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Three scale homogenization methods applied to cardiac bidomain model

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In our work, the three-scale homogenization methods are proposed to study the electrical behavior of the cardiac tissue structure with multiple heterogeneities at two different levels. The first level associated with the mesoscopic structure such that the cardiac tissue is composed at extracellular Ω_e and intracellular Ω_i domains separated by cellular membrane $\Gamma = \partial\Omega_e \cap \partial\Omega_i$. The second level associated with the microscopic structure in such a way that the intracellular medium can only be viewed as periodical layout of unit cells. We know that the cardiac tissue is viewed at macroscopic scale as a single domain to be the superposition of the intracellular and extracellular media. Finally, we obtain the macroscopic bidomain model independent on ε and δ describing the electrical behavior of the heart.

We consider the microscopic bidomain model which consists of two quasi-static equations, for the electrical potential in the extracellular medium and intracellular media, coupled through a dynamic boundary equation at the interface of these two media (the membrane Γ) depending on two small scaling parameters ε and δ:

\begin{equation}
\begin{aligned}
-\nabla_{x}\cdot\left(\mathrm{M}*{e}\left(x,\dfrac{x}{\varepsilon}\right)\nabla*{x} u_{e}^{\varepsilon} \right) &=0
&\text{ in } \Omega_{e}, &\quad \text{(extra quasi-stationnary conduction)}
\\ -\nabla_{x}\cdot\left(\mathrm{M}*{i}\left(x,\dfrac{x}{\varepsilon},\dfrac{x}{\varepsilon \delta}\right)\nabla*{x} u_{i}^{\varepsilon,\delta}\right
&=0 &\text{ in } \Omega_{i}, & \quad \text{(intra quasi-stationnary conduction)}
\\-\mathrm{M}*{i}^{\varepsilon,\delta}\nabla*{x} u_{i}^{\varepsilon,\delta} \cdot n_i=\mathrm{M}*{e}^{\varepsilon}\nabla*{x}
u_{e}^{\varepsilon} \cdot n_e & =\mathcal{I}*m &\ \text{on} \ \Gamma, & \quad \text{(continuity equation)}
\\ \varepsilon\left(\partial*{t} v_\varepsilon+\mathcal{I}*{ion}(v\varepsilon,w_\varepsilon)-\mathcal{I}*{app,\varepsilon}\right)
&=\mathcal{I}_m &\ \text{on} \ \Gamma, & \quad \text{(reaction onface condition)}
\\ \partial*{t} w_\varepsilon-H(v_\varepsilon,w_\varepsilon) &=0 & \text{ on } \Gamma,& \quad \text{(dynamic
coupling)}
\end{aligned}
\end{equation}
where the slow variable x describes the macroscopic scale, the fast variables x/ε describes the mesoscopic one while $x/\varepsilon\delta$ describes the microscopic one. Here, $\mathrm{M}_j,\ u_j$ are respectively the corresponding conductivities and electrical potentials of the cardiac tissue for $j = i, e$ and the \textit{transmembrane} potential $v = (u_i - u_e)_{|\Gamma}$. In addition, $\mathcal{I}_m$ represents the sum of all current densities across the membrane Γ.

The first homogenization method is based on a power series expansion which allows to determine the macroscopic (homogenized) bidomain model from the microscopic bidomain problem. First, we use the two-scale asymptotic expansion to homogenize the extracellular problem. Next, we apply a new three-scale asymptotic expansion in the intracellular problem to obtain its homogenzied equation at two levels. The second method based on unfolding operators which not only derive the homogenized equation but also prove the convergence and rigorously justify the mathematical writing of the preceding formal method.

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