

On Asymptotic Preserving schemes for some SDEs and SPDEs in the diffusion approximation regime.

Friday, December 4, 2020 3:00 PM (30 minutes)

We introduce and study a notion of Asymptotic Preserving schemes, related to convergence in distribution, for a class of slow-fast Stochastic Differential Equations (SDE). We focus on an example in the so-called diffusion approximation regime: $dX_t^\epsilon = \frac{\sigma(X_t^\epsilon)m_t^\epsilon}{\epsilon} dt$, where $dm_t^\epsilon = -\frac{m_t^\epsilon}{\epsilon^2} dt + \frac{1}{\epsilon} d\beta_t$. The solution X^ϵ then converges in distribution when $\epsilon \rightarrow 0$ to the solution diffusion equation $dX_t = \sigma(X_t) \circ dW_t$, with a Stratonovitch interpretation of the noise W . The natural schemes fail to capture the correct limiting equation, as they give a limit scheme consistent with an Itô interpretation of the noise ($dX_t = \sigma(X_t)dW_t$). We propose an Asymptotic Preserving scheme, in the sense that the scheme converges when $\epsilon \rightarrow 0$, and that the limit scheme is consistent with the limiting equation with the correct interpretation of the noise. We also present a kinetic stochastic PDE $\partial_t f^\epsilon + \frac{1}{\epsilon} v \cdot \nabla_x f^\epsilon = \frac{1}{\epsilon^2} L f^\epsilon + \frac{1}{\epsilon} m^\epsilon f^\epsilon$, which also converge to a diffusion equation $\partial_t \rho = \text{div}(K\rho) + \rho \circ QdW$, and some ideas on how to construct AP schemes for this SPDE.

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