

## Some aspects of the analysis of MsFEM methods

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A multi-scale finite element method (MsFEM) is a finite element approach that allows to solve partial differential equations (PDEs) with highly oscillatory coefficients on a coarse mesh, that is, a mesh with elements of size much larger than the characteristic scale of the oscillations [1, 2]. To do so, MsFEMs use pre-computed basis functions adapted to the differential operator that comprizes the small scales of the problem. The numerical analysis of these methods exploits homogenization theory to obtain an error estimate in terms of the small parameter of the problem and the mesh size used. Not all specific settings are covered by the numerical analyses in the existing literature (see e.g. [1, 3]). The main purpose of this contribution is to consider, for the specific, classical variant of MsFEM called "linear MsFEM", the case of rectangular meshes and to comment on the Hölder continuity assumption for the coefficients.

To start with, we will recall in this talk the main ideas of MsFEM and the theoretical background necessary to study its convergence. Then we will state a precise convergence result and outline its proof, in a way that differs from the existing literature. The convergence can next be deduced from standard arguments of finite element analysis and a suitable application of homogenization results to the exact solution and to the basis functions of the MsFEM. Our treatment includes explicitly the differences between triangular and rectangular meshes. An ongoing research effort is devoted to further improving the error estimate. The basic ingredients this is based on will be briefly approached toward the end of the talk. We refer to [4] for more details.

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### References

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