Applications of Hecke and related algebras: Representations, Integrability and Physics

February 26, 2023 - March 3, 2023

Program

	Monday	Tuesday	Wednesday	Thursday	Friday
9h - 10h20		M. De Visscher 2	P. Martin 2	M. De Visscher 3	P. Martin 3
10h40 - 12h	M. De Visscher 1 (10h-11h) E. Mukhin 1 (11h-12h)	E. Mukhin 2	C. Lecouvey 2	E. Mukhin 3	C. Lecouvey 3
	Lunch (12h30) + Break	Lunch (12h30) + Break	Lunch (12h30) + Break	Lunch (12h30) + Break	Lunch (12h30) *Departure (before 15:00)*
16h30 - 17h30	P. Martin 1	D. Chernyak J. Lamers	*Free Afternoon*	M. Nurcombe T. Pinet	
17h45 - 18h45	C. Lecouvey 1	M. Matushko B. Morris		S. Rostam F. Torzewska	
	Welcome Drink (19h00)	Dinner (19h30)	Dinner (19h30)	Dinner (19h30)	

Mini-courses

• **Maud de Visscher** (City University London)

Title: *Representations of centraliser algebras*

In this series of lecture, we will consider centraliser algebras of the action of classical groups (and their quantum analogues) on tensor spaces. The most studied and best understood is the Temperley–Lieb algebra and we will start by reviewing its representation theory. We will then turn to the Brauer, walled Brauer and partition algebra and explain how their representation theory can be studied in a uniform way by generalising the techniques used for the Temperley–Lieb algebra. We will show how their decomposition matrices can all be described by certain parabolic Kazhdan–Lusztig polynomials. These, in turn, have very nice combinatorial description in terms of oriented generalised Temperley–Lieb algebras.

• **Cédric Lecouvey** (Université de Tours)

Title: *The Kostka polynomials and their generalizations*

The topic of this mini-course will be focused on the notion of Kostka polynomial, a natural quantization of the Kostka numbers. These polynomials appear in many different contexts related to representation theory of symmetric groups (Green functions), Lie algebras (graded multiplicities), algebraic combinatorics (crystal graphs) and quantum groups (energy function and one-dimensional sums). After recalling their definition and certain of the main properties of the Kostka polynomials in type A, we shall introduce more recent results on their generalizations to the other root systems.

• **Paul Martin** (University of Leeds)

Title: *Statistical Mechanics, Representation Theory and diagram categories*

In this mini course we are interested in how computation in statistical mechanics, and representation theory, fit together. Each one informs and provides assistance for, and a certain path through, the other. A short course must lean heavily on examples, and this is where the diagram categories come in. For us, a diagram category is a subcategory of the partition category (of Potts model transfer matrix algebras), or a deformation thereof. So this can include Brauer categories, blob categories, Temperley–Lieb, and so on.

• **Evgeny Mukhin** (Indiana University)

Lecture 1. Gaudin models and Bethe ansatz.
I will define the Gaudin models and discuss a few basic properties. Then I will explain the Bethe ansatz - the method invented by physicists to look for the eigenvalues and eigenvectors of the Gaudin Hamiltonians. I will discuss the power and limitations of the Bethe ansatz method.

Lecture 2. The $gl(m)$-$gl(n)$ duality of Gaudin models.
I will briefly discuss the classical Schur-Weyl and $gl(n)$-$gl(m)$ dualities and then prove the corresponding statement for the Gaudin models. The proof is reduced to a generalization of the famous Schur formula in linear algebra to the case of particular matrices with non-commuting entries. Time permitting, I will explain the relation to the property of bispectrality in the KP hierarchy.

Lecture 3. The Gaudin models and the Schubert Calculus.
I will use the Bethe ansatz to explain that the algebra of $gl(n)$ Gaudin Hamiltonians acting in a module coincides with the corresponding scheme theoretic intersection of Schubert varieties in a Grassmannian. This can be thought of as a version of Geometric Langlands duality. I will discuss the implications of this duality for both geometry and integrable systems.

Prerequisites. I will do my best to make the lectures self-contained. However, to follow comfortably, I suggest refreshing the basics of $gl(n)$ representation theory.

Short Talks

• **Dmitry Chernyak** (LPENS)

Title: *$U_q sl_2$-invariant non-compact boundary conditions for the XXZ spin chain*

Using the $U_q sl_2$ symmetry of the open XXZ spin chain, we introduce new boundary conditions by coupling the bulk Hamiltonian to an infinite-dimensional Verma module on one or both boundaries. We show that for generic values of the parameters the new boundary coupling provides a faithful representation of the blob algebra which is Schur–Weyl dual to the $U_q sl_2$ action. Modifying the boundary conditions on both the left and the right, we obtain a representation of the (universal) two-boundary Temperley–Lieb algebra. These representations then naturally define various boundary loop models on the lattice. As an example, we specialise the deformation parameter to $q = i$ where the model can be explicitly solved by free fermions and compute its conformal scaling limit. If time permits, we will explain how to solve this model and obtain its conformal scaling limit for arbitrary q using Bethe ansatz. Based on joint work with A. Gainutdinov, J. Jacobsen and H. Saleur (2207.12772 + unpublished work).

• **Jules Lamers** (IPhT Saclay)

Title: *Hecke algebras, spin-Macdonald model and long-range spin chains*

Affine Hecke algebras are intimately related to Macdonald polynomials. I will discuss a generalisation including spins that naturally arises from a quantum-affine version of Schur–Weyl duality. The resulting Ruijsenaars–Macdonald-type model is integrable, and in a special limit gives rise to an integrable long-range spin chain known as the q-deformed Haldane–Shastry spin chain. I will review joint work with Vincent Pasquier and Didina Serban, as well as new results.

• **Maria Matushko** (Stekhlov Mathematical Institute)

Title: *Anisotropic spin generalization of elliptic Macdonald–Ruijsenaars operators and R-matrix identities*

We propose commuting set of matrix-valued difference operators in terms of the elliptic Baxter–Belavin R-matrix in the fundamental representation of gl_M. In the scalar case $M = 1$ these operators are the elliptic Macdonald–Ruijsenaars operators, while in the general case they can be viewed as anisotropic versions of the quantum spin Ruijsenaars Hamiltonians. In scalar case Ruijsenaars proved that commutativity of the operators written in the form with arbitrary function is equivalent to the system of functional equations and the elliptic Kronecker function solves this system. We show that commutativity of the operators for any M is equivalent to a set of R-matrix identities. The proof of identities is based on the properties of elliptic R-matrix including the quantum and the associative Yang–Baxter equations. We also discuss the trigonometric degenerations and some applications to long-range spin chains of Haldane–Shastry type. Based on joint works with Andrei Zotov arxiv:2201.05944 and arxiv:2202.01177.

• **Benjamin Morris** (University of Leeds)

Title: *Towards a factorised solution of the Yang–Baxter equation with $U_q(\mathfrak{sl}_n)$-symmetry*

I will discuss the parameter permutation method for constructing factorised solutions of the Yang–Baxter equation related to the algebras $\mathcal{A} = U(\mathfrak{sl}_n)$, or $U_q(\mathfrak{sl}_n)$. Solutions are R-matrices which act in the tensor product of differential or q-difference representations of $\mathcal{A}$ on a complex function space in $n(n-1)/2$ variables. The defining relation for R can be interpreted as a permutation condition allowing for its factorisation into elementary transposition operators. We will focus on some partial results in the q-deformed case. Firstly, we attempt to write a factorised L-operator that generalises a known expression for the undeformed case, however, by studying the $n = 4$ case we will see this fails in general. Secondly, we obtain most transposition operators for the $U_q(\mathfrak{sl}_n)$ L-operator and prove that they obey the necessary symmetric group relations. This proof introduces a terminating q-series identity between q-Lauricella series.

• **Madeline Nurcombe** (University of Queensland)

Title: *The Ghost Algebra*

The Temperley–Lieb algebra has many diverse applications in mathematics, from physical models of polymers and percolation in statistical mechanics, to knot theory. It is also a diagram algebra; its basis elements can be expressed as string diagrams, which are multiplied by concatenation. The one-boundary Temperley–Lieb algebra is similar, but its basis diagrams have an additional boundary line that the strings may be connected to. There is also a two-boundary Temperley–Lieb algebra, with a second boundary, but its diagrams require an even number of strings connected to each boundary, and it is infinite-dimensional, unlike the zero- and one-boundary algebras. In this talk, I will introduce the ghost algebra, a finite-dimensional two-boundary version of the Temperley–Lieb algebra that allows diagrams with odd numbers of connections to each boundary. Its diagrams contain ghosts: dots on the boundaries that act as bookkeeping devices to ensure associativity of multiplication. I will discuss some structural properties of this algebra, and how they relate to its representation theory.

- **Théo Pinet** (Université de Paris - Université de Montréal)

Title: *Spin chains as modules over the affine Temperley–Lieb algebra*

There are natural commuting actions of the Temperley–Lieb algebra $TL_N(q)$ and of the quantum group $U_q(sl_2)$ on the open XXZ spin-$\frac{1}{2}$ chains. This phenomenon, called quantum Schur–Weyl duality, ceases however to hold for periodic chains where the action of $TL_N(q)$ is replaced by an action of the affine Temperley–Lieb algebra $aTL_N(q)$. In this talk, we will nevertheless recover part of the duality by showing that all $aTL_N(q)$-morphisms between periodic chains are induced by the action of specific divided powers inside $U_q(sl_2)$ (for q is a root of unity). This enables us in particular to obtain the structure of the periodic spin chains as $aTLN$-modules and to obtain an explicit realization of the morphisms between these chains. Joint work with Yvan Saint-Aubin.

- **Salim Rostam** (ENS Rennes)

Title: *Skew cellularity of the Hecke algebras of complex reflection groups*

In 1996, Graham and Lehrer introduced the notion of cellular algebra. Graham and Lehrer proved that many known algebras are cellular, for instance: the Ariki-Koike algebra (this includes the Iwahori-Hecke algebras of type A and B), the Brauer algebra, the Temperley–Lieb algebra. Namely, the latter algebra has been introduced in connection with statistical mechanics. Later, using some deep properties of Kazhdan–Lusztig bases, Geck proved that the Iwahori–Hecke algebra associated with any finite Coxeter group (or, equivalently, any real reflection group) is cellular. In this talk, I will introduce the notion of skew cellular algebra that we developed in a joint work with Jun Hu and Andrew Mathas. This notion generalises Graham and Lehrer's definition of cellular algebra; our main result is that the Hecke algebra associated with any irreducible complex reflection group $G(r, p, n)$ of the infinite series (that is, all but finitely many irreducible complex reflection groups) is a skew cellular algebra.

- **Fiona Torzewska** (University of East Anglia)

Title: *Classification of charge conserving loop-braid representations*

(w/ Eric Rowell, Paul Martin) A loop-braid representation is a monoidal functor from the loop-braid category LB. Given a monoidal category C, a rank-N charge-conserving representation (or *spin-chain* representation) is a monoidal functor from C to the category MatchN of rank-N charge-conserving matrices. In this work we construct all charge-conserving loop braid representations, and classify up to suitable notions of isomorphism.