Determinants

$$\begin{aligned}\det \mathcal{T}_{H_n} &\cong (\det \mathcal{F}_n^{\otimes -3} \otimes \det \mathcal{E}_n^{\otimes 2})^{\otimes 4} \otimes \det(\pi_{1*}\mathcal{O}_{\Pi_n}(1)))^{\otimes 6} \\ &\qquad \otimes \mathcal{O}_{H_n}(-6 \cdot H_n^{\mathrm{ncm}}) \otimes \mathcal{O}_{\mathrm{Gr}(4,n+1)}(n+1) \\ &= \mathcal{M}^{\otimes 2} \otimes \mathcal{O}_{\mathrm{Gr}(4,n+1)}(n+1)\end{aligned}$$

$$\begin{aligned}\det \mathcal{E}_{m,n} &= \det \mathcal{E}_n^{\otimes -\binom{m+1}{3}} \otimes \det \mathcal{F}_n^{\otimes \binom{m}{3}} \\ &\qquad \otimes \mathcal{O}_{H_n}(K_m H_n^{\mathrm{ncm}}) \otimes \mathcal{O}_{\mathrm{Gr}(4,n+1)}(M_m) \\ &= \mathcal{L}^{\otimes 2} \otimes \mathcal{O}_{H_n}(K_m H_n^{\mathrm{ncm}}) \otimes \mathcal{O}_{\mathrm{Gr}(4,n+1)}(M_m)\end{aligned}$$

with

$$M_m = \frac{(3m-1)m}{2},\ K_m = \frac{(3m-1)m}{2},\ 3m+1 = 4n.$$

$3m+1=4n$, n even, $1 \le i < j \le n/2$.

$$f_1(a_i,a_j) = \pm \left[\prod_{\ell=0}^{m-1} ((m-\ell)a_i - \ell a_j) \right] \cdot m!! \cdot a_j^{\frac{m+1}{2}}$$

$$\begin{aligned} g_1(a_i,a_j) = \pm 4a_i a_j (a_i - a_j)^2 (a_i + 3a_j)(a_i + a_j) a_i a_j \\ \cdot \prod_{\ell \ne i,j} (a_\ell^2 - a_i^2) \cdot \prod_{\ell \ne i,j} (a_\ell^2 - a_j^2). \end{aligned}$$

$$m!! := m \cdot (m-2) \cdots 3 \cdot 1$$

n	degree(s)	signature	rank
4	(5)	765	317206375
5	(3,3)	90	6424326
10	(13)	768328170191602020	794950563369917462703511361114326425387076
11	(3,11)	4407109540744680	3119084496832138244550288073698704091 6
11	(5,9)	313563865853700	16348587834933290273869035353880090 0
11	(7,7)	136498002303600	31226586782010349970656128100205356
12	(3,3,9)	43033957366680	35502236537604625191071472539252 04
12	(3,5,7)	5860412510400	6794415721803210746415212176890 0
12	(5,5,5)	1833366298500	680759542596051491774185981250 0