Towards Tensor Models for Quantum Gravtiy

Johannes Lumma

10.06.2020

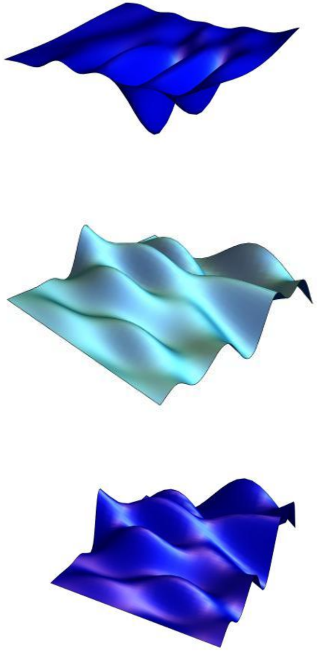
“Universal critical behaviour in tensor models for four-dimensional quantum gravity”
JHEP 2002 (2020) 110

In collab with: A. Eichhorn, A. D. Pereira Jr & A. Sikandar

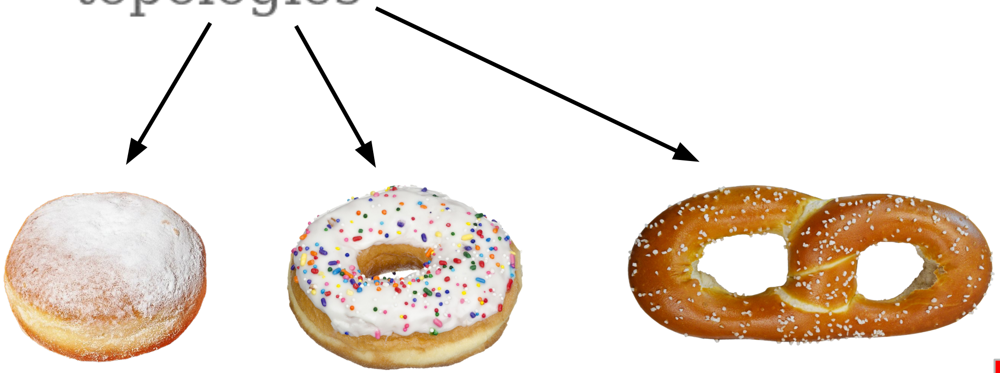
UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Goal: Make sense of



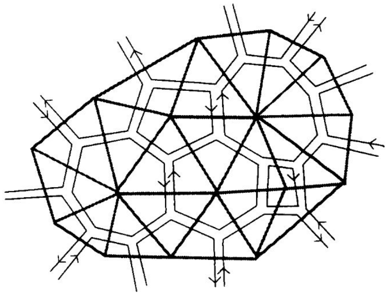
$$\sum_{\text{topologies}} \int [\mathcal{D}g][\mathcal{D}X]\, e^{-S_{\text{EH}}-S_{\text{SM}}-S_{\text{other}}}$$



challenging
(using traditional tools)

Below Planck scale:
Cosmological perturbation theory &
EFT formulation of QG by Donoghue et al.

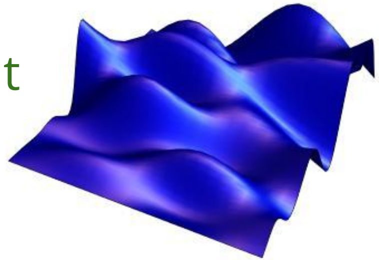
A discrete strategy



Discretize the continuum Path integral

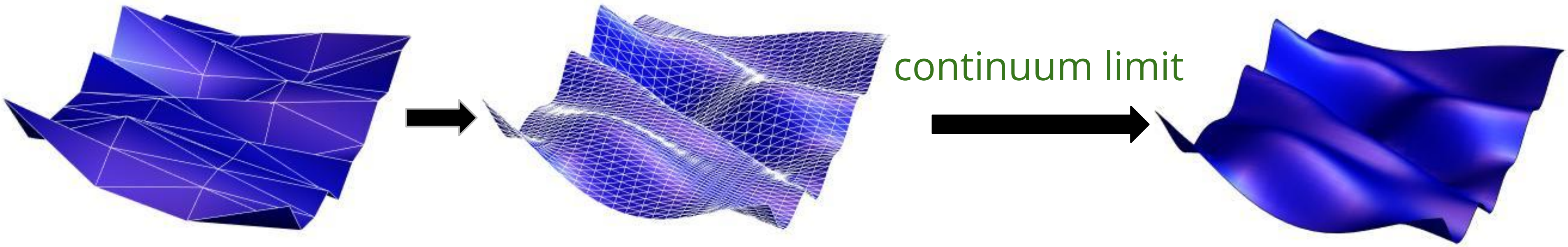
$$\sum_{\text{topologies}} \int [\mathcal{D}g] \longrightarrow \sum_{\text{triangulations}}$$

continuum limit



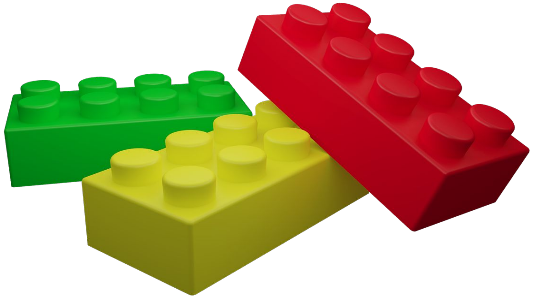
A discrete strategy

Discretize the continuum Path integral

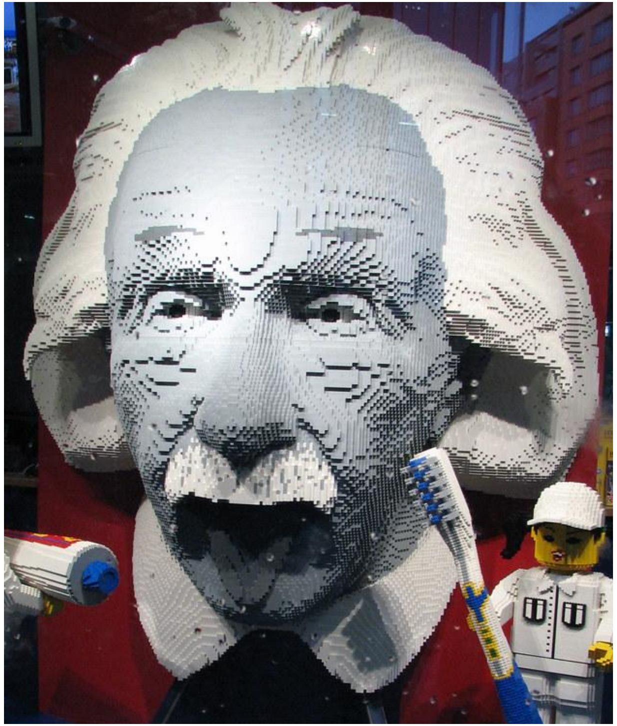


A discrete strategy

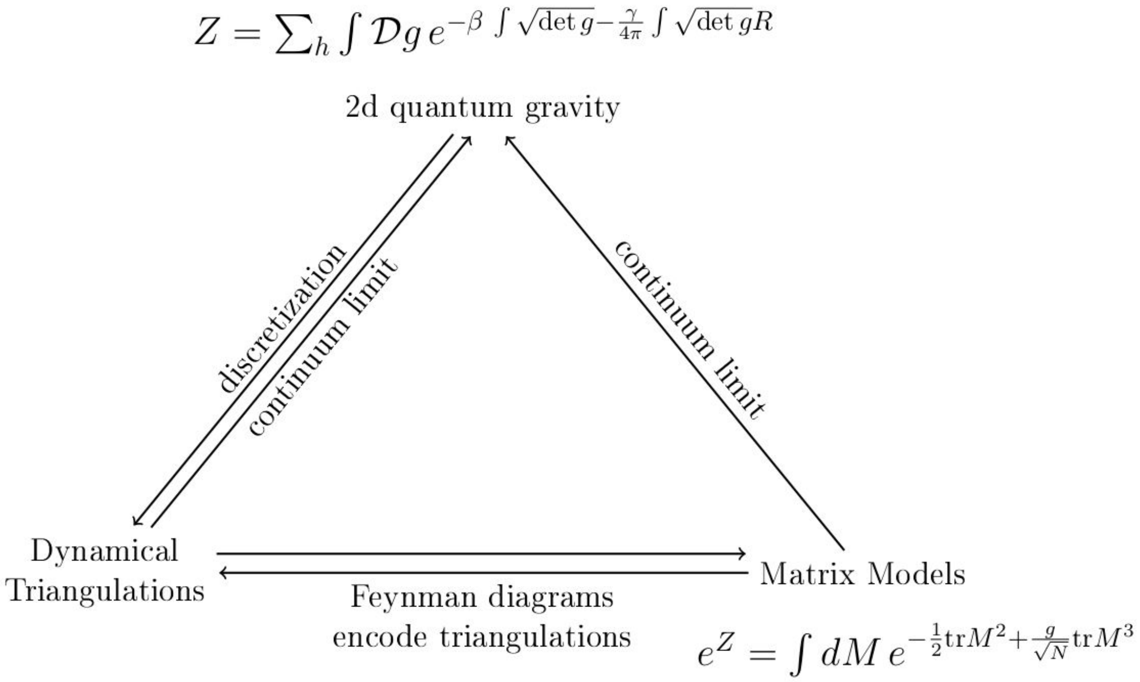
"Gas Phase"



"Fluid Phase"

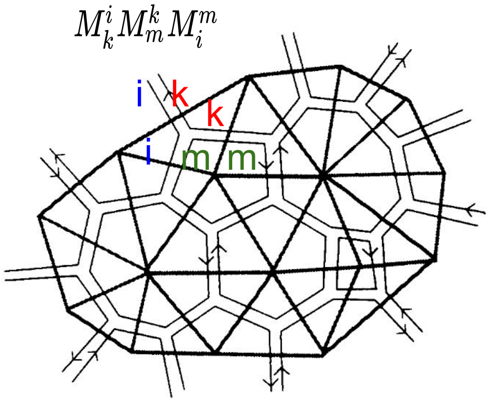


Start with 2D: The "holy trinity"



Start with 2D: Random Matrices

- Random Matrices are dual to triangulations in 2D



Picture from 2D Gravity and Random Matrices, Di Francesco, Ginsparg, Zinn-Justin

2D Euclidean QG

$$\mathcal{Z} = \sum_h \int \mathcal{D}g\, e^{-\beta A + \gamma \chi} \quad (1)$$

Sum over topologies = Sum over handles h

A=Area = $\int \sqrt{g}$

$\chi = 2 - 2h$ is the Euler character

Random Matrices

$$e^{\mathcal{Z}} = \int dM\, e^{-\frac{1}{2} trM^2 + \frac{g}{\sqrt{N}} trM^3} \quad (2)$$

$M \to \frac{M}{\sqrt{N}}$: $N = e^{\gamma}$ and $g = e^{-\beta}$

$\Longrightarrow$ Continuum Limit is 2D Eucl. QG

How to take the continuum limit ?

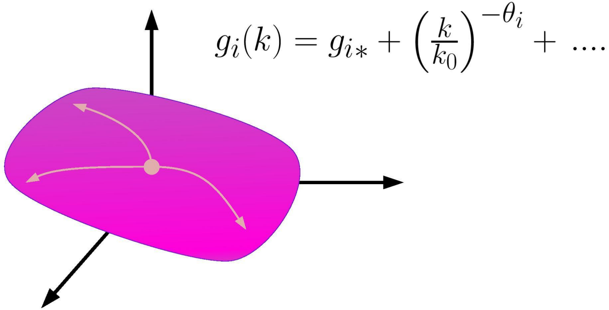
Double-scaling limit

- Continuum limit from double-scaling limit (=contributions from all topol.)

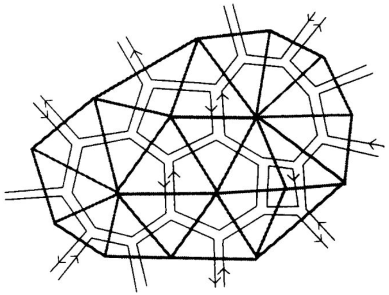
$$(g-g_c)^{\frac{1}{\theta}}\, N = N_0$$

- Linearized "RG Flow" in matrix size N [Brezin, Zinn-Justin '92] [Eichhorn, Koslowski '13]

$$g = g_c + \left(\frac{N}{N_0}\right)^{-\theta}$$



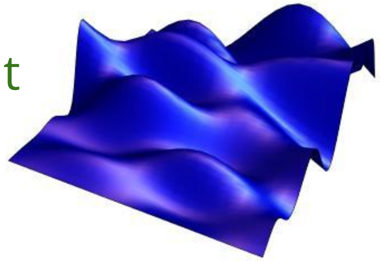
A discrete strategy



Discretize the continuum Path integral

$$\sum_{\text{topologies}} \int [\mathcal{D}g] \longrightarrow \sum_{\text{triangulations}}$$

continuum limit



- Works in Two Dimensions but what about 4D

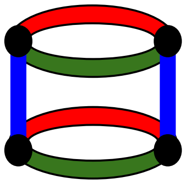
From Matrix to Tensor Models

- Tensor Models = # of indices ≥ 3 [Ambjorn, Durhuus, Jonsson ‘91; Sasakura ‘91; Godfrey, Gross ‘91]
- Feynman Amplitdues are dual to Pseudo-Manifolds
- Large-N limit exists for “colored” Tensor Models (indexed by Gurau degree)

[Gurau ‘09, ’10, ‘11; Bonzom, Gurau, Rivasseau ‘12]

$$\vdots$$

$$T_{a_1 a_2 a_3} T_{a_1 b_2 b_3} T_{b_1 a_2 a_3} T_{b_1 b_2 b_3}$$



So Far

- Matrix Models discretize 2D QG
- Generalize to Tensor Models
- Search for continuum limit resembling our universe

Connection to Asymptotic Safety?

Use Functional RG as an exploratory tool

[taken from the internet

Canonical Scaling Dimension in background-independent RG flow

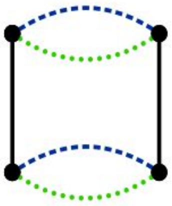
Local Flows

- RG step: local averaging
- Notion of canonical dimensionality defined by mass dimension

Background-independent flows

- RG step: Non-local averaging
- No units, no (a priori) scaling dimension
- Canonical Dimension from autonomous system of beta functions in large-N limit

$\bar{g}^{2,1}_{4,1}$



$$\beta_{\bar{g}^{2,1}_{4,1}} = 2N^2 \left(\bar{g}^{2,1}_{4,1}\right)^2 + \dots \qquad (1)$$

$$g^{2,1}_{4,1} = \bar{g}^{2,1}_{4,1} N^2$$

$$\beta_{g^{2,1}_{4,1}} = 2g^{2,1}_{4,1} + 2\left(g^{2,1}_{4,1}\right)^2 \qquad (2)$$

Towards the fundamental nature of space-time

- Use a mathematical Microscope to zoom into the nature of space-time

$$N\frac{\partial\Gamma_N}{\partial N} = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_N}{\delta\mathbf{T}\delta\mathbf{T}} + R_N\right)^{-1} N\frac{\partial R_N}{\partial N}\right]$$

→ From many degrees of freedom to few

$$N\partial_N\Gamma_N = 0$$

→ Indicates Phase Transition

- Γ_N includes all possible interactions allowed by symmetries of underlying theory

How to cook with the FRG - A recipe

Ingredients:

- A truncation of the effective action
- A regulator R_N

1) $a_1 + a_2 + a_3 < N: \quad R_N > 0$

2) $N < a_1 + a_2 + a_3: \quad R_N = 0$

3) $N \to N' \to \infty: \quad R_N \to \infty$

Instructions:

- Compute $\Gamma_N^{(2)}$
- Specify R_N
- Cook β-fcts (or compute) using FRG
- Fix scaling of couplings

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation (up to 30 coup.)

  1. Starting Scheme: Quartic Order with perurbative approximation for η
  2. Quartic Order with polynomial approximation for η
  3. Quartic Order with full non-polynomial η
  4. Sixth & Eight Order, same procedure for η

- Regulator bound $\eta < 1$
- Assumption: Canonical guiding principle

Regulator Bound($\eta < 1$)

[Meibohm, Pawlowski, Reichert, 2015]

- Rewrite anomalous dimension
  - $\eta = \frac{-N\partial_N Z_N}{Z_N} \Longrightarrow Z_N \sim N^{-\eta}$
- Consider now our regulator
  - $R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\, \delta_{a_2 b_2}\, \delta_{a_3 b_3}\, \delta_{a_4 b_4} \left(\frac{N}{a_1+a_2+a_3+a_4} - 1\right) \theta\left(\frac{N}{a_1+a_2+a_3+a_4} - 1\right)$
- Consider $N \to N' \to \infty$ (Recover Bare Action)
  - $\lim_{N \to N' \to \infty} R_N \sim Z_N\, N \sim N^{1-\eta}$
- Regulator needs to diverge in that limit

N: IR-cutoff

N': UV-cutoff

⟹ $\eta < 1$

Canonical Guiding Principle

- Idea: Increasing truncation should not induce new relevant directions

$$\Longrightarrow \max(\theta) - \max(d_{\bar{g}}) \leq 5$$

$$\left[\bar{g}^2_{4,1}\right] = -2 \quad \left[\bar{g}^{0,(i,j)}_{4,1}\right] = -2 \quad \left[\bar{g}^{0,(1,i)}_{4,2}\right] = -2$$

$$\left[\bar{g}^2_{4,2}\right] = -3 \quad \ldots$$

All couplings with scaling dimension > -5 are included

Next coupling with largest scaling dimension $[\bar{g}^0_{8,1}] = -5$

$$\xrightarrow{\max(d_{\bar{g}}) = -2} \quad \max(\theta) \leq 3$$

Trusting the Fixed Point?

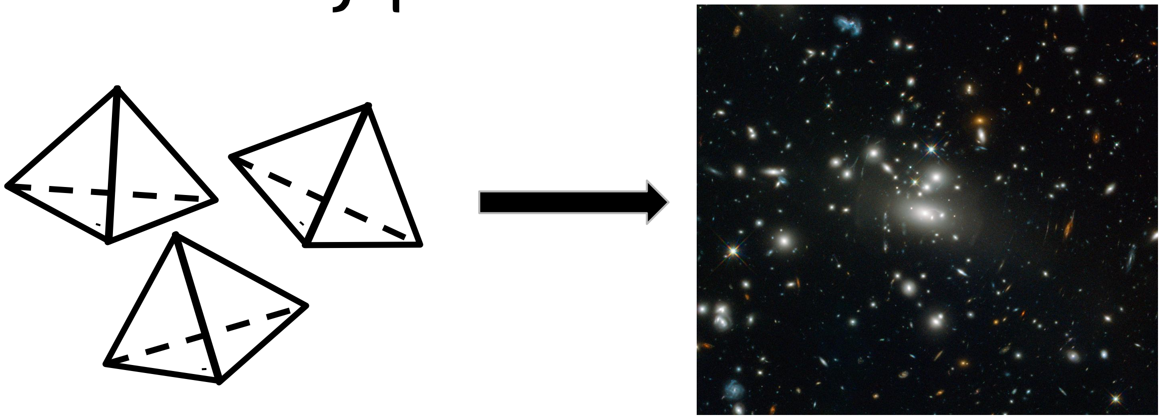
- Ensure stability of results by successively increasing truncation (up to 30 coup.)

  1. Starting Scheme: Quartic Order with perurbative approximation for η
  2. Quartic Order with polynomial approximation for η
  3. Quartic Order with full non-polynomial η
  4. Sixth & Eight Order, same procedure for η

- Regulator bound $\eta < 1$

- Assumption: Canonical guiding principle

Is there a Phase Transition from a discrete Quantum Gravity phase to a continuous one?

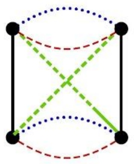


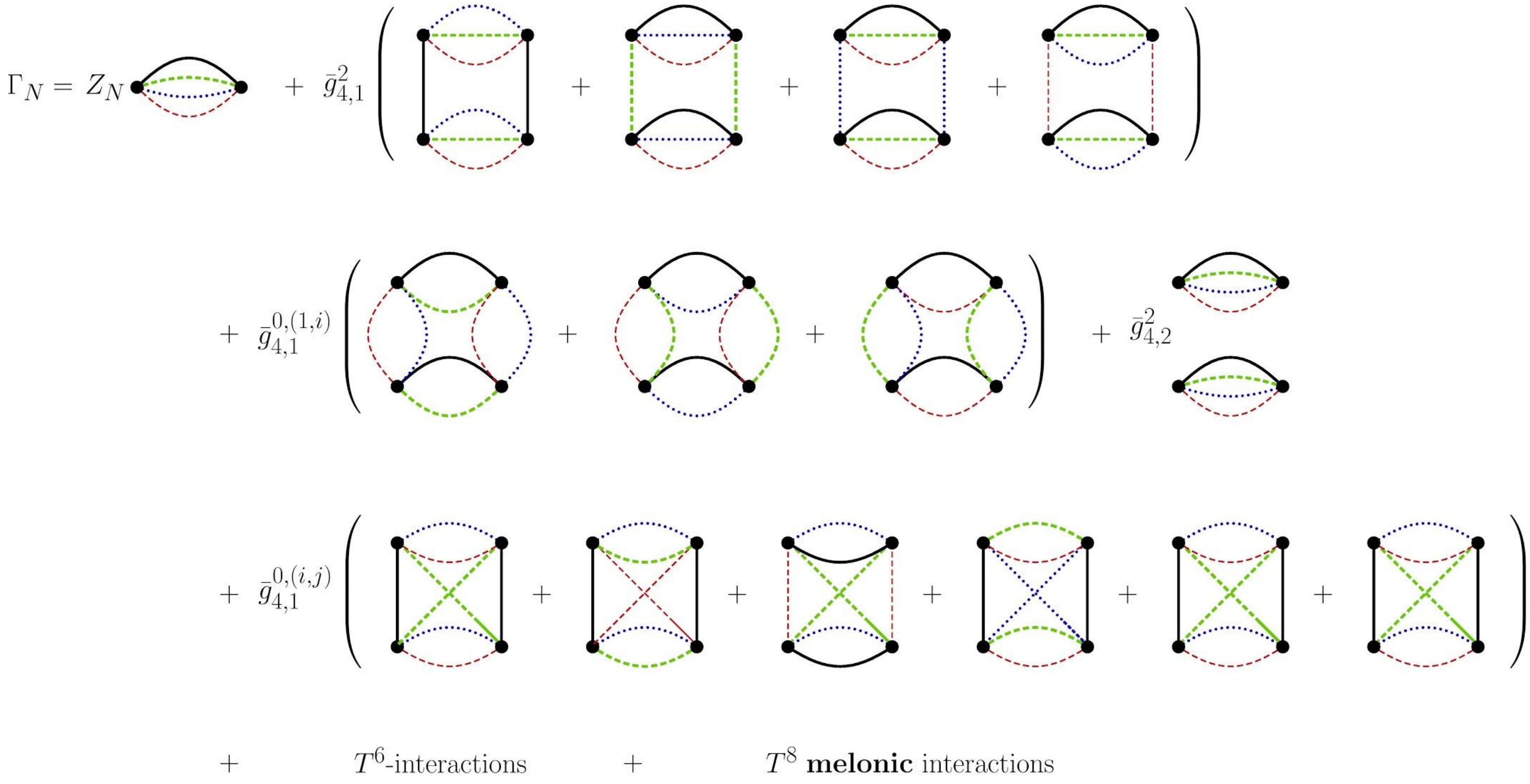
The Model

- Consider a real rank-4 Tensor model s.t.

$$T_{a_1a_2a_3a_4} \rightarrow O^{(1)}_{a_1b_1}O^{(2)}_{a_2b_2}O^{(3)}_{a_3b_3}O^{(4)}_{a_3b_3}T_{b_1b_2b_3b_4}$$

- Enlarged Theory space compared to $U(N)^{\otimes 4}$

E.g.  at quartic level



At T^6 : 132 interactions (belonging to 20 combinatorial families) [Avohou, Ben Geloun, Dub '19]

At T^8 : 4154 interactions (belonging to 188 combinatorial families) [Avohou, Ben Geloun, Dub '19]

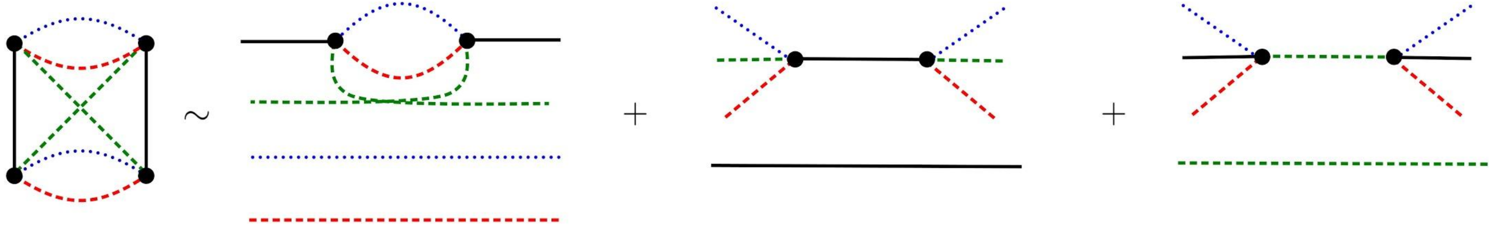
An Algorithm to set up the truncation

- Finding all the interactions by hand/drawing is VERY time consuming and annoying

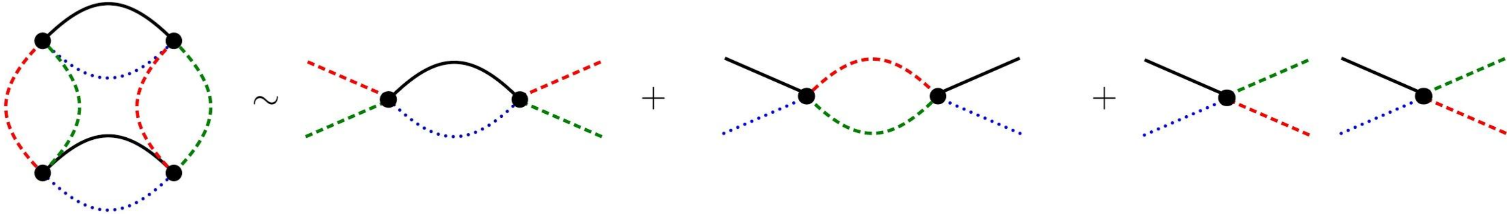
- Alternative: Construct the interactions from bottom up using a computer

- How? $\rightarrow$ Gluing together fundamental Lego blocks, only input are T^4 interactions

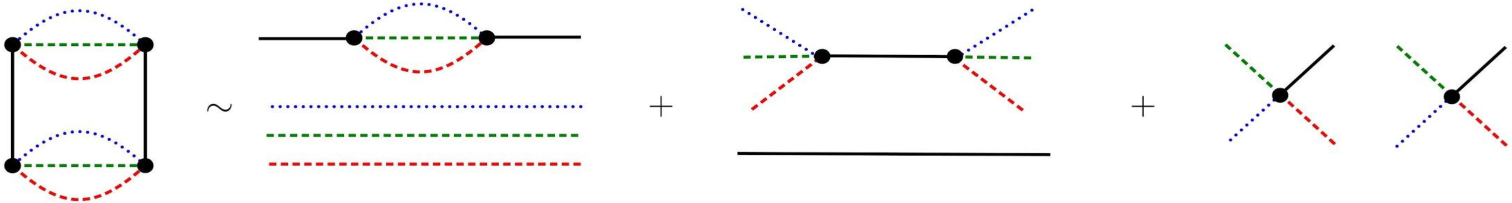
$$\frac{\partial^2}{\partial T_{\{\alpha_i\}} \partial T_{\{\beta_i\}}}$$



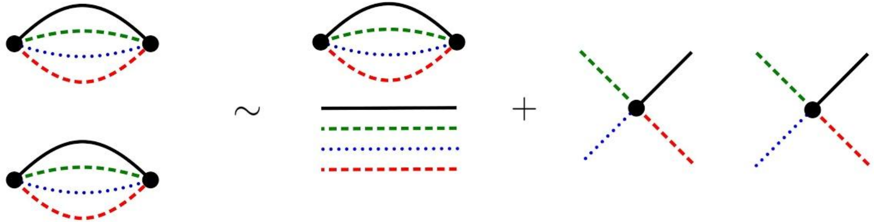
$$\frac{\partial^2}{\partial T_{\{\alpha_i\}} \partial T_{\{\beta_i\}}}$$



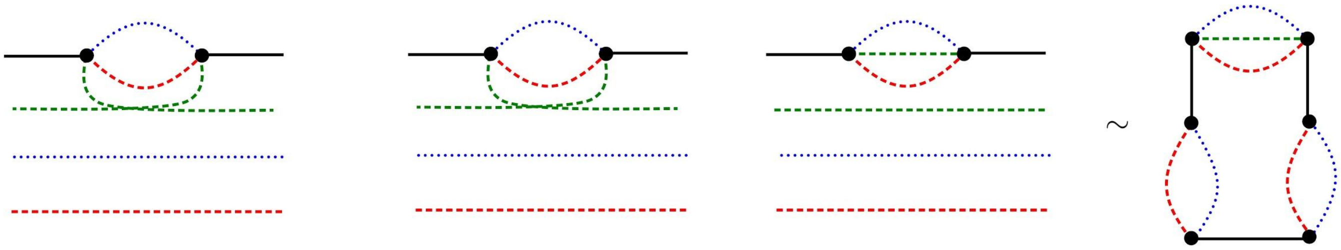
$$\frac{\partial^2}{\partial T_{\{\alpha_i\}} \partial T_{\{\beta_i\}}}$$



$$\frac{\partial^2}{\partial T_{\{\alpha_i\}} \partial T_{\{\beta_i\}}}$$



A little example

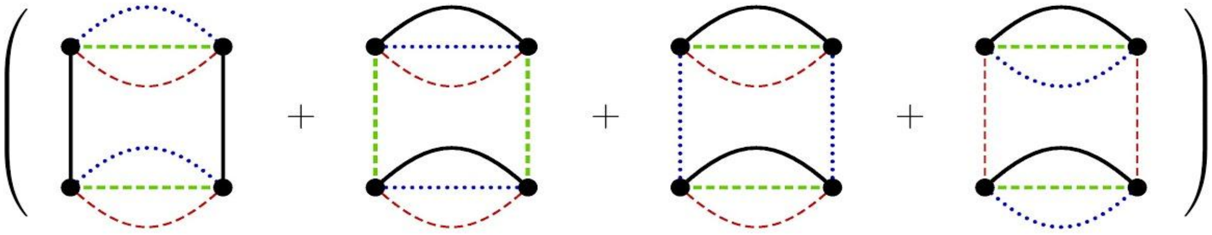


Do this for all possible combinations to obtain all possible interactions

(up to degeneracies)

Finding color-symmetric families

$$\Gamma_N \sim \bar{g}^2_{4,1}$$



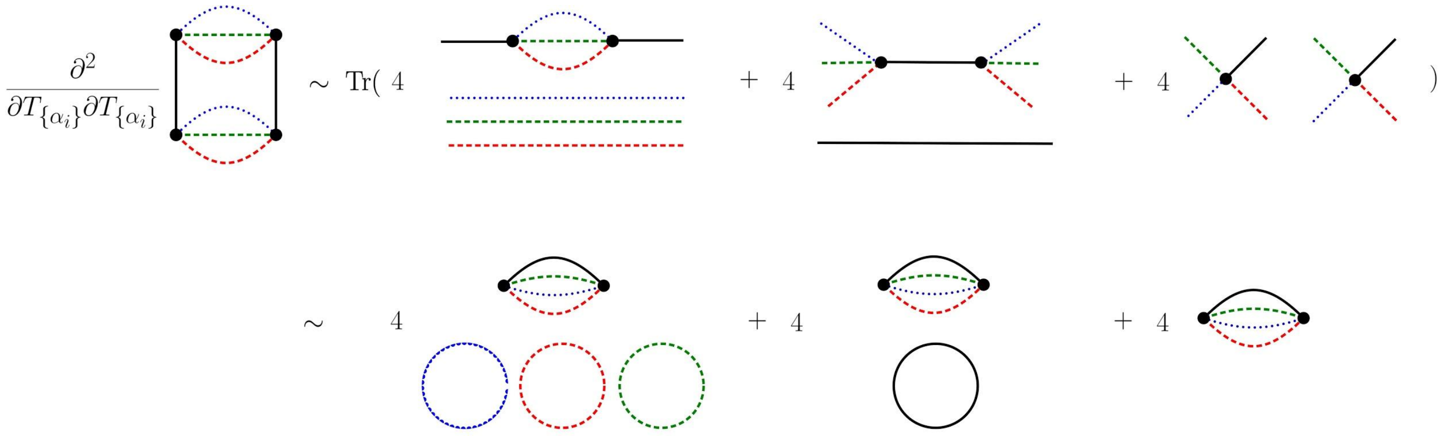
- Mathematica gives: T[-a1,-a2,-a3,-a4] T[a1,a2,a3,b4] T[b1,b2,b3,a4] T[-b1,-b2,-b3,-b4]

Introduce the Indicator

The Indicator

$$\frac{\partial^2}{\partial T_{\{\alpha_i\}} \partial T_{\{\alpha_i\}}} \mathrm{Tr}(T^n)$$

Set of couples of numbers capturing the combinatorial structure of an interaction



$$\Rightarrow \quad \{(4,3),(4,1),(4,0)\}$$

Applying the FRG

- Coarse-graining in *N*

$$N\partial_N\Gamma_N = \frac{1}{2}\mathrm{Tr}\left(\Gamma_N^{(2)} + R_N\right)^{-1}\partial_t R_N$$

→ One equation for every interaction

→ Have to derive and solve 30 coupled non-linear equations

Ingredient II: Regulator

$$R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\, \delta_{a_2 b_2}\, \delta_{a_3 b_3}\, \delta_{a_4 b_4} \left(\frac{N^r}{a_1+a_2+a_3+a_4} - 1\right) \theta\left(\frac{N^r}{a_1+a_2+a_3+a_4} - 1\right)$$

$$a_1 + a_2 + a_3 < N^r : \quad R_N > 0$$

$$N^r < a_1 + a_2 + a_3 : \quad R_N = 0$$

$$N \to N' \to \infty : \quad R_N \to \infty$$

No notion of mass dimensionality

⟹ All choices $r \geq 0$ seem to be allowed

r is fixed by the dual geometric interpretation

Scaling and the FRG

- Crucial point is that there is no a priori scaling

- Scaling is fixed by demanding that the beta functions be autonomous

$$\beta_{g_3} \sim g_1 g_2 N^{\alpha} \Rightarrow \beta_{\bar{g}_3} \sim \bar{g}_1 \bar{g}_2 N^{\alpha+[g1]+[g2]-[g3]}$$

$$\Rightarrow \alpha + [g1] + [g2] - [g3] \leq 0$$

- Need to solve a bunch of inequalities (Upper Bound=Enhanced Scaling)

A little insight...

$$\begin{aligned}\beta_{g^0_{4,1}} &= \left(-[\bar{g}^0_{4,1}] + 2\eta\right) g^0_{4,1} + 16\,\mathcal{I}^1_2\, \left(g^{2,1}_{4,1} + g^{2,2}_{4,1} + g^{2,3}_{4,1}\right) g^0_{4,1}\, N^{[\bar{g}^{2,i}_{4,1}]} - \mathcal{I}^2_1 \left(g^{1,1}_{6,1} + g^{1,2}_{6,1} + g^{1,3}_{6,1}\right) \times \\ &\quad \times N^{[\bar{g}^{1,i}_{6,1}]-[\bar{g}^0_{4,1}]} - \mathcal{I}^3_1\, g^1_{6,2} N^{[\bar{g}^1_{6,2}]-[\bar{g}^0_{4,1}]} - 6\mathcal{I}^1_1 g^{0,p}_{6,1} N^{[\bar{g}^{0,p}_{6,1}]-[\bar{g}^0_{4,1}]}\,,\end{aligned}$$

$$\begin{aligned}\beta_{g^{2,i}_{4,1}} &= \left(-[\bar{g}^{2,i}_{4,1}] + 2\eta\right) g^{2,i}_{4,1} + 8\mathcal{I}^1_2\, \left(g^0_{4,1}\right)^2\, N^{2[\bar{g}^0_{4,1}]-[\bar{g}^{2,i}_{4,1}]} + 16\,\mathcal{I}^1_2\, \left(g^{2,i}_{4,1}\, g^0_{4,1}\right) N^{[\bar{g}^0_{4,1}]} + 8\,\mathcal{I}^2_2 \left(g^{2,i}_{4,1}\right)^2 \times \\ &\quad \times N^{[\bar{g}^{2,i}_{4,1}]} - 5\,\mathcal{I}^1_1\, g^{1,i}_{6,1}\, N^{[\bar{g}^{1,i}_{6,1}]-[\bar{g}^{2,i}_{4,1}]} - \mathcal{I}^3_1\, g^{3,i}_{6,2}\, N^{[\bar{g}^{3,i}_{6,2}]-[\bar{g}^{2,i}_{4,1}]} - 3\,\mathcal{I}^2_1\, g^{3,i}_{6,1} N^{[\bar{g}^{3,i}_{6,1}]-[\bar{g}^{2,i}_{4,1}]} - 3\,\mathcal{I}^1_1\, g^{0,np}_{6,1} \times \\ &\quad \times N^{[\bar{g}^{0,np}_{6,1}]-[\bar{g}^{2,i}_{4,1}]} - 2\,\mathcal{I}^2_1\, g^{2,i}_{6,1}\, N^{[\bar{g}^{2,i}_{6,1}]-[\bar{g}^{2,i}_{4,1}]} - \mathcal{I}^1_1 g^{0,p}_{6,1} N^{[\bar{g}^{0,p}_{6,1}]-[\bar{g}^{2,i}_{4,1}]}\,,\end{aligned}$$

$$\begin{aligned}\beta_{g^2_{4,2}} &= \left(-[\bar{g}^2_{4,2}] + 2\eta\right) g^2_{4,2} + 16\,\mathcal{I}^1_2 \left(g^{2,1}_{4,1}\, g^{2,2}_{4,1} + g^{2,1}_{4,1}\, g^{2,3}_{4,1} + g^{2,2}_{4,1}\, g^{2,3}_{4,1}\right) N^{2[\bar{g}^{2,i}_{4,1}]-[\bar{g}^2_{4,2}]} \\ &\quad + 16\,\mathcal{I}^2_2\, g^2_{4,2} \left(g^{2,1}_{4,1} + g^{2,2}_{4,1} + g^{2,3}_{4,1}\right)\, N^{[\bar{g}^{2,i}_{4,1}]} + 8\,\mathcal{I}^3_2 \left(g^2_{4,2}\right)^2\, N^{[\bar{g}^2_{4,2}]} - 3\,\mathcal{I}^3_1\, g^3_{6,3}\, N^{[\bar{g}^3_{6,3}]-[\bar{g}^2_{4,2}]} \\ &\quad - 2\,\mathcal{I}^2_1 \left(g^{3,1}_{6,2} + g^{3,2}_{6,2} + g^{3,3}_{6,2}\right) N^{[\bar{g}^{3,i}_{6,2}]-[\bar{g}^2_{4,2}]} - \mathcal{I}^1_1 \left(g^{2,1}_{6,1} + g^{2,2}_{6,1} + g^{2,3}_{6,1}\right) N^{[\bar{g}^{2,i}_{6,1}]-[\bar{g}^2_{4,2}]} \\ &\quad - 6\,\mathcal{I}^1_1\, g^1_{6,2}\, N^{[\bar{g}^1_{6,2}]-[\bar{g}^2_{4,2}]} + 48\,\mathcal{I}^1_2\, g^2_{4,2}\, g^0_{4,1}\, N^{[\bar{g}^0_{4,1}]}\,,\end{aligned}$$

$$\begin{aligned}\beta_{g^{3,i}_{6,1}} &= \left(-[\bar{g}^{3,i}_{6,1}] + 3\eta\right) g^{3,i}_{6,1} + 8\,\mathcal{I}^1_2\, g^{1,i}_{6,1}\, g^0_{4,1} N^{[\bar{g}^{1,i}_{6,1}]+[\bar{g}^0_{4,1}]-[\bar{g}^{3,i}_{6,1}]} + 32\,\mathcal{I}^1_2\, g^{1,i}_{6,1}\, g^{2,i}_{4,1} N^{[\bar{g}^{1,i}_{6,1}]+[\bar{g}^{2,i}_{4,1}]-[\bar{g}^{3,i}_{6,1}]} \\ &\quad + 48\,\mathcal{I}^1_2\, g^{3,i}_{6,1}\, g^0_{4,1}\, N^{[\bar{g}^0_{4,1}]} - 8\,\mathcal{I}^1_2 \sum_{j,k}^{3} g^{2,i}_{4,1} \left(g^{2,j}_{6,1} + g^{2,k}_{6,1}\right) (\delta_{ij} - 1)(\delta_{jk} - 1)(\delta_{ik} - 1)\, N^{[\bar{g}^{2,i}_{4,1}]+[\bar{g}^{2,i}_{6,1}]-[\bar{g}^{3,i}_{6,1}]} \\ &\quad - 192\,\mathcal{I}^1_3 \left(g^{2,i}_{4,1}\right)^2\, g^0_{4,1}\, N^{2[\bar{g}^{2,i}_{4,1}]+[\bar{g}^0_{4,1}]-[\bar{g}^{3,i}_{6,1}]} - 32\,\mathcal{I}^2_3\, \left(g^{2,i}_{4,1}\right)^3 N^{3[\bar{g}^{2,i}_{4,1}]-[\bar{g}^{3,i}_{6,1}]} + 24\,\mathcal{I}^2_2\, g^{3,i}_{6,1}\, g^{2,i}_{4,1} N^{[\bar{g}^{2,i}_{4,1}]}\,,\end{aligned}$$

$$\vdots$$

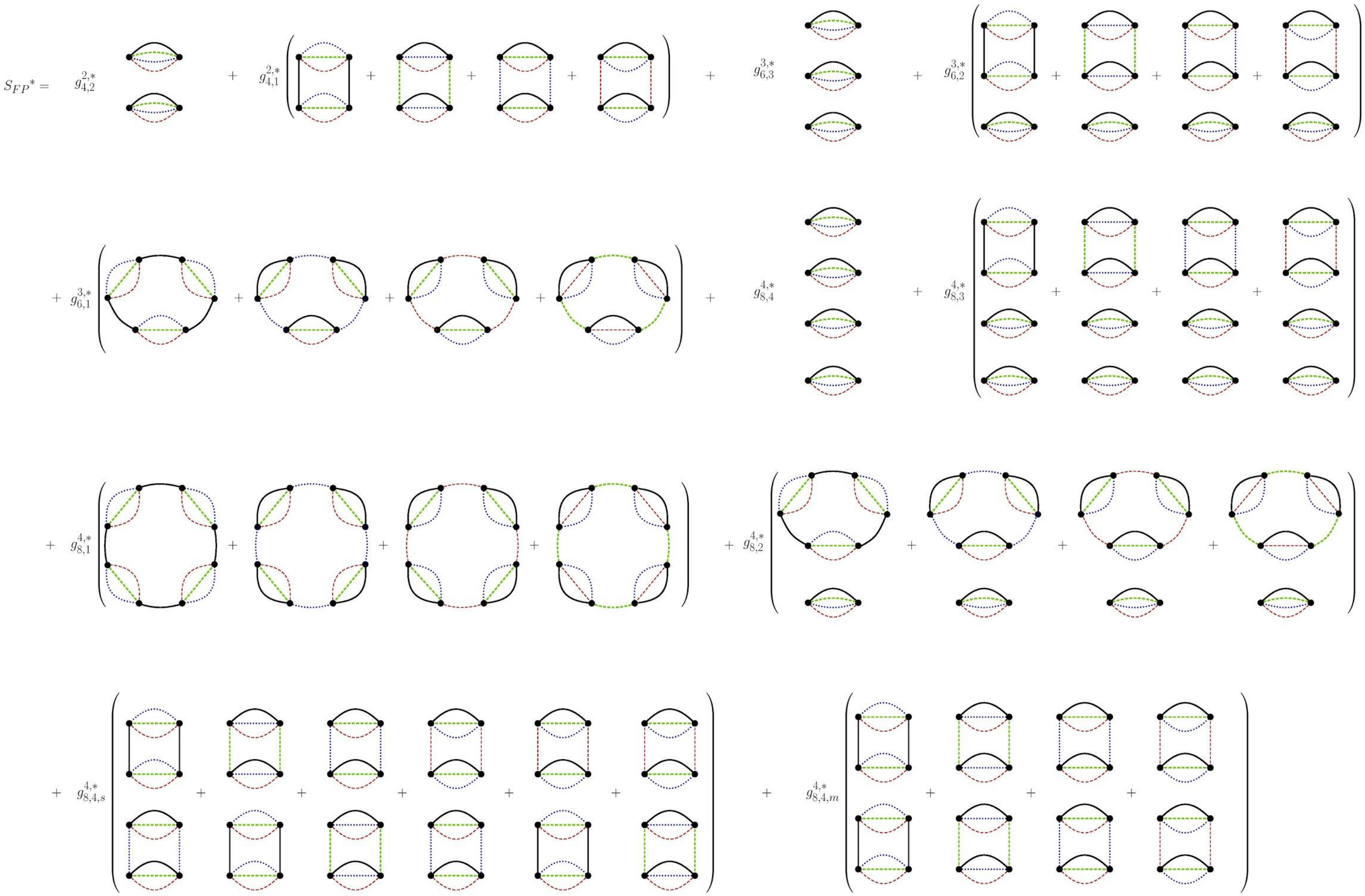
Interesting Continuum Limit?

truncation	$g^2_{4,2}$	$g^2_{4,1}$	$g^3_{6,3}$	$g^3_{6,1}$	$g^2_{6,2}$	$g^4_{8,4}$	$g^4_{8,3}$	$g^4_{8,1}$	$g^4_{8,2}$	$g^4_{8,2,s}$	$g^4_{8,2,m}$	η	$\theta_{1,2}$	θ_3	θ_4
T^4	11.3	-1.61	-	-	-	-	-	-	-	-	-	0.86	$2.996 \pm$ i 1.227	-0.288	-0.288
T^6	5.36	-0.98	230.1	-1.42	-12.43	-	-	-	-	-	-	-0.62	$2.984 \pm$ i 1.369	-0.752	-0.752
T^8	3.50	-0.73	219.6	-1.68	-12.29	-300.2	272.8	-2.4	-19.0	-23.6	-6.1	-0.49	$2.793 \pm$ i 1.478	-0.21	-1.01

gauge	cutoff	operators included beyond Einstein-Hilbert	# rel. dir.	# irrel. dir.	Reθ_1	Reθ_2	Reθ_3
$\alpha = 1, \beta = 0$	exp.	-	2	-	1.94	1.94	-
$\alpha = 0$	Litim [209, 210]	-	2	-	1.67	1.67	-
$\alpha = 0, \beta = 0$	exp.	$\sqrt{g}R^2$	3	0	28.8	2.15	2.15
$\beta = 1, \alpha = 0$	Litim	$\sqrt{g}R^2, \sqrt{g}R^3$	3	1	2.67	2.67	2.07
$\alpha = 1, \beta = 1$	Litim	$\sqrt{g}R^2, \sqrt{g}R^3$	3	1	2.71	2.71	2.07
$\beta = 1, \alpha = 0$	Litim	$\sqrt{g}R^2, \sqrt{g}R^6$	3	1	2.39	2.39	1.51
$\alpha = 1, \beta = 1$	Litim	$\sqrt{g}R^2, ..., \sqrt{g}R^8$	3	6	2.41	2.41	1.40
$\alpha = 0, \beta = 0$	Litim	$\sqrt{g}R^2, ..., \sqrt{g}R^{34}$	3	32	2.50	2.50	1.59
$\alpha = 0$, h/o	Litim	$\sqrt{g}R^2, \sqrt{g}R_{\mu\nu}R^{\mu\nu}$	3	1	8.40	2.51	1.69
$\beta = \alpha = 1$	Litim	$\sqrt{g}C^{\mu\nu\kappa\lambda}C_{\kappa\lambda\rho\sigma}C^{\rho\sigma}{}_{\mu\nu}$	2	1	1.48	1.48	-

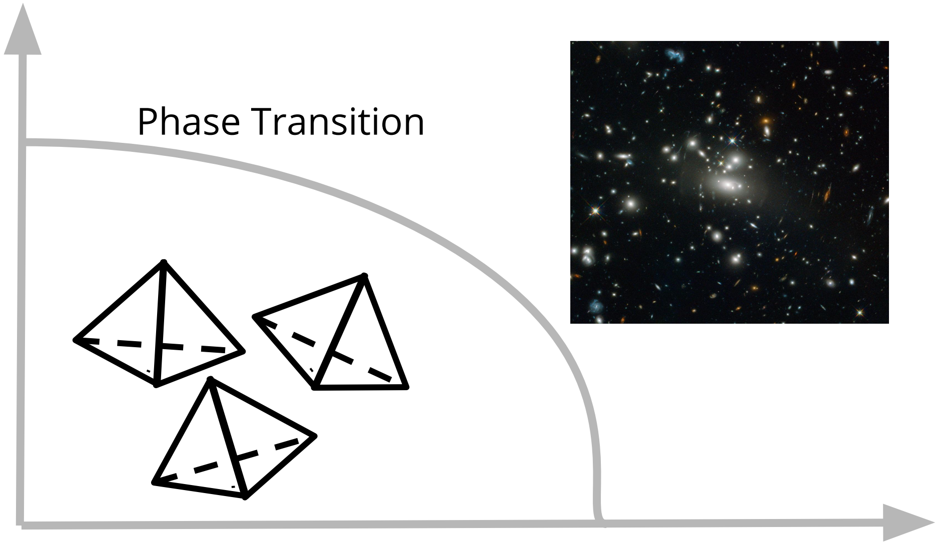
Not imcopatible with Reuter Fixed point found in Asymptotic Safety

[stolen from Astrid, “An asymptotically safe guide to quantum gravity and matter”]


$S_{FP}^* =$
$g_{4,2}^{2,*}$
$+ \; g_{4,1}^{2,*}$
$+ \; g_{6,3}^{3,*}$
$+ \; g_{6,2}^{3,*}$
$+ \; g_{6,1}^{3,*}$
$+ \; g_{8,4}^{4,*}$
$+ \; g_{8,3}^{4,*}$
$+ \; g_{8,1}^{4,*}$
$+ \; g_{8,2}^{4,*}$
$+ \; g_{8,4,s}^{4,*}$
$+ \; g_{8,4,m}^{4,*}$

Phase Transition

- Possibility of a (second-order) phase transition from a discrete phase of Quantum Gravity to a continouos phase of Quantum Gravity ?



Summary

- Explore theory space of Tensor Models

- Study Composite Operators → Connection to AS

- Spectral Dimension?

Geometrical Properties ?

- Asymptotic Safety and Dynamical Triangulations two sides of the same medal?