Convergence of an MPFA-O finite volume scheme for a seawater intrusion model with sharp-diffuse interfaces

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Abstract: We consider a sharp-diffuse interfaces seawater intrusion model [3] in coastal aquifers. This process leads to a coupled system of two nonlinear parabolic partial differential equations simulating two immiscible fluids considering the dynamics of transition zones.
To the descritization in time, we apply a cell-centred Multi-Point Flux Approximation (MPFA-O) finite volume scheme [1] for the nonlinear system on an unstructured mesh and for the time discretization, we use an implicit Euler scheme, which allows the use of large time steps and then the reduction of CPU time. It is shown that this scheme ensures the non-negativity of the discrete solution of the freshwater and seawater thicknesses, taking into account the anisotropy and heterogeneity of the coastal aquifer. Based on an apriori estimate and using a fixed point theorem, we have established the existence of the discrete solution issued from the MPFA-O scheme. After that, we proved the strong convergence of the numerical solution to the weak solution of the continuous problem due to some recent compactness arguments.
We have developed and implemented a new module in the open-source platform DuMux [4]. The efficiency and the robustness of our method are studied numerically by comparing our results with an analytical solution presented in [2]. Moreover, the comparison between our $2D$ numerical simulations and those of [3] have shown that this approach yields performant results. Our new module based on recent numerical tools is accurate, efficient and able to solve a $2D$ seawater intrusion model tacking into account the dynamics of the transition zones. Some numerical results will be presented. Future works will mainly focus on the validation of this approach for more realistic test cases.

Références

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