

Quotients of Symmetric Polynomial Rings Deforming the Cohomology of the Grassmannian

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One of the many connections between Grassmannians and combinatorics is cohomological: The cohomology ring of a Grassmannian $\text{Gr}(k, n)$ is a quotient of the ring S of symmetric polynomials in k variables. More precisely, it is the quotient of S by the ideal generated by the k consecutive complete homogeneous symmetric polynomials $h_{n-k}, h_{n-k+1}, \dots, h_n$. We deform this quotient, by replacing the ideal by the ideal generated by $h_{n-k} - a_1, h_{n-k+1} - a_2, \dots, h_n - a_k$ for some k fixed elements a_1, a_2, \dots, a_k of the base ring. This generalizes both the classical and the quantum cohomology rings of $\text{Gr}(k, n)$. We find three bases for the new quotient, as well as an S_3 -symmetry of its structure constants, a “rim hook rule” for straightening arbitrary Schur polynomials, and a fairly complicated Pieri rule. We conjecture that the structure constants are nonnegative in an appropriate sense (treating the a_i as signed indeterminate), which suggests a geometric or combinatorial meaning for the quotient.

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