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Conformal Symmetry and Composite Operators in the $O(N)^3$ Tensor Field Theory

Kenta Suzuki

D. Benedetti, R. Gurau & KS; 2002.07652 [hep-th]

MOTIVATIONS

- A simple quantum mechanical many-body system with all-to-all quenched disorder (i.e. SYK model) provides a solvable example of AdS/CFT and helps better understanding of quantum gravity problems [Kitaev '15].

- Due to the quenched disorder, SYK is not genuine quantum system. On the other hand, tensor models share many aspects of SYK (at least in large N) and they are genuine quantum systems.

- Due to the all-to-all couplings, SYK does not have spacial locality. On the other hand, it is straightforward to introduce space-time dependence in tensor models (i.e. tensor field theories).

- It is tempting to search tensor model CFT's with potential AdS dual theories as well.

Outline

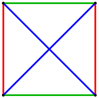
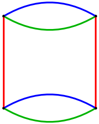
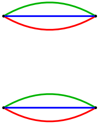
1. **THE MODEL**
2. **CONPOSITE OPERATORS**
3. **CONFORMAL SYMMETRY**
4. **2pt and 3pt FUNCTIONS**
5. **CONCLUSIONS**

1. THE MODEL

- We consider the model

$$S[\phi] = \frac{1}{2}\int d^dx\,\phi_{\mathbf{a}}(x)(-\partial^2)^{\zeta}\phi_{\mathbf{a}}(x) + \frac{1}{4}\int d^dx\left[\frac{\lambda_t}{N^{3/2}}\,\delta^t_{\mathbf{abcd}} + \frac{\lambda_p}{N^2}\,\delta^p_{\mathbf{abcd}} + \frac{\lambda_d}{N^3}\,\delta^d_{\mathbf{abcd}}\right]\phi_{\mathbf{a}}\phi_{\mathbf{b}}\phi_{\mathbf{c}}\phi_{\mathbf{d}}$$

with $\phi_{\mathbf{a}}(x) \equiv \phi_{a_1a_2a_3}(x)$ are the bosonic $O(N)^3$ tensor fields. The scaling of large N is chosen to be the optimal scaling [Carrozza & Tanasa '15].

$\delta^t_{\mathbf{abcd}}$ =  , $\delta^p_{\mathbf{abcd}}$ =  , $\delta^d_{\mathbf{abcd}}$ = 

- In the following we consider $N \gg 1$.

$\zeta = 1$ **Case:** [Giombi, Klebanov & Tarnopolsky '17]

- The dimensional regularization $(d = 4 - \epsilon)$ in large N leads to

$$\begin{aligned}
\beta_t &= -\epsilon\, g_t + 2\, g_t^3 \\
\beta_p &= -\epsilon\, g_p + \left(6\, g_t^2 + \tfrac{2}{3} g_p^2\right) - 2\, g_t^2 g_p \\
\beta_d &= -\epsilon\, g_d + \left(\tfrac{4}{3}\, g_p^2 + 4 g_p g_d + 2\, g_d^2\right) - 2\, g_t^2 \left(4\, g_p + 5\, g_d\right)
\end{aligned}$$

- Solving $\beta_a = 0$, we find the critical couplings:

$$g_t^\star = \pm(\epsilon/2)^{1/2}\,, \qquad g_p^\star = \pm 3i(\epsilon/2)^{1/2}\,, \qquad g_d^\star = \mp i(3 \pm \sqrt{3})(\epsilon/2)^{1/2}\,,$$

and for the bi-linear operator $\phi^2 \equiv \sum_{\mathbf{a}} \phi_{\mathbf{a}}\phi_{\mathbf{a}}$, its scaling dimension is

$$\Delta_{\phi^2} = d - 2 + 2(g_p^\star + g_d^\star) = 2 \pm i\sqrt{6\epsilon} + \mathcal{O}(\epsilon)$$

- The complex dimension of ϕ^2 leads to an instability of the model.

$\zeta = d/4$ **Case:** [Benedetti, Gurau & Harribey '19]

• By choosing $\zeta = d/4$, the IR scaling dimension of ϕ^2 is matched to the UV dimension (i.e. a line of fixed point):

$$G(p) = Z\, p^{-d/2}$$

• The beta function of the tetrahedron coupling is identically: $\beta_t = 0$.

• By introducing $\lambda_1 = \lambda_p/3$ and $\lambda_2 = \lambda_p + \lambda_d$, the other two beta functions are solved as

$$g_1^\star = \pm\sqrt{-g_t^2} + \mathcal{O}(g_t^2)\,, \qquad g_2^\star = \pm\sqrt{-3g_t^2} + \mathcal{O}(g_t^2)$$

and for the bi-linear operator ϕ^2, its scaling dimension is

$$\Delta_{\phi^2} = d/2 + 2\,\frac{\Gamma(d/4)^2}{\Gamma(d/2)}\, g_2^\star + \mathcal{O}(g_t^2)$$

• If $g_t^\star$ is pure imaginary, $g_{1,2}^\star$ and Δ_{ϕ^2} are real.

Long-range Ising Model

• In fact when $N = 1$, this model (with $\zeta = d/4$) is an effective theory of the long-range Ising model:

$$H_{\text{LRI}} = -J \sum_{i,j=\text{all}} \hat{S}_i \hat{S}_j / r_{ij}^{d+2\zeta}$$

• This is a contrast to the fact that the $\zeta = 1$ & $N = 1$ case (standard ϕ^4 theory) is the effective theory of the short-range Ising model:

$$H_{\text{SRI}} = -J \sum_{i,j=\substack{\text{nearest}\\ \text{neighbors}}} \hat{S}_i \hat{S}_j$$

• For the LRI model, the conformal symmetry is proven in the range of the parameter [Paulos, Rychkov, Rees & Zan '15]:

$$\frac{d}{4} \leq \zeta \ll \frac{d+1}{4}$$

Operator Product Expansion

- Operator Product Expansion (OPE)

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) \sim \sum_k \frac{C_{ijk}}{(x_1 - x_2)^{\Delta_i+\Delta_j-\Delta_k}} \left[1 + \#\, x_{12}\partial_{x_2} + \cdots\right]\mathcal{O}_k(x_2)$$

with the OPE coefficient C_{ijk}.

- It also appears in the three-point function

$$\left\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\right\rangle = \frac{C_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_k+\Delta_i-\Delta_j}}$$

- For a unitary CFT, all OPE coefficients are real and positive.

4-point Function

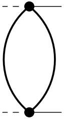
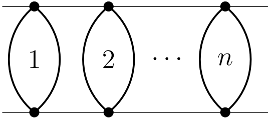
- We consider the 4-point function:

$$\mathcal{F} = \langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle \,.$$

- They are determined by [Maldacena & Stanford '16]

$$\mathcal{F} = \sum_{n=0}^{\infty} \mathcal{F}_n = \sum_{n=0}^{\infty} K^n \mathcal{F}_0 = \frac{\mathcal{F}_0}{1-K} \,,$$

with the 4-point kernel K:

$K = 3\lambda^2 \times$  , $\mathcal{F}_n =$ 

Conformal Block Expansion

- Let us for now assume the fixed-point is indeed conformally symmetric.

- The connected 4-point function with scalar fields of dimension Δ_ϕ can be expanded on the conformal blocks $G^{\Delta_\phi}_{h,J}$:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_{\text{connected}} = \sum_J \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{dh}{2\pi i} \frac{\mu^d_{\Delta_\phi}(h,J)}{1-k(h,J)} \, G^{\Delta_\phi}_{h,J}(x_i)$$

with an appropriate measure $\mu^d_{\Delta_\phi}$ and the 4-point kernel eigenvalue $k(h,J)$.

- The spectrum of bilinear operators (of the form $\phi\partial_{\mu_1}\cdots\partial_{\mu_J}\partial^{2m}\phi$) is determined by $1 = k(h,J)$ and the solutions are given by

$$h_{m,J} = \frac{d}{2} + J + 2m + \cdots$$

OPE Coefficients

- On the other hand, OPE's of (1,2) and (3,4) lead to [Dolan & Osborn '00]

$$\begin{aligned}
&\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_{\text{connected}} \\
&= \sum_{h,h',J,J'} c_{h,J} c_{h',J'} \hat{C}(x_{12},\partial_2)\hat{C}(x_{34},\partial_4)\langle \mathcal{O}_{h,J}(x_2)\mathcal{O}_{h',J'}(x_4)\rangle \\
&= \sum_{m,J} c^2_{m,J}\, G^{\Delta_\phi}_{h_m,J}(x_i)\,.
\end{aligned}$$

- Comparing with the previous expression, we find the OPE coefficients:

$$c^2_{m,J} = -\mu^d_{\Delta_\phi}(h_{m,J},J)\text{Res}\left[\frac{1}{1-k(h,J)}\right]_{h=h_{m,J}} = \frac{\mu^d_{\Delta_\phi}(h_{m,J},J)}{k'(h_{m,J},J)}\,,$$

where $h_{m,J}$ are the solutions of $k(h,J)=1$.

Conjecture

- The tensor model with ϕ^4 interactions and Laplacian $\mathcal{L} = (-\partial^2)^{d/4}$ is unitary at large N limit at the IR fixed-point with a pure imaginary $g_t^\star$.

- The bilinear operator (of the form $\phi \partial_{\mu_1} \cdots \partial_{\mu_J} \partial^{2m} \phi$) spectrum is given by

$$h_{m,J} = \frac{d}{2} + J + 2m + \cdots$$

where the small coupling corrections are $\mathcal{O}(\sqrt{-g_t^2})$ or $\mathcal{O}(g_t^2)$. They are real for pure imaginary coupling g_t.

- The free limit of the OPE coefficients are all positive real for pure imaginary coupling g_t.

- The real conformal dimensions and positive real OPE coefficients of all bilinear primaries are strong indication of the unitarity of the model.

2. CONPOSITE OPERATORS

- Operator Product Expansion (OPE) tells

$$\phi(x_1)\phi(x_2) \sim \sum_i \frac{C_{\phi\phi\mathcal{O}_i}}{(x_1-x_2)^{2\Delta_\phi-\Delta_{\mathcal{O}^{(\ell)}}}}\Big[1+\#\,x_{12}\partial_{x_2}+\cdots\Big]\mathcal{O}_i(x_2)$$

so $x_1 \to x_2$ limit leads to a UV divergence.

- In order to regulate these divergence, we introduce the UV regulator ϵ by $\zeta=(d+\epsilon)/4$ and define composite operators by

$$[\phi^2] = Z_{\phi^2}\,\phi^2\,, \qquad [\phi_1^4] = Z_{\phi_1^4}\,\phi_1^4\,, \qquad \text{and so on}$$

where Z's are renormalization constants will be given by the form

$$Z = \left(1+\frac{\#}{\epsilon}+\cdots\right)$$

ϕ^4 Operator Mixing (1)

- Composite operators $[\phi_a^4]$ is defined by [Brown '79]

$$\frac{\partial}{\partial g_a}\left\langle \phi(x_1)\ldots\phi(x_n)\right\rangle_c = -\mu^{d-\Delta_a}\int d^d x\left\langle [\phi_a^4](x)\phi(x_1)\ldots\phi(x_n)\right\rangle_c$$

where μ is the IR regulator.

- We can write the bare operator in terms of a linear combination of the composite operators with the mixing matrix M_{ab}:

$$\phi_a^4 = \sum_b M_{ab}\,[\phi_b^4]\,, \qquad [\phi_a^4] = \sum_b \left(M^{-1}\right)_{ab}\phi_b^4$$

- From the previous definition of the composite operators

$$[\phi_a^4] = \mu^{\Delta_a - d}\sum_b \frac{\partial \lambda_b}{\partial g_a}\,\phi_b^4 \quad \Rightarrow \quad \left(M^{-1}\right)_{ab} = \mu^{\Delta_a - d}\,\frac{\partial \lambda_b}{\partial g_a}$$

ϕ^4 Operator Mixing (2)

- Since the bare couplings are independent of μ:

$$0 = \mu \frac{d\lambda_a}{d\mu} = \mu \frac{\partial\lambda}{\partial\mu} + \mu \sum_b \frac{\partial g_b}{\partial\mu} \frac{\partial\lambda_a}{\partial g_b}$$
$$= (d - \Delta_a)\lambda_a + \sum_b \mu^{d-\Delta_b} \beta_b \left(M^{-1}\right)_{ba}$$

where $\beta_b = \mu\frac{dg_b}{d\mu}$. This fixies

$$\beta_b = \sum_a (\Delta_a - d)\mu^{\Delta_a - d}\lambda_a M_{ab}$$

- Finally, we find

$$\sum_a (\Delta_a - d)\lambda_a \phi_a^4 = \sum_a \mu^{d-\Delta_a} \beta_a \left[\phi_a^4\right]$$

This linear combination always appears in the action and the energy-momentum tensor (will be defined later).

One-loop Mixing

- At one-loop order, we can compute ($Q = 2(4\pi)^{-d/2}/\Gamma(d/2)$)

$$m_0^2 = \mu^{d-2\Delta_\phi} m^2 \left(1 + Q\,\frac{g_2}{\epsilon}\right), \qquad \lambda_t = \mu^{d-4\Delta_\phi} g_t\,,$$

$$\lambda_1 = \mu^{d-4\Delta_\phi}\left(g_1 - Q\,\frac{g^2}{\epsilon} + Q\,\frac{g_1^2}{\epsilon}\right), \quad \lambda_2 = \mu^{d-4\Delta_\phi}\left(g_2 - 3Q\,\frac{g^2}{\epsilon} + Q\,\frac{g_2^2}{\epsilon}\right)$$

- This leads to the composite operators

$$[\phi^2] = \left(1 + Q\,\frac{g_2}{\epsilon}\right)\phi^2\,, \qquad [\phi_1^4] = \left(1 + 2Q\,\frac{g_1}{\epsilon}\right)\phi_1^4\,,$$

$$[\phi_2^4] = \left(1 + 2Q\,\frac{g_2}{\epsilon}\right)\phi_2^4\,, \qquad [\phi_t^4] = \phi_t^4(x) - 2Q\frac{g}{\epsilon}\,\phi_1^4 - 6Q\frac{g}{\epsilon}\,\phi_2^4$$

where we suppressed ϕ^2 mixing into ϕ^4 operators.

3. CONFORMAL SYMMETRY

- Strictly speaking $\beta_a = 0$ means scaling symmetry, but not conformal symmetry. [Polchinski '87]

- Because of the non-local kinetic term

$$S_0[\phi] = \frac{1}{2}\int d^dx\, \phi_{\mathbf{a}}(x)(-\partial^2)^{d/4}\phi_{\mathbf{a}}(x)\,,$$

the naive definition

$$T^{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}}\,,$$

leads to a non-local energy-momentum tensor, even if it exists.

$D = d + p$ Dimensional Embedding

• We can localize the kinetic term by embedding it in $D = d + p$ $(p = 2 - 2\zeta)$ dimensional space [Caffarelli & Silvestre '06]:

$$S[\Phi] = \frac{1}{2}\int d^D X \left(\partial_M \Phi_{\mathbf{a}}(X)\right)^2 + \frac{1}{4}\int_{y=0} d^d x \left[\lambda_t \hat{\delta}^t_{\mathbf{abcd}} + \lambda_1 \hat{P}^{(1)}_{\mathbf{ab;cd}} + \lambda_2 \hat{P}^{(2)}_{\mathbf{ab;cd}}\right] \Phi_{\mathbf{a}}\Phi_{\mathbf{b}}\Phi_{\mathbf{c}}\Phi_{\mathbf{d}}$$

where the D-dimensional coordinates are labeled by $X^M = (x^\mu, y^m)$.

• The original field is obtained by $\phi = \Phi|_{y\to 0}$.

$D = d + p$ Dimensional EM Tensor

- In this D dimensional space, we have a local EM tensor [Paulos, Rychkov, Rees & Zan '15]:

$$T_{MN} = \sum_{\mathbf{a}} \left[\partial_M \Phi_{\mathbf{a}} \partial_N \Phi_{\mathbf{a}} - \frac{1}{2} \delta_{MN} (\partial_K \Phi_{\mathbf{a}})^2 \right] - \frac{\delta^{\|}_{MN} \delta^p(y)}{4} \sum_{\mathbf{a,b,c,d}} \left(\lambda_t \hat{\delta}^t_{\mathbf{abcd}} + \lambda_1 \hat{P}^{(1)}_{\mathbf{ab;cd}} + \lambda_2 \hat{P}^{(2)}_{\mathbf{ab;cd}} \right) \Phi_{\mathbf{a}} \Phi_{\mathbf{b}} \Phi_{\mathbf{c}} \Phi_{\mathbf{d}}$$

where $\delta^{\|}_{MN} = \delta_{\mu\nu}$ if both indices are in the d-dimensional space, and zero otherwise.

Dilatation and Special Conformal Transformation

- Considering the Renormalization of the ϕ^4 operators, trace of the EM tensor is

$$T^M{}_M = -\Delta_\phi E + \left(\frac{1}{2} - \frac{D}{4}\right)\sum_{\mathbf{a}} \partial_K^2 \Phi_{\mathbf{a}}^2 + \frac{\mu^\epsilon\, \delta^p(y)}{4}\Big[\beta_t\, [\phi_t^4](x) + \beta_1\, [\phi_1^4](x) + \beta_2\, [\phi_2^4](x)\Big]$$

where E represents the equation of motion of Φ.

- The dilatation and special conformal transformation (SCT) currents are constructed

$$\mathcal{D}_M = T_{MN} X^N\,, \qquad \mathcal{C}_M{}^N = T_{MK}(2X^K X^N - \delta^{KN} X^2)$$

and the divergence of the currents $\mathcal{J}_M = \{\mathcal{D}_M, \mathcal{C}_M{}^N\}$ obey

$$\int d^D X \left\langle \Big(\partial^M \mathcal{J}_M(X)\Big)\Phi(X_1)\cdots\Phi(X_n)\right\rangle = 0$$

Ward Identities

• Since the n-point function behaves continuously in the limit $y \to 0$, we have the Ward Identities

$$\sum_{i=1}^{n} \Big[x_i \cdot \partial_{x_i} + \Delta_\phi \Big] \Big\langle \phi(x_1) \cdots \phi(x_n) \Big\rangle$$
$$= \frac{\mu^\epsilon}{4} \int d^d x \, \Big\langle \Big[\beta_t \, \big[\phi_t^4\big](x) + \beta_1 \, \big[\phi_1^4\big](x) + \beta_2 \, \big[\phi_2^4\big](x) \Big] \phi(x_1) \cdots \phi(x_n) \Big\rangle$$

and

$$\sum_{i=1}^{n} \left[(2 x_i^\mu x_i^\nu - \delta^{\mu\nu} x_i^2) \frac{\partial}{\partial x_i^\mu} + 2 \Delta_\phi \, x_i^\nu \right] \Big\langle \phi(x_1) \cdots \phi(x_n) \Big\rangle$$
$$= \frac{\mu^\epsilon}{2} \int d^d x \, x^\nu \, \Big\langle \Big[\beta_t \, \big[\phi_t^4\big](x) + \beta_1 \, \big[\phi_1^4\big](x) + \beta_2 \, \big[\phi_2^4\big](x) \Big] \phi(x_1) \cdots \phi(x_n) \Big\rangle$$

• At the fixed-point ($\beta_t = \beta_1 = \beta_2 = 0$), the conformal symmetry (dilatation + SCT) is restored.

4. 2pt and 3pt FUNCTIONS

• Conformal symmetry significantly restrict the two- and three-point functions of conformal operators [Polyakov '70].

• For scalar conformal operators $\mathcal{O}_i$, two-point functions are

$$\left\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\right\rangle = \frac{\delta_{\Delta_i\Delta_j}}{|x-y|^{2\Delta_i}} ,$$

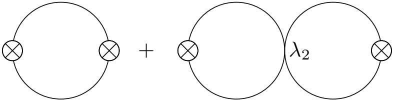
and three-point functions are

$$\left\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\right\rangle = \frac{C_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_k+\Delta_i-\Delta_j}}$$

Two-point Function $\langle[\phi^2](x)[\phi^2](y)\rangle$ (1)

- Up to the first order of the couplings, we have

$$\left\langle[\phi^2](x)[\phi^2](y)\right\rangle =$$



This reads

$$\left\langle[\phi^2](x)[\phi^2](y)\right\rangle \propto \left[Z_{\phi^2}^2\, C(x-y)^2 - \lambda_2 \int d^d z\, C(x-z)^2 C(z-y)^2\right]$$

where the bare propagator is given by ($\zeta = (d+\epsilon)/4$)

$$C(x-y) = \frac{c(\zeta)}{|x-y|^{d-2\zeta}}\,, \qquad c(\zeta) = \frac{\Gamma(\frac{d}{2}-\zeta)}{4^\zeta \pi^{d/2}\Gamma(\zeta)}$$

Two-point Function $\langle[\phi^2](x)[\phi^2](y)\rangle$ (2)

- After the integration, we obtain

$$\Big\langle[\phi^2](x)[\phi^2](y)\Big\rangle \propto \frac{1}{|x-y|^{4\Delta_\phi}}\left[Z_{\phi^2}^2 \;-\; 2\,Q\,g_2\left(\frac{1}{\epsilon}+\log|x-y|+\cdots+\mathcal{O}(\epsilon)\right)\right]$$

- In order to cancel the UV divergence piece (ϵ^{-1}), we need $Z_{\phi^2}=1+\frac{Qg_2}{\epsilon}+\cdots$. This agrees with the previous β function computation result.

- Finally exponentiating the log term, we obtain ($\Delta_\phi^\star=d/4$ and $\delta h_{\phi^2}=Q\,g_2^\star$)

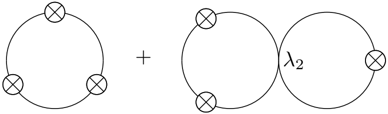
$$\Big\langle[\phi^2](x)[\phi^2](y)\Big\rangle \propto \frac{1}{|x-y|^{4\Delta_\phi^\star+2\delta h_{\phi^2}}}$$

- We find the conformal form of the 2pt function!

Three-point Function $\langle[\phi^2](x)[\phi^2](y)[\phi^2](z)\rangle$ (1)

- For the three-point function, up to the first order of the couplings

$$\left\langle[\phi^2][\phi^2][\phi^2]\right\rangle \;=$$



This reads

$$\left\langle[\phi^2](x)[\phi^2](y)[\phi^2](z)\right\rangle \,\propto\, N^{-\frac{3}{2}}\Big[Z_{\phi^2}^3\, C(x-y)C(y-z)C(z-x) \\ -\,\lambda_2\, C(x-y)\int d^du\, C(x-u)C(y-u)C(z-u)^2+\cdots\Big]$$

where $\cdots$ contains the permutations of the last term (in x, y, z).

Three-point Function $\langle[\phi^2](x)[\phi^2](y)[\phi^2](z)\rangle$ (2)

- After the integration, we obtain

$$\begin{aligned}&\left\langle[\phi^2](x)[\phi^2](y)[\phi^2](z)\right\rangle\\ &\propto N^{-\frac{3}{2}}\,\frac{Z_{\phi^2}^3-Qg_2\left[\frac{3}{\epsilon}+\log\left(|x'-y'||y'-z'||z'-x'|\right)+\cdots\right]}{|x'-y'|^{2\Delta_\phi}|y'-z'|^{2\Delta_\phi}|z'-x'|^{2\Delta_\phi}}\,,\end{aligned}$$

- With $Z_{\phi^2}=1+\frac{Qg_2}{\epsilon}+\cdots$ and exponentiating the log term, we obtain

$$\left\langle[\phi^2](x)[\phi^2](y)[\phi^2](z)\right\rangle\propto\frac{N^{-\frac{3}{2}}}{|x-y|^{2\Delta_\phi^\star+\delta h_{\phi^2}}\,|y-z|^{2\Delta_\phi^\star+\delta h_{\phi^2}}\,|z-x|^{2\Delta_\phi^\star+\delta h_{\phi^2}}}$$

- We find the conformal form of the 3pt function!

5. CONCLUSIONS

- We studied a tensor model with Laplacian $\mathcal{L} = (-\partial^2)^{d/4}$ and ϕ^4 interactions.

- The bilinear operator spectrum is real for pure imaginary coupling g_t and the free limit of the OPE coefficients are all positive real for pure imaginary coupling g_t.

- The real OPE coefficients of all bilinear primaries are strong indication of the unitarity of the model.

- Even though there is no local energy-momentum tensor, we have the Ward identities of the dilatation and special conformal transformation. The conformal symmetry is preserved at the IR fixed-point.

- 2pt and 3pt functions of ϕ^2 and ϕ_t^4 obey the expected conformal forms.

Thank you!