

# **Mini-rencontre ANR MoDiff**

**Monday, October 26, 2020 - Wednesday, October 28, 2020**

**Labri**

## **Scientific Program**

**Nicolas Curien** : *Surfaces discrètes aléatoires*

À quoi ressemble une géométrie 2D aléatoire ? Un modèle très simple est obtenu en recollant (au hasard !)  $2n$  triangles le long de leur bords pour former une surface. Quand  $n$  est grand on peut se poser diverses questions :

- Quelle est le genre de la surface ? son nombre de sommets ? son diamètre ?
- Ces propriétés sont-elles universelles (au sens où les détails du modèle discret n'influent guère) ?
- Quid d'autres modèles de géométrie (hyperbolique) aléatoire ?

La première partie, introductive, sera destinée à un public large. Dans la seconde partie de cet exposé nous parlerons plus en détail des explorations Markoviennes (ou "peeling") qui sont un des outils privilégiés pour l'étude des propriétés géométriques de ces graphes aléatoires.

**Eduard Duryev** : *Masur-Veech volumes of strata with odd zeros and their friends*

I will review some steps of progress that we made with Elise towards computing Masur-Veech volumes of strata of quadratic differentials with only odd zeros and simple poles. Our approach follows the ideas of Delecroix-Goujard-Zograf-Zorich, which relate volumes of principal strata (only simple zeros and poles) to combinatorial moduli spaces. I will discuss some of the ingredients of their approach (Kontsevich polynomials, intersection numbers, ribbon graphs), how they interact and what are the obstacles we meet when we try to run the same ideas for higher orders of zeros.

**Mingkun Liu** : *Length statistics of random multicurves on closed hyperbolic surfaces*

Fix a topological type of a simple closed multicurve on a hyperbolic surface. There is a finite number of geodesic multicurves of total length at most  $L$  having this prescribed topological type. Letting  $L$  tend to infinity, one can collect asymptotic statistics of length distribution of components of such multicurves.

M. Mirzakhani found an explicit form of such length distribution in the case when the multicurve is a pants decomposition and showed that the length distribution does not depend on the hyperbolic metric. We generalize this result to arbitrary multicurves and illustrate it in the particular example of multicurves having two components. Similar results were independently obtained by F. Arana-Herrera.

**Duc-Manh Nguyen** : *Variation of Hodge structure and enumerating triangulations and quadrangulations of surfaces*

Since the work of Eskin-Okounkov (in 2001), it has been known that in any stratum of translation surfaces the number of square-tiled surfaces constructed from at most  $n$  squares grows like  $c\pi^{2g}n^d$ , where  $d$  is the (complex) dimension of the stratum,  $g$  is the genus of the surfaces, and  $c$  is a rational number. Similar phenomenon also occurs in strata of quadratic differentials. Counting square-tiled surfaces in a given stratum is more or less the same as counting quadrangulations of a topological surface, with some prescribed conditions on the singularities and the holonomy of the associated flat metric. More recently, Engel showed that the asymptotics of the numbers of quadrangulations and triangulations, satisfying some prescribed conditions at the singularities, with at most  $n$  tiles are of the form  $\alpha n^d$ , where  $\alpha$  is a constant in  $Q[\pi]$  or  $Q[\sqrt{3}\pi]$ .

In this talk, we will explain how the asymptotics above can be related to the computation of the volume of some moduli spaces, and how one can show that in some situations the constant  $\alpha$  belongs actually to either  $Q\pi^d$ , or  $Q(\sqrt{3}\pi)^d$  by using tools from complex algebraic geometry. This is joint work with Vincent Koziarz.

**Bram Petri** : *Random hyperbolic geometry*

Random constructions of (hyperbolic) manifolds can be used to study the shape of a "typical" manifold. Moreover, random manifolds are a good testing ground for conjectures that are still out of

reach and can also sometimes be used to prove the existence of manifolds with certain extremal properties. In the first of these talks I will give an overview of the different models of random manifolds that are out there and what we do and don't know about them, without assuming any a priori familiarity with hyperbolic geometry. In the second talk I will focus on a specific problem: finding hyperbolic surfaces of small diameter.

Projection de l'exposé BISTRO de **Laura Monk** : *Geometry and spectrum of random hyperbolic surfaces*

The aim of this talk is to describe the geometry and spectrum of most random hyperbolic surfaces, picked with the Weil-Petersson probability measure.

In this model, one can get a good understanding of the geometry of a typical surface: Cheeger constant, diameter (Mirzakhani), injectivity radius, number of short closed geodesics (Mirzakhani-Petri), length of the shortest non-simple closed geodesic, improved collar theorem (joint work with Joe Thomas), Benjamini-Schramm convergence.

I will explain how these geometric properties, together with the Selberg trace formula, lead to precise estimates on the distribution of the eigenvalues of the Laplacian on a typical surface.