Numerical algorithm for two-phase flows drift-flux model in a porous media on a staggered grid

Anouar MEKKAS [1]
Sami KOURAICHI [2]

anouar.mekkas@cea.fr
sami.kouraichi@math.univ-paris13.fr

(1) CEA PARIS-SACLAY
(2) PARIS 13 UNIVERSITY, SUPGALILÉE

web

Context

FLICA4 is a 3D compressible code specially devoted to reactor core analysis which solves a compressible drift-flux model for two-phase flows in a porous medium [1]. To define convective fluxes, FLICA4 uses a specific finite volume numerical method based on an extension of the Roe's approximate Riemann colocated solver [2]. Nevertheless, an analysis of this type of method shows that in low-Mach number, it is necessary to apply modifications to the 2D or 3D geometries on a cartesian mesh otherwise this method does not converge to the right solution when the mach number tends to zero [3]. For this reason, we apply a so-called "pressure correction". Although this correction is necessary to reach the required precision, it may produces some checkerboard oscillations in space, especially in the 1D case.

Since these checkerboard oscillations are sometimes critical and may lead to unstable resolutions or even divergence in some cases, we also investigate another numerical algorithm to solve this compressible drift-flux model in the low Mach regime. The key point is to develope a compressible solver on staggered grid since checkerboard oscillations cannot exist on this type of discretisation. The aim of this work is to present such a compressible scheme and to validate it in low Mach regime with test cases describing a simplified nuclear core.

Four equation Drift-flux model in porous medium

The four equations model $(\mathcal{M})$ is established from the six equations equations model in porous medium [1]. A Drift-flux model is used to take into account the slip between the vapor and the liquid phase.

$$(\mathcal{M})\begin{cases}\phi\dfrac{\partial\rho}{\partial t}+\nabla\cdot(\phi\rho\vec{V})=0\\ \phi\dfrac{\partial(\rho C)}{\partial t}+\nabla\cdot(\phi\rho C\vec{V})+\nabla\cdot(\phi\rho C(1-C)\vec{V}_r)-\nabla\cdot(\phi K_{cv}\nabla C)=\phi(\Gamma_{lv}+\Gamma_{wv})\\ \phi\rho\left(\dfrac{\partial\vec{V}}{\partial t}+\vec{V}\nabla\vec{V}\right)+\nabla\cdot\left(\phi\rho C(1-C)\vec{V}_r\otimes\vec{V}_r\right)+\phi\nabla P=\nabla\cdot(\bar{\bar{\sigma}})+\phi(\tau_s+\tau_w)+\phi\rho\vec{g}\\ \phi\dfrac{\partial(\rho e)}{\partial t}+\nabla\cdot\left(\phi\rho e\vec{V}\right)+P\nabla\cdot\left(\phi\vec{V}\right)+\nabla\cdot\left(\phi\rho C(1-C)(H_v-H_l)\vec{V}_r\right)=\phi Q+\nabla\cdot(q)\end{cases}$$

➢ **Unknows:**

- $\vec{V}$: mixture velocity
- $C=\frac{\alpha_v\rho_v}{\rho}$: concentration
- P: mixture pressure
- e or h: internal mixture energy or specific mixture enthalpy

➢ **Given quantities:**

- ϕ: porosity
- $\vec{g}$: gravity
- Q: power density
- K (τ_s): singular pressure loss coefficient

➢ **Simplifying hypothesis (neglected terms):**

- Γ_{wv}: vapor production at the heating surfaces,
- τ_w: friction between the wall and the fluid,
- K_{cv}: phase mass diffusion term due to two-phase flow turbulence,
- $\bar{\bar{\sigma}}$: viscous stress tensor and turbulence effects modeling,
- q: fluid heat conduction and energy turbulence diffusion terms.

➢ **Closure models:**

- Equation of state: Stiffened gas EOS [5] $\Rightarrow\mathcal{E}(P,\rho,h$ ou $e)=0$,
- Non-equilibrium thermodynamics: the vapor phase is assumed to be saturated $\Rightarrow h_v(\rho_v,P)=h_v^{sat}(P)$,
- Non-equilibrium Kinematics: the relative velocity between liquid and vapor phases is taken into account by a kinetic constitutive equation $\Rightarrow\vec{V}_r=\vec{V}_v-\vec{V}_l=\vec{V}_r(\vec{V},h,C,P)$. Here, we choose Ishii drift-model [4].
- Interphase mass exchange Γ_{lv}: we choose F3 model [1].

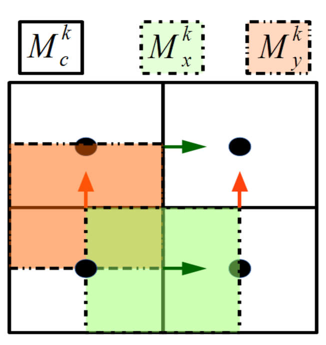
➢ **Boundary conditions:**

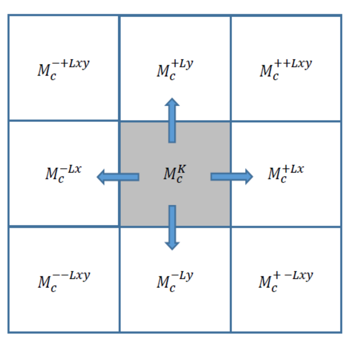
- Inlet: concentration, velocity and specific enthalpy,
- Outlet: pressure.

Time-Space discretization

STAGGERED GRID

A principal grid is used for scalar transport equations (mass, internal energy, vapor mass). The so-called scalar variables are defined at the center of every mesh cell: pressure, internal energies, concentration. Three other grids are used, one for each velocity component. Velocity nodes are located on the center of the faces normal to the velocity component.





$\mathcal{X}\in\{x,y,z\}$
$\mathbb{V}_{M_{\mathcal{X}}^k}=(\phi\Delta x\Delta y\Delta z)_{M_{\mathcal{X}}^k}$
σ: common interface between two neighboring cells
$\mathbb{S}_\sigma$: surface of σ

In the equations of mixture momentum, we use the centered value for the evaluation of scalar variables to the faces. To ensure the stability of the numerical scheme, we use a donor (upwind) scheme approximation in each convection terms for each balance equations.

➢ **MIXTURE MASS** F^1: $\frac{\rho^{n+1}_{M_c^K}-\rho^n_{M_c^K}}{\Delta t}+\frac{1}{\mathbb{V}_{M_c^K}}\sum_{\sigma\in\varepsilon_K}\mathscr{F}_\sigma^{n+1}=0$

$$\mathscr{F}_\sigma^{n+1}=\mathbb{S}_\sigma\rho_\sigma^n\vec{V}_\sigma^{n+1}.\vec{n}_\sigma$$

➢ **VAPOR MASS** F^2: $\frac{(\rho C)^{n+1}_{M_c^K}-(\rho C)^n_{M_c^K}}{\Delta t}+\frac{1}{\mathbb{V}_{M_c^K}}\sum_{\sigma\in\epsilon_K}\left[\mathscr{F}_\sigma^{n+1}+\mathscr{G}_\sigma^{n+1}\right]-(\Gamma_{lv})^{n+1}_{M_c^K}=0$

$$\mathscr{F}_\sigma^{n+1}=\mathbb{S}_\sigma[\rho C]_\sigma^n\vec{V}_\sigma^{n+1}.\vec{n}_\sigma\qquad\mathscr{G}_\sigma^{n+1}=\mathbb{S}_\sigma[\rho_r C_r(1-C_r)]_\sigma^n(\vec{V}_r)_\sigma^{n+1}.\vec{n}_\sigma$$

➢ **MIXTURE INTERNAL ENERGY** F^3: $\frac{(\rho e)^{n+1}_{M_c^K}-(\rho e)^n_{M_c^K}}{\Delta t}+\frac{1}{\mathbb{V}_{M_c^K}}\sum_{\sigma\in\epsilon_K}\left[\mathscr{F}_\sigma^{n+1}+P^{n+1}_{M_c^K}\mathscr{G}_\sigma^{n+1}+\mathscr{H}_\sigma^{n+1}\right]-Q^n_{M_c^K}=0$

$$\mathscr{F}_\sigma^{n+1}=\mathbb{S}_\sigma[\rho e]_\sigma^n\vec{V}_\sigma^{n+1}.\vec{n}_\sigma\qquad\mathscr{G}_\sigma^{n+1}=\mathbb{S}_\sigma\vec{V}_\sigma^{n+1}.\vec{n}_\sigma\qquad\mathscr{H}_\sigma^{n+1}=\mathbb{S}_\sigma[\rho_r C_r(1-C_r)(H_v-H_l)]_\sigma^n(\vec{V}_r)_\sigma^{n+1}.\vec{n}_\sigma$$

➢ **MIXTURE MOMENTUM** F^4,F^5 and F^6: $\rho^n_{M_{\mathcal{X}}^K}\frac{(V^{\mathcal{X}})^{n+1}_{M_{\mathcal{X}}^K}-(V^{\mathcal{X}})^n_{M_{\mathcal{X}}^K}}{\Delta t}+\frac{1}{\mathbb{V}_{M_{\mathcal{X}}^K}}\sum_{\sigma\in\varepsilon_K}\left[\mathscr{F}_\sigma^{n+1}-(V^{\mathcal{X}})^{n+1}_{M_{\mathcal{X}}^K}\mathscr{G}_\sigma^{n+1}+\mathscr{H}_\sigma^{n+1}\right]+\left[\mathbb{S}_{M_c^{+Lx}}P^{n+1}_{M_c^{+L\mathcal{X}}}-\mathbb{S}_{M_c^K}P^{n+1}_{M_c^K}\right]-(\rho g^{\mathcal{X}})^n_{M_{\mathcal{X}}^K}+\frac{K^{\mathcal{X}}}{2}\rho^n_{M_{\mathcal{X}}^K}(V^{\mathcal{X}})^{n+1}_{M_{\mathcal{X}}^K}|(V^{\mathcal{X}})^{n+1}_{M_{\mathcal{X}}^K}|=0$

$$\mathscr{F}_\sigma^{n+1}=\mathbb{S}_\sigma\rho_\sigma^n(V^{\mathcal{X}})_\sigma^{n+1}\vec{V}_\sigma^{n+1}.\vec{n}_\sigma\qquad\mathscr{G}_\sigma^{n+1}=\mathbb{S}_\sigma\rho_\sigma^n\vec{V}_\sigma^{n+1}.\vec{n}_\sigma\qquad\mathscr{H}_\sigma^{n+1}=\mathbb{S}_\sigma(\rho_r C_r(1-C_r))_\sigma^n(V_r^{\mathcal{X}})_\sigma^{n+1}(\vec{V}_r)_\sigma^{n+1}.\vec{n}_\sigma$$

Solution algorithm

Let (S) denote the non linear system we ought to solve at each physical time step. This system is linearized using a Newton-Raphson iterative method. This method gives a linear system of equations for the increments of the principal variables $\mathcal{U}=(P,h,C,\vec{V})^t$:

$$(S)\begin{cases}F^1(P,h,\vec{V})=0\\F^3(P,h,\vec{V})=0\\F^2(P,h,C,\vec{V})=0\\F^4(P,\vec{V})=0\\F^5(P,\vec{V})=0\\F^6(P,\vec{V})=0\end{cases}\Rightarrow\underbrace{\begin{pmatrix}\frac{\partial F^1}{\partial P}&\frac{\partial F^1}{\partial h}&\frac{\partial F^1}{\partial C}&\frac{\partial F^1}{\partial V^x}&\frac{\partial F^1}{\partial V^y}&\frac{\partial F^1}{\partial V^z}\\\frac{\partial F^2}{\partial P}&\frac{\partial F^2}{\partial h}&\frac{\partial F^2}{\partial C}&\frac{\partial F^2}{\partial V^x}&\frac{\partial F^2}{\partial V^y}&\frac{\partial F^2}{\partial V^z}\\\frac{\partial F^3}{\partial P}&\frac{\partial F^3}{\partial h}&\frac{\partial F^3}{\partial C}&\frac{\partial F^3}{\partial V^x}&\frac{\partial F^3}{\partial V^y}&\frac{\partial F^3}{\partial V^z}\\\frac{\partial F^4}{\partial P}&\frac{\partial F^4}{\partial h}&\frac{\partial F^4}{\partial C}&\frac{\partial F^4}{\partial V^x}&\frac{\partial F^4}{\partial V^y}&\frac{\partial F^4}{\partial V^z}\\\frac{\partial F^5}{\partial P}&\frac{\partial F^5}{\partial h}&\frac{\partial F^5}{\partial C}&\frac{\partial F^5}{\partial V^x}&\frac{\partial F^5}{\partial V^y}&\frac{\partial F^5}{\partial V^z}\\\frac{\partial F^6}{\partial P}&\frac{\partial F^6}{\partial h}&\frac{\partial F^6}{\partial C}&\frac{\partial F^6}{\partial V^x}&\frac{\partial F^6}{\partial V^y}&\frac{\partial F^6}{\partial V^z}\end{pmatrix}}_{\mathcal{A}(\mathcal{U}^k)\,:\,\text{jacobian of nonlinear system }(S)}\underbrace{\begin{pmatrix}\Delta\mathcal{U}_1\\\Delta\mathcal{U}_2\\\Delta\mathcal{U}_3\\\Delta\mathcal{U}_4\\\Delta\mathcal{U}_5\\\Delta\mathcal{U}_6\end{pmatrix}}_{\Delta\mathcal{U}^{k+1}\,:\,\text{increments}}=\underbrace{\begin{pmatrix}\mathcal{S}_1=-F^1\\\mathcal{S}_2=-F^2\\\mathcal{S}_3=-F^3\\\mathcal{S}_4=-F^4\\\mathcal{S}_5=-F^5\\\mathcal{S}_6=-F^6\end{pmatrix}}_{\mathcal{S}(\mathcal{U}^n,\mathcal{U}^k)\,:\,\text{residuals}}.$$

• **Step 1: Eliminating the velocity increments**
We consider vector (momentum) balance equations of system (S):

➢ $F^4(P^{n+1}_{\mathscr{M}_c^K},P^{n+1}_{\mathscr{M}_c^{+Lx}},(V^x)^{n+1}_{\mathscr{M}_x^K})=0$
➢ $F^5(P^{n+1}_{\mathscr{M}_c^K},P^{n+1}_{\mathscr{M}_c^{+Ly}},(V^y)^{n+1}_{\mathscr{M}_y^K})=0$
➢ $F^6(P^{n+1}_{\mathscr{M}_c^K},P^{n+1}_{\mathscr{M}_c^{+Lz}},(V^z)^{n+1}_{\mathscr{M}_z^K})=0$

$$\Rightarrow\begin{pmatrix}\frac{\partial F^4}{\partial P}&0&0&\frac{\partial F^4}{\partial V^x}&0&0\\\frac{\partial F^5}{\partial P}&0&0&0&\frac{\partial F^5}{\partial V^y}&0\\\frac{\partial F^6}{\partial P}&0&0&0&0&\frac{\partial F^6}{\partial V^z}\end{pmatrix}\begin{pmatrix}\Delta\mathcal{U}_1\\\Delta\mathcal{U}_2\\\Delta\mathcal{U}_3\\\Delta\mathcal{U}_4\\\Delta\mathcal{U}_5\\\Delta\mathcal{U}_6\end{pmatrix}=\begin{pmatrix}S^4\\S^5\\S^6\end{pmatrix}$$

$$\Rightarrow\begin{cases}(\Delta V^x)_{M_x^K}=\frac{1}{\frac{\partial F^4_{M_x^K}}{\partial V^x_{M_x^K}}}[S^4_{M_x^K}-\frac{\partial F^4_{M_x^K}}{\partial P_{M_c^K}}(\Delta P)_{M_c^K}-\frac{\partial F^4_{M_x^K}}{\partial P_{M_c^{+Lx}}}(\Delta P)_{M_c^{+Lx}}]\\(\Delta V^y)_{M_y^K}=\frac{1}{\frac{\partial F^5_{M_y^K}}{\partial V^y_{M_y^K}}}[S^5_{M_y^K}-\frac{\partial F^5_{M_y^K}}{\partial P_{M_c^K}}(\Delta P)_{M_c^K}-\frac{\partial F^5_{M_y^K}}{\partial P_{M_c^{+Ly}}}(\Delta P)_{M_c^{+Ly}}]\\(\Delta V^z)_{M_y^K}=\frac{1}{\frac{\partial F^6_{M_y^K}}{\partial V^z_{M_y^K}}}[S^6_{M_y^K}-\frac{\partial F^6_{M_y^K}}{\partial P_{M_c^K}}(\Delta P)_{M_c^K}-\frac{\partial F^6_{M_y^K}}{\partial P_{M_c^{+Lz}}}(\Delta P)_{M_c^{+Lz}}]\end{cases}\quad(1)$$

By analogy, we obtain $(\Delta V^x)_{M_x^{-Lx}}$, $(\Delta V^y)_{M_y^{-Ly}}$ et $(\Delta V^z)_{M_z^{-Lz}}$.

• **Step 2: Pressure solver**
We consider scalar balance equations of system (S):

➢ $F^1(P^{n+1}_{\mathscr{M}_c^K},h^{n+1}_{\mathscr{M}_c^K},\vec{V}_\sigma^{n+1})=0$
➢ $F^2(P^{n+1}_{\mathscr{M}_c^K},h^{n+1}_{\mathscr{M}_c^K},C^{n+1}_{\mathscr{M}_c^K},\vec{V}_\sigma^{n+1})=0$
➢ $F^3(P^{n+1}_{\mathscr{M}_c^K},h^{n+1}_{\mathscr{M}_c^K},\vec{V}_\sigma^{n+1})=0$

$$\Rightarrow\begin{pmatrix}\frac{\partial F^1}{\partial P}&\frac{\partial F^1}{\partial h}&0&\frac{\partial F^1}{\partial V^x}&\frac{\partial F^1}{\partial V^y}&\frac{\partial F^1}{\partial V^z}\\\frac{\partial F^2}{\partial P}&\frac{\partial F^2}{\partial h}&\frac{\partial F^2}{\partial C}&\frac{\partial F^2}{\partial V^x}&\frac{\partial F^2}{\partial V^y}&\frac{\partial F^2}{\partial V^z}\\\frac{\partial F^3}{\partial P}&\frac{\partial F^3}{\partial h}&0&\frac{\partial F^3}{\partial V^x}&\frac{\partial F^3}{\partial V^y}&\frac{\partial F^3}{\partial V^z}\end{pmatrix}\begin{pmatrix}\Delta\mathcal{U}_1\\\Delta\mathcal{U}_2\\\Delta\mathcal{U}_3\\\Delta\mathcal{U}_4\\\Delta\mathcal{U}_5\\\Delta\mathcal{U}_6\end{pmatrix}=\begin{pmatrix}S_1\\S_2\\S_3\end{pmatrix}\quad(2)$$

At this stage applying the operations $(L^1)\longleftarrow(L^1)\times\frac{\partial F^2}{\partial h}-(L^3)\times\frac{\partial F^1}{\partial h}$ and then $(L^2)\leftrightarrow(L^3)$ on the system (2) results in:

$$\begin{pmatrix}J^{1,1}&0&0&J^{1,4}&J^{1,5}&J^{1,6}\\\frac{\partial F^3}{\partial P}&\frac{\partial F^3}{\partial h}&0&\frac{\partial F^3}{\partial V^x}&\frac{\partial F^3}{\partial V^y}&\frac{\partial F^3}{\partial V^z}\\\frac{\partial F^2}{\partial P}&\frac{\partial F^2}{\partial h}&\frac{\partial F^2}{\partial C}&\frac{\partial F^2}{\partial V^x}&\frac{\partial F^2}{\partial V^y}&\frac{\partial F^2}{\partial V^z}\end{pmatrix}\begin{pmatrix}\Delta\mathcal{U}_1\\\Delta\mathcal{U}_2\\\Delta\mathcal{U}_3\\\Delta\mathcal{U}_4\\\Delta\mathcal{U}_5\\\Delta\mathcal{U}_6\end{pmatrix}=\begin{pmatrix}D^1=S_1\frac{\partial F^2}{\partial h}-S_2\frac{\partial F^1}{\partial h}\\S_3\\S_2\end{pmatrix}\quad(3)$$

We develop the first row of the system (3), we get the following equation:

$$(J^{1,1})_{M_c^K}(\Delta P)_{M_c^K}+(J^{1,4})_{M_x^K}(\Delta V^x)_{M_x^K}+(J^{1,4})_{M_x^{-Lx}}(\Delta V^x)_{M_x^{-Lx}}+(J^{1,5})_{M_y^K}(\Delta V^y)_{M_y^K}+(J^{1,5})_{M_y^{-Ly}}(\Delta V^y)_{M_y^{-Ly}}$$
$$+(J^{1,6})_{M_z^K}(\Delta V^z)_{M_z^K}+(J^{1,6})_{M_z^{-Lz}}(\Delta V^z)_{M_z^{-Lz}}=D^1_{M_c^K}\quad(4)$$

We make use of the velocity increments calculated in the last step. Their integration in the equation (4) gives us:

$$\mathbb{A}(\Delta P)_{M_c^K}+\mathbb{B}(\Delta P)_{M_c^{+Lx}}+\mathbb{C}(\Delta P)_{M_c^{-Lx}}+\mathbb{D}(\Delta P)_{M_c^{+Ly}}+\mathbb{E}(\Delta P)_{M_c^{-Ly}}$$
$$+\mathbb{F}(\Delta P)_{M_c^{+Lz}}+\mathbb{G}(\Delta P)_{M_c^{-Lz}}=\mathbb{S}\quad(5)$$

• **Step 3: Other increments**

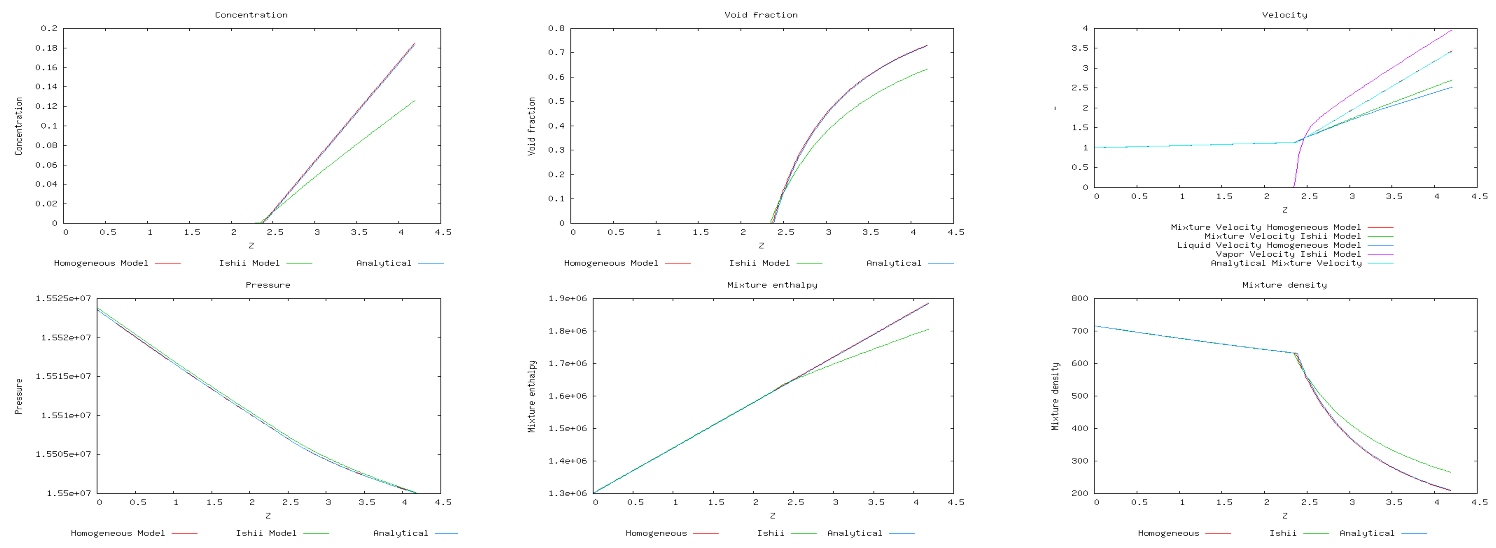
➢ **Velocity increments** As a solution of the pressure equation (5), the pressure increments will be used to compute the velocity increments (step 1).

➢ **Enthalpy increments** To compute the enthalpy increments we need is to develop the second row of the system (3).

➢ **Concentration increments** The same method as above is applied to calculate the concentration increments that can be computed using the third row of the system (3).

Numerical tests

The physical quantities that we use in these tests match the functioning of the Pressurized Water Reactors. We consider a 4.2 meter long channel heated by a uniform thermal flux $Q=1.E8\ w/m^3$ on which we impose the following conditions: inlet concentration $C_i=0$, inlet enthalpies $h_l=1.3E^6\ J/kg$ and $h_v=2.6E^6\ J/kg$, inlet velocities $u_l=u_v=1.0\ m/s$ and outlet pressure $P_s=155\ bar$. In the figure below, we compare the results of homogeneous and Ishii models to an analytical solution obtained with low Mach mixture model [5].



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