

Mean field limits for interacting diffusions with colored noise: phase transitions and spectral numerical methods

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Systems of interacting particles appear in a wide variety of applications, ranging from plasma physics and galactic dynamics to mathematical biology, the social sciences, dynamical density functional theory (DDFT) and machine learning. In this presentation, we study systems of interacting particles of the type

$$dX_i = -\left(V'(X_i) + \theta \left(X_i - \frac{1}{N} \sum_{j=1}^N X_j\right)\right) dt + \sqrt{2\beta^{-1}} \xi_t^i, \quad i=1, \dots, N,$$

where N is the number of particles, $V(\cdot)$ is a confining potential, θ is the interaction strength and β is the inverse temperature of the system. In contrast with most studies on the subject, and in order to better model realistic scenarios, we assume that ξ_t^i are independent *colored noise processes*, i.e. noise processes with a nonzero correlation time.

There is extensive literature studying the behavior of systems of this type in the case of white noise, and it is well-known that these systems exhibit phase transitions when the confining potential V admits several local minima: the associated McKean–Vlasov equation, which describes the evolution of the empirical measure of the particle system in the mean field limit, admits a unique steady-state solution for sufficiently small β , but it admits more than one steady-state solution when β exceeds a critical value.

Our goal is to investigate the effect of the correlation time of the noise on the bifurcation diagram for the equilibrium states in the case where V is the standard double-well potential. To this end, we develop a spectral numerical method based on Hermite polynomials for solving the McKean–Vlasov equation corresponding to the particle system with colored noise. We prove the spectral convergence of the numerical method in a simple setting and we corroborate our numerical findings in the small correlation time regime using perturbation theory.

Reference: S. N. GOMES, G. A. PAVLIOTIS and U. VAES, *Mean field limits for interacting diffusions with colored noise: phase transitions and spectral numerical methods*, Multiscale Model. Simul., 18 (3), pp. 1343-1370 (2020).

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