

On a second-order well-balanced Lagrange-Projection scheme for blood flow equations with varying parameters

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Our work deals with the construction of a second-order well-balanced Lagrange-projection scheme applied to the 1D Blood Flow Equations [Formaggia, Quarteroni, Veneziani 2009]. We study the model in the particular case in which the cross-sectional area at rest and the wall stiffness of the blood vessel could be not constant in space. Indeed, there exist physiological and pathological situations in which geometrical and mechanical parameters can vary locally, as in presence of stenoses or aneurysm and tapering of blood vessels [Ghigo, Delestre, Fullana, Lagrée 2017]. However, to consider non-constant parameters leads to the presence of a non-null source term and, consequently, we aim to develop a numerical scheme for the resulting hyperbolic system of balance laws.

The Lagrange-projection formalism [Morales De Luna, Castro Díaz, Chalons 2020] entails a decomposition of the mathematical model into two different systems: the acoustic/Lagrangian one, which takes into account the (fast) acoustic waves, and the transport/projection step based on the (slow) transport waves. This proves to be useful and very efficient when considering subsonic or low-Mach number flows. In such situations the CFL restriction of Godunov-type schemes is driven by the acoustic waves and can be very restrictive. Thus, this kind of decomposition allows the design of very efficient implicit-explicit numerical schemes.

In particular, in order to solve the acoustic system, we delineate an approximate Riemann solver. To be able to do that, first we exploit the Suliciu relaxation approach [Bouchut 2004]. Then, following Gallice [Chalons, Kestener, Kokh, Stauffert 2017], we easily solve the associated Riemann problem as the resulting hyperbolic system is composed of only linearly-degenerate waves. The numerical source term is included in such a way that it is consistent in the integral sense and, at the same time, the scheme is well-balanced, namely able to preserve the zero-velocity stationary solution, the so-called “man at eternal rest” solution.

Finally, in order to reach second-order of accuracy in space and time, we make respectively use of polynomial reconstruction and Runge-Kutta TVD scheme. However, as the usual reconstructed polynomial would prevent the scheme to be well-balanced, we modify the slopes in such a way that they cancel when the “man at eternal rest” condition is satisfied. This is achieved by making use of the so-called fluctuations [Morales De Luna, Castro Díaz, Chalons 2020].

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