) \; a_i(\mathrm{x}')$$

where $$\lambda_i(\mathrm{x}, \mathrm{x}') = \sum_{\text{neuron } j \text{ using } a_i} \frac{df_\theta(\mathrm{x})}{db_j} \frac{df_\theta(\mathrm{x}')}{db_j}$$

- For parameter-sharing networks:

$$k_\theta(\mathrm{x}, \mathrm{x}') = \sum_{\text{params } i} \left(\sum_{(j,k) \in \mathcal{S}_i} a_k(\mathrm{x}) \frac{df_\theta(\mathrm{x})}{db_j} \right) \left(\sum_{(j,k) \in \mathcal{S}_i} a_k(\mathrm{x}') \frac{df_\theta(\mathrm{x}')}{db_j} \right)$$

$\Longrightarrow$ reflects network invariances (e.g., translation-inv for convnets)

Counting neighbors

- Similarity measure $k_\theta \implies$ notion of neighborhood
- Number of neighbors of point x ?
- Hard-thresholding, for a given threshold $\tau \in [0,1]$:

$$N_\tau(\mathrm{x}) = \sum_{\mathrm{x}'} 1_{k_\theta(\mathrm{x},\mathrm{x}') \geqslant \tau}$$

- Soft estimate:

$$N_S(\mathrm{x}) = \sum_{\mathrm{x}'} k_\theta(\mathrm{x},\mathrm{x}')$$

- The two are linked:

$$\int_{\tau=0}^{1} N_\tau(\mathrm{x}) d\tau = \sum_{\mathrm{x}'} \int_{\tau=0}^{1} 1_{k_\theta(\mathrm{x},\mathrm{x}') \geqslant \tau}\, d\tau = \sum_{\mathrm{x}'} k_\theta(\mathrm{x},\mathrm{x}')\, 1_{k_\theta(\mathrm{x},\mathrm{x}') \geqslant 0} \simeq N_S(\mathrm{x})$$

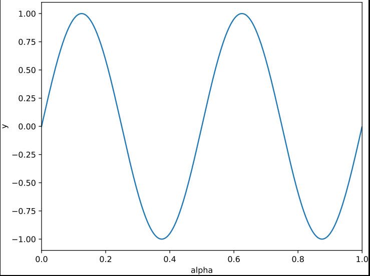
- Low complexity of the soft estimate:

$$N_S(\mathrm{x}) = \sum_{\mathrm{x}'} k_\theta(\mathrm{x},\mathrm{x}') = \sum_{\mathrm{x}'} \frac{\nabla_\theta f_\theta(\mathrm{x})}{\|\nabla_\theta f_\theta(\mathrm{x})\|} \cdot \frac{\nabla_\theta f_\theta(\mathrm{x}')}{\|\nabla_\theta f_\theta(\mathrm{x}')\|} = \frac{\nabla_\theta f_\theta(\mathrm{x})}{\|\nabla_\theta f_\theta(\mathrm{x})\|} \cdot \mathrm{g} \quad \text{with } \mathrm{g} = \sum_{\mathrm{x}'} \frac{\nabla_\theta f_\theta(\mathrm{x}')}{\|\nabla_\theta f_\theta(\mathrm{x}')\|}$$

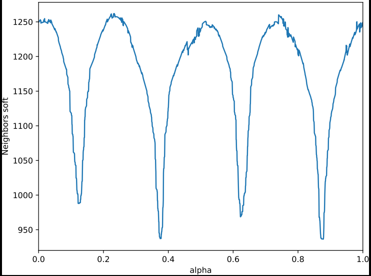
- Very fast to compute! Estimate density at every point in 2 passes over the dataset

Testing these density estimators

- Experiment design: train networks to imitate sinusoids of various frequencies



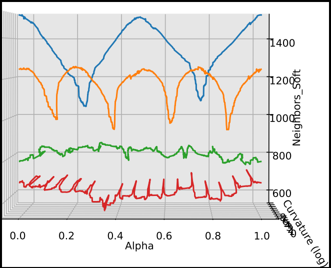
(a) Function to predict.

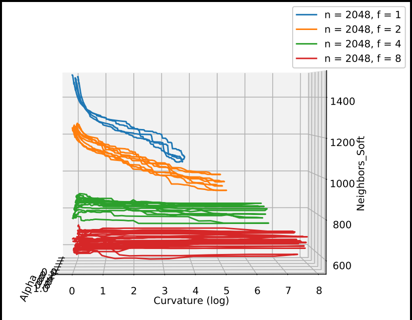


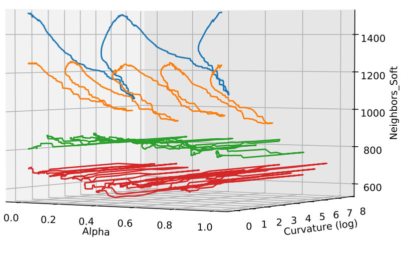
(b) Neighbors soft estimate.

Figure: Toy problem with the frequency $f = 2$.

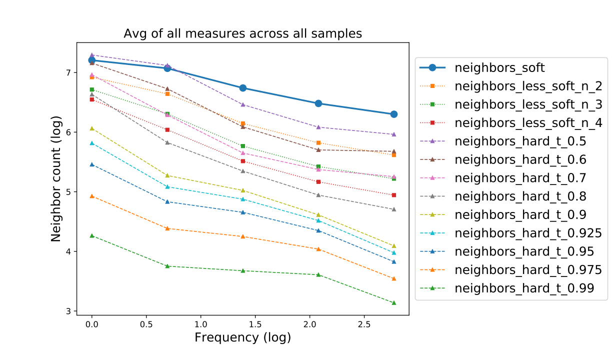
Better viewed in 3D: with curvature







Color: frequency
Depth: curvature
Height: density



Density estimation using the various approaches (log scale). All approaches behave similarly and show good results, except the ones with extreme thresholds.

Density, so what?

- Test at point x: very low density? $\Longrightarrow$ not reliable prediction!
  - as no neighbor $\Longrightarrow$ independent of training set
  - quantify prediction uncertainty
- Very high density? Might underfit.
  - useful to know during training
- Differentiable quantities... $\Longrightarrow$ possible to optimize them while training!

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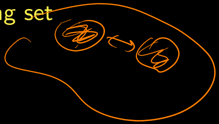


By the way...

- Differentiable similarity estimate $\Longrightarrow$ possible to enforce while training that some examples should be perceived as similar (or different) by the network
- Enforcing similarity on a classification task: small boosting effect (on MNIST...)

Part IV

Back to remote sensing image registration

What do neighbors look like?

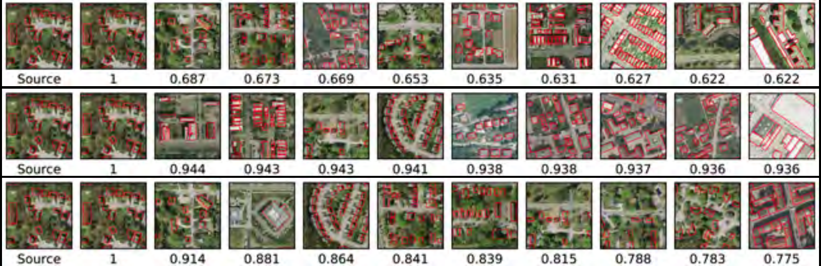


Figure: Example of nearest neighbors for a patch. Each line corresponds to a round. Each patch has its similarity written under it.

(a) Round 1 (b) Round 2 (c) Round 3

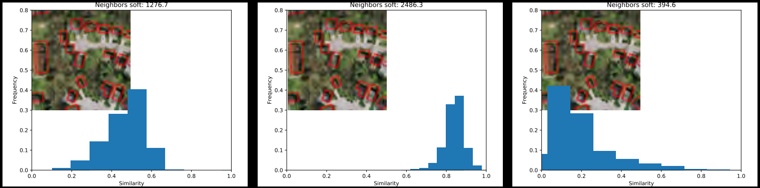


Figure: Histograms of similarities for one patch across rounds.

Back to remote sensing image registration

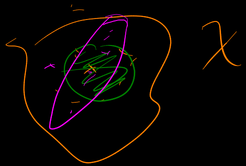
Figure: Closest neighbors to the leftmost patch, using the perceptual loss (first row) and our similarity definition (second row).

Part IV - bis

Back to dataset self-denoising

From similarity statistics to self-denoising effect estimation

$$
\begin{aligned}
\text{input} &: \mathrm{x}_i \\
\text{true (unknown) label} &: y_i \\
\text{(unknown) noise} &: \varepsilon_i \text{ (iid, centered)} \\
\text{noisy (available) label} &: \widetilde{y}_i = y_i + \varepsilon_i \\
\text{predicted label} &: \widehat{y}_i = f_\theta(\mathrm{x}_i) \\
\text{training loss} &: L(\theta) = \textstyle\sum_j ||\widehat{y}_j - \widetilde{y}_j||^2
\end{aligned}
$$



- at convergence $\nabla_\theta L = 0 \implies \mathbb{E}_k[\widehat{y}] = \mathbb{E}_k[\widetilde{y}]$
- $\mathbb{E}_k[a] := \sum_j a_j\, k_\theta(\mathrm{x}_i, \mathrm{x}_j)$: mean value around x_i

$$\widehat{y}_i - \mathbb{E}_k[y] = \mathbb{E}_k[\varepsilon] + (\,\widehat{y}_i - \mathbb{E}_k[\widehat{y}]\,)$$

- $\widehat{y}_i - \mathbb{E}_k[y]$: prediction error to smoothed true labels
- $\mathbb{E}_k[\varepsilon] \;\propto\; \sigma_\varepsilon \, ||k_\theta(\mathrm{x}_i, \cdot)||_{L2} \implies$ denoising factor: 0.02 ($\simeq$ constant)
- Shift: $(\,\widehat{y}_i - \mathbb{E}_k[\widehat{y}]\,)$: 4.4 px (varying) /50

Conclusion

Conclusion

- Defined input similarity as perceived by the neural network
- Skipped the maths for the higher-dim case
- Fast similarity / density estimation
  $\Longrightarrow$ opens the door to underfit/overfit/uncertainty analyses and control
- Similarity enforced during training: dataset-dependent boosting effect (cf supp.mat.)
- Extended Noise2Noise to non-identical inputs: self-denoising effect as a function of inputs similarities
- Links with Neural Tangent Kernel [4]: same concept! used differently
- Code available on GitHub:
  `http://github.com/Lydorn/netsimilarity`

Recent news:

- Our first citation! [Hanawa et al.]
- Comparison of several criteria for similar image retrieval $\Longrightarrow$ ranked first!
- It seems they did not compute the right quantity...

Papers

Guillaume Charpiat, Nicolas Girard, Loris Felardos, and Yuliya Tarabalka.
Input similarity from the neural network perspective.
In Thirty-third Conference on Neural Information Processing Systems (NeurIPS), Vancouver, Canada, December 2019.

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