Combinatorics and Arithmetic for Physics: special days

Report of Contributions

https://indico.math.cnrs.fr/e/6181

Contribution ID: **1** Type: **not specified**

Tracelet Algebras

Wednesday, December 2, 2020 10:30 AM (50 minutes)

Stochastic rewriting systems evolving over graph-like structures are a versatile modeling paradigm that covers in particular biochemical reaction systems. In fact, to date rewriting-based frameworks such as the Kappa platform [1] are amongst the very few known approaches to faithfully encode the enormous complexity in both molecular structures and reactions exhibited by biochemical reaction systems in living organisms. Since in practice experimental constraints permit to track only very limited information about a given reaction system (typically the concentrations of only a handful of molecules), a fundamental mathematical challenge arises: which types of information are meaningful to derive and computable from a stochastic rewriting system in view of the limited empirical data? Traditionally, the main focus of the mathematical theory of stochastic rewriting theory has been upon the derivation of ODE systems describing the evolution of averages and higher moments of pattern counts (i.e. the concentrations of molecular species). In this talk, we present an alternative approach based upon so-called tracelets [2]. The latter are the precise mathematical encoding of the heuristic notion of pathways in biochemistry. We demonstrate a novel mathematical concept of tracelet algebras and highlight a computational strategy that permits to derive structural, high level insights into the dynamics of pattern counts. In view of the focus of CAP on combinatorial aspects, we will illustrate this mathematical approach with an analysis of planar rooted binary trees in a rewriting-based formulation utilizing the Rémy generator.

[1] Pierre Boutillier et al., ”The Kappa platform for rule-based modeling.”, Bioinformatics 34.13 (2018): pp. 583-592.

[2] Nicolas Behr, ”Tracelets and Tracelet Analysis Of Compositional Rewriting Systems”, Electronic Proceedings in Theoretical Computer Science 323 (2020), pp. 44-71.

Presenter: BEHR, Nicolas (Université de Paris, IRIF)

Contribution ID: **2** Type: **not specified**

Dialogue Games and Logical Proofs in String Diagrams

Wednesday, December 2, 2020 11:30 AM (50 minutes)

After a short introduction to the functorial approach to logical proofs and programs initiated by Lambek in the late 1960s, based on the notion of free cartesian closed category, we will describe a recent convergence with the notion of ribbon category introduced in 1990 by Reshetikhin and Turaev in their functorial study of quantum groups and knot invariants. The connection between proof theory and knot theory relies on the notion of ribbon dialogue category, defined by relaxing the traditional assumption that duality is involutive in a ribbon category. We will explain first how to construct the free such dialogue category using a logic of tensor and negation inspired by the work by Girard on linear logic. A coherence theorem for ribbon dialogue categories will be then established, which ensures that two tensorial proofs are equal precisely when their underlying ribbon tangles are equivalent modulo deformation. At the end of the talk, we will show how to understand these ribbon tangles as interactive Opponent/Player strategies tracking the flow of negation functors in dialogue games. The resulting diagrammatic description of tensorial proofs as interactive strategies is performed in the 3-dimensional language of string diagrams for monoidal 2-categories (or more generally weak 3-
categories) initiated in the mid 1990s by Street and Verity, McIntyre and Trimble.

A few references:
https://www.irif.fr/~mellies/hdr-mellies.pdf
https://www.irif.fr/~mellies/tensorial-logic/1-game-semantics-in-string-diagrams.pdf
https://www.irif.fr/~mellies/papers/lics2018-ribbon-tensorial-logic.pdf

Presenter: MELLIÈS, Paul-André (Université de Paris, IRIF)

Contribution ID: **3** Type: **not specified**

Untyped Linear Lambda Calculus and the Combinatorics of 3-valent Graphs

Wednesday, December 2, 2020 1:30 PM (50 minutes)

The lambda calculus was invented by Church in the late 1920s, as part of an ambitious project to build a foundation for mathematics around the concept of function. Although his original system turned out to be logically inconsistent, Church was able to extract from it two separate usable systems, with a typed calculus for logic and an untyped calculus for pure computation. Through the work of Lawvere and Lambek in the 1970s, a close connection was established between typed lambda calculus and the theory of cartesian closed categories (cccs). Around the same time, Dana Scott discovered the first non-trivial mathematical models of untyped lambda calculus, which he later axiomatized using the notion of reflexive object in a ccc. After Jean-Yves Girard's formulation of Linear Logic in the 1980s, some renewed attention was paid to the linear subsystem of lambda calculus, which has similar relationships with the theory of symmetric monoidal closed categories, in

particular untyped linear lambda calculus may be modelled as the endomorphism operad of a reflexive object in a symmetric closed multicategory. In the talk, I will analyze a surprising bijection originally presented by Bodini, Gardy, and Jacquot (2013) between untyped linear lambda terms and rooted 3-valent maps (= 3-valent graphs embedded on oriented surfaces). Rather than being a mere coincidence, this bijection appears to be part of a deeper connection between the combinatorics of lambda calculus and the theory of map enumeration initiated by Tutte in the 1960s, as witnessed by a host of correspondences between different natural subsystems of linear lambda calculus and different natural families of maps.

Presenter: ZEILBERGER, Noam (Ecole Polytechnique)

Contribution ID: **4** Type: **not specified**

Constructive Matrix Theory for Hermitian Higher Order Interaction

Wednesday, December 2, 2020 2:20 PM (50 minutes)

In this seminar we study the constructive loop vertex expansion for stable matrix models with (single trace) interactions of arbitrarily high even order in the Hermitian and real symmetric cases. It relies on a new and simpler method which can also be applied in the previously treated complex case. We prove analyticity in the coupling constant of the free energy for such models in a domain uniform in the size of the matrix.

Presenter: RIVASSEAU, Vincent (Laboratoire de Physique Théorique, Université de Paris-Sud)

Contribution ID: **5** Type: **not specified**

Quantum Mechanics of Bipartite Ribbon Graphs: A Combinatorial Interpretation of the Kronecker Coefficient.

Wednesday, December 2, 2020 3:20 PM (50 minutes)

The action of subgroups on a product of symmetric groups allows one to enumerate different families of graphs. In particular, bipartite ribbon graphs (with at most edges) enumerate as the orbits of the adjoint action on two copies of the symmetric group (of order n!). These graphs form a basis of an algebra, which is also a Hilbert space for a certain sesquilinear form. Acting on this Hilbert space, we define operators which are Hermitians. We are therefore in the presence of a quantum mechanical model. We show that the multiplicities of the eigenvalues of these operators are precisely the Kronecker coefficients, well known in representation theory. We then prove that there exists an algorithm that delivers the Kronecker coefficients and allow us to interpret those as the dimension of a sub-lattice of the lattice of the ribbon graphs.Thus, this provides an answer to Murnaghan's question (Amer. J. Math, 1938) on the combinatorial interpretation of the Kronecker coefficient.

Presenter: BENGELOUN, Joseph (Université de Paris 13, LIPN)

Contribution ID: **6** Type: **not specified**

Quotients of Symmetric Polynomial Rings Deforming the Cohomology of the Grassmannian

Wednesday, December 2, 2020 4:30 PM (50 minutes)

One of the many connections between Grassmannians and combinatorics is cohomological: The cohomology ring of a Grassmannian $\mathrm{Gr}(k,n)$ is a quotient of the ring S of symmetric polynomials in k variables. More precisely, it is the quotient of S by the ideal generated by the k consecutive complete homogeneous symmetric polynomials $h_{n-k}, h_{n-k+1}, \ldots, h_n$. We deform this quotient, by replacing the ideal by the ideal generated by $h_{n-k} - a_1, h_{n-k+1} - a_2, \ldots, h_n - a_k$ for some k fixed elements $a_1, a_2, \ldots, a_k$ of the base ring. This generalizes both the classical and the quantum cohomology rings of $\mathrm{Gr}(k,n)$. We find three bases for the new quotient, as well as an S_3-symmetry of its structure constants, a "rim hook rule"for straightening arbitrary Schur polynomials, and a fairly complicated Pieri rule. We conjecture that the structure constants are nonnegative in an appropriate sense (treating the a_i as signed indeterminate), which suggests a geometric or combinatorial meaning for the quotient.

Presenter: GRINBERG, Darij (Drexel University, Philadelphia, US / currently Germany)

Contribution ID: **7** Type: **not specified**

On a Tropical Version of the Jacobian Conjecture

Thursday, December 3, 2020 11:00 AM (50 minutes)

We prove that, for a tropical rational map if for any point the convex hull of Jacobian matrices at smooth points in a neighborhood of the point does not contain singular matrices then the map is an isomorphism. We also show that a tropical polynomial map on the plane is an isomorphism if all the Jacobians have the same sign (positive or negative). In addition, for a tropical rational map we prove that if the Jacobians have the same sign and if its preimage is a singleton at least at one regular point then the map is an isomorphism. This is a joint work with Danylo Radchenko, ETH (Zürich).

Presenter: GRIGORYEV, Dimitri (CNRS Painlevé Lab, Univ. Lille)

Contribution ID: **8** Type: **not specified**

From Reflection Equation Algebra to Matrix Models

Wednesday, December 2, 2020 5:20 PM (50 minutes)

Reflection Equation Algebra is one of the Quantum matrix algebras, associated with a given Hecke symmetry, i.e. a braiding of Hecke type. I plan to explain how to introduce analogs of Hermitian Matrix Models arising from these algebras. Some other applications of the Reflection Equation Algebras will be discussed.

Presenter: GUREVICH, Dimitry (Valenciennes University, France)

Contribution ID: **9** Type: **not specified**

Maximal Green Sequences for Certain Triangle Products

Thursday, December 3, 2020 11:50 AM (50 minutes)

Bernhard Keller introduced maximal green sequences as a combinatorial tool for computing refined Donaldson-Thomas invariants in the framework of cluster algebras. Maximal green sequences furthermore can be used to prove the existence of nice bases of cluster algebras and play a prominent role in the work on the full Fock-Goncharov conjecture due to Gross-Hacking-Keel-Kontsevich. In Physics, maximal green sequences appear in the computation of spectra of BPS states. We report on joint work with Gleb Koshevoy introducing maximal green sequences for certain triangle products of quivers. As an application we comment on the consequences regarding the full Fock-Goncharov conjecture for double Bruhat cells.
Joint work with Gleb Koshevoy.

Presenter: GENZ, Volker (Bochum)

Contribution ID: **10** Type: **not specified**

Kleene Stars in Shuffle Algebras

Thursday, December 3, 2020 1:30 PM (50 minutes)

We present some bialgebras and their monoid of characters. We entend, to the case of some rings, the well-known theorem (in the case when the scalars form a field) about linear independence of characters. Examples of algebraic independence of subfamilies and identites derived from their groups (or monoids) of characters are *provided. In this framework, we detail the study of one-parameter groups of characters. It is a joint work (arXiv:2009.10970) with Darij Grinberg (Drexel Universiy, Philadelphia, US / currently Germany) and Hoang Ngoc Minh (LIPN, Paris XIII University).

Presenter: DUCHAMP, Gérard H. E. (Université de Paris 13, LIPN)

Contribution ID: **11** Type: **not specified**

MRS Factorisations and Applications

Thursday, December 3, 2020 2:20 PM (50 minutes)

We review simultaneously the essential steps to establish the equation bridging the algebraic structures of converging polyzetas, via their noncommutative generating series put in factorised form MRS. This equation then allows us to describe polynomial relations, homogenous in weight, among these polyzetas, via an identification of local coordinates.

Presenter: NGOC MINH, Hoang (Université de Paris, LIPN)

Contribution ID: **12** Type: **not specified**

Highly Noncommutative Words and Noncommutative Poisson Structures

Thursday, December 3, 2020 3:20 PM (50 minutes)

I will talk on homology calculations for the higher cyclic Hochschild complex and on combinatorial description of Lie structure on highly noncommutative words.
It is based on the texts: Arxiv:1906.07134 (J. ALgebra, 2020), preprints IHES M/19/14.

Presenter: IYUDU, Natalja K. (Research Fellow, University of Edinburgh)

Contribution ID: **13** Type: **not specified**

Hopf-Algebraic Renormalization of Multiple Zeta Values and their q-analogues

Thursday, December 3, 2020 4:30 PM (50 minutes)

After a brief introductory account, I'll explain how a quasi-shuffle compatible definition (by no means unique) of multiple zeta values can be given for integer arguments of any sign, through Connes-Kreimer's Hopf-algebraic renormalization. Finally, I'll introduce the Ohno-Okuda-Zudilin model of q-analogues for multiple zeta values, describe the algebraic structure which governs it, and explain how it could open a way to the more delicate renormalization of shuffle relations.

Presenter: MANCHON, Dominique (LMBP, CNRS (UMR 6620) Université de Clermont Auvergne)

Contribution ID: **14** Type: **not specified**

Unifying Colour $SU(3)$ with $\mathbb{Z}_3$-Graded Lorentz-Poincaré Algebra

Thursday, December 3, 2020 5:20 PM (50 minutes)

A generalization of Dirac's equation is presented, incorporating the three-valued colour variable in a way which makes it intertwine with the Lorentz transformations. We show how the Lorentz-Poincaré group must be extended to accomodate both $SU(3)$ and the Lorentz transformations. Both symmetries become intertwined, so that the system can be diagonalized only after the sixth iteration, leading to a six-order characteristic equation with complex masses similar to those of the Lee-Wick model. The spinorial representation of the $\mathbb{Z}_3$-graded Lorentz algebra is presented, and its vectorial counterpart acting on a $\mathbb{Z}_3$-graded extension of the Minkowski space-time is also constucted. Application to new formulation of the QCD and its gauge-field content is briefly evoked.

Presenter: KERNER, Richard (LPTMC, Sorbonne Université, Paris)

Contribution ID: **15** Type: **not specified**

Tropical Convexity, Mean Payoff Games and Nonarchimedean Convex Programming

Thursday, December 3, 2020 10:00 AM (50 minutes)

Convex sets can be defined over ordered fields with a non-archimedean valuation. Then, tropical convex sets arise as images by the valuation of non-archimedean convex sets. The tropicalization of polyhedra and spectrahedra are of special in- terest, since they can be described in terms of deterministic and stochastic games with mean payoff. In that way, one gets a correspondence between classes of zero- sum games, with an unsettled complexity, and classes of semilagebraic convex op- timization problems over non-archimedean fields. We shall discuss applications of this correspondence, including a counter example concerning the complexity of interior point methods, and the fact that non-archimedean spectrahedra have precisely the same images by the valuation as convex semi-algebraic sets. This is based on works with Allamigeon, Benchimol, Joswig and Skomra.

Presenter: GAUBERT, Stéphane (INRIA and CMAP Ecole Polytechnique)