Reeb flows in dimension three with exactly two periodic orbits

Joint with Cristofaro-Gardiner, Hutchings and Liu

Umberto L. Hryniewicz

(RWTH Aachen)

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Main Question

Question. Can we understand a Reeb flow on a closed 3-manifold with precisely two periodic orbits?

Irrational rotation numbers

Reeb flows with precisely two periodic orbits the analogues in dimension three of pseudo-rotations of the 2-disk.

Definition

A pseudo-rotation of the closed disk is an area-preserving and orientation-preserving homeomorphism of the closed disk with precisely one interior periodic point.

Question. What can we say about the boundary rotation number of a pseudo-rotation?

Theorem (Franks)

It is irrational!

Irrational rotation numbers

Theorem (Franks)

Let the homeomorphism

$$f : \mathbb{R}/\mathbb{Z} \times (0,1] \to \mathbb{R}/\mathbb{Z} \times (0,1]$$

preserve area and be isotopic to the identity.

If f has no interior periodic point then its boundary rotation number is irrational.

Proof.

Step 1.

Theorem (Franks)

Let f be an area- and orientation-preserving homeomorphism of $\mathbb{R}/\mathbb{Z} \times (0,1)$.
If some lift $\tilde{f}$ *to* $\mathbb{R} \times (0,1)$ *has positively and negatively returning disks, then f has a fixed point.*

Step 2.

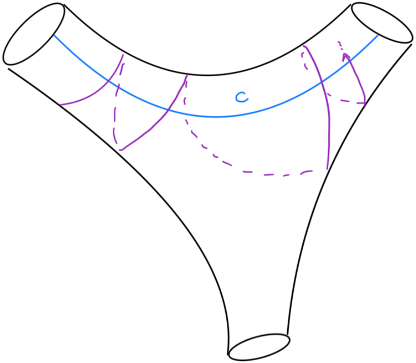
Theorem (Franks)

$M = S^2 \setminus \{k \text{ points}\}$, $k \geq 2$, $f : M \to M$ *homeomorphism isotopic to the identity preserving a Borel probability measure* μ *positive on open sets, with no atoms.*
If for some lift $\tilde{f}$ *to* $\tilde{M}$ *we have* $y \cdot \mu = 0$ *for all* $y \in H^1(M;\mathbb{R})$, *then f has a fixed point.*

Irrational rotation numbers

Irrational rotation numbers

$M = S^2 \setminus 3\text{pts}$



$\gamma = c^* \in H^1(M;\mathbb{R})$

Step 3.

Let $\tilde{f}$ be a lift to $\mathbb{R} \times (0,1]$, and let $\rho \in \mathbb{R}$ be the boundary rotation number.

If $\rho = p/q$ then $g = \tilde{f}^q - (p,0)$ has zero boundary rotation number.

If g satisfies $\mathrm{hor} \cdot \mathrm{area} = 0$, then apply Step 2 to get an interior fixed point of f^q.

If g satisfies $\mathrm{hor} \cdot \mathrm{area} \neq 0$, choose n/m between 0 and $\mathrm{hor} \cdot \mathrm{area}$. Then $g^m - (n,0)$ has positively and negatively returning disks. Apply Step 1 to get an interior periodic point. □

Irrational rotation numbers

What is the analogous statement for Reeb flows?

Theorem (Cristofaro-Gardiner, H., Hutchings, Liu)

Let a Reeb flow on a closed 3-manifold have exactly two periodic orbits γ_1, γ_2. Let $\rho(\gamma_j) \in \mathbb{R}/\mathbb{Z}$ be their rotation numbers.

Then these orbits are irrationally elliptic:

$$\rho(\gamma_1), \rho(\gamma_2) \notin \mathbb{Q}/\mathbb{Z} .$$

In other words, the contact form is non-degenerate and $\mathrm{CZ}(\gamma_j^n) = \text{odd}$ for all $n \geq 1$.

Corollary

Let M = closed 3-manifold, λ = contact form on M.

Assume that λ has exactly two periodic Reeb orbits γ_1, γ_2. Denote their primitive periods by

$$T_1, T_2 > 0$$

the contact volume by

$$\mathrm{vol}(\lambda) = \int_M \lambda \wedge d\lambda$$

and the contact structure by $\xi = \ker \lambda$.

Then:

Characterizing the Reeb flow

- $(M, \xi) \simeq (L(p,q), \xi_{\mathrm{std}})$, for some p, q.
- γ_1, γ_2 are the core circles of a genus one Heegaard decomposition, hence are p-unknotted, $\mathrm{link}_{\mathbb{Q}}(\gamma_1, \gamma_2) = 1/p$. Moreover, $\mathrm{sl}_{\mathbb{Q}}(\gamma_j) = -1/p$.
- The Seifert rotation numbers ϕ_1, ϕ_2 are irrational.
- We have identities

$$\mathrm{vol}(\lambda) = pT_1T_2 = \frac{T_1^2}{\phi_1} = \frac{T_2^2}{\phi_2} .$$

- λ is dynamically convex.
- Both γ_j span rational disk-like GSS, and Reeb dynamics can be described by a pseudo-rotation.

Characterizing the Reeb flow

Proof.

- Hutchings-Taubes $\Rightarrow$ M is a lens space, γ_1, γ_2 are the core circles of a genus one Heegaard decomposition, $\mathrm{link}_{\mathbb{Q}}(\gamma_1, \gamma_2) = 1/p$ where $p = |\pi_1(M)|$.
  Both γ_1, γ_2 are p-unknotted.
  Each γ_j has a unique lift $\tilde{\gamma}_j$ to $\tilde{M} = S^3$, there are exactly two $\tilde{\lambda}$-Reeb orbits, $\mathrm{link}(\tilde{\gamma}_1, \tilde{\gamma}_2) = 1$, both are unknotted.

- The contact form $\tilde{\lambda}$ on $\tilde{M} = S^3$ has no hyperbolic orbits.
  Hofer-Wysocki-Zehnder $\Rightarrow$ $(\tilde{M}, \tilde{\xi})$ is tight.
  Honda $\Rightarrow$ $(M, \xi) = (L(p, q), \xi_{\mathrm{std}})$ (some q).

Characterizing the Reeb flow

- Hofer-Wysocki-Zehnder $\Rightarrow$ one of the lifted orbits, say $\tilde{\gamma}_1$, has $\mathrm{sl}(\tilde{\gamma}_1) = -1$ and $\mathrm{CZ}(\tilde{\gamma}_1) = 3$. In particular, $0 < p\phi_1 < 1$.
  From the identities $p^2\phi_1\phi_2 = 1 \Rightarrow p\phi_2 > 1$.
  H.-Salomão $\Rightarrow \mathrm{sl}(\tilde{\gamma}_2) = -1 \Rightarrow \mathrm{CZ}(\tilde{\gamma}_2) \geq 5$, hence dynamical convexity.
  In particular $\mathrm{sl}_{\mathbb{Q}}(\gamma_1) = \mathrm{sl}_{\mathbb{Q}}(\gamma_2) = -1/p$.
  H.-Licata-Salomão $\Rightarrow$ both γ_1, γ_2 span rational disk-like GSS.
  Return maps extend to closed disk and are conjugated (by a homeomorphism) to a pseudo-rotation.

$\square$

The structure of the proof

The proof is based on Hutchings' ECH.

$(M, \xi) =$ closed contact 3-manifold
$\Gamma \in H_1(M)$

$ECH_*(\xi, \Gamma)$ is a vector space over $\mathbb{Z}/2\mathbb{Z}$ graded by $\mathbb{Z}/d\mathbb{Z}$ where d is the divisibility of $c_1(\xi) + 2\mathrm{PD}(\Gamma)$.

If λ is a non-degenerate contact form, $\xi = \ker \lambda$, and J is an admissible almost complex structure on $\mathbb{R} \times M$, then the chain complex $ECC_*(\lambda, \Gamma)$ is generated by orbit sets

$$\alpha = \{(\alpha_i, m_i)\} \qquad m_i \in \mathbb{N} \qquad \alpha_i \text{ is a (prime) closed Reeb orbit}$$

satisfying

$$\alpha_i \text{ hyperbolic} \qquad \Rightarrow \qquad m_i = 1\,.$$

The structure of the proof

There is a degree -1 differential

$$\delta_J : ECC_*(\lambda, \Gamma) \to ECC_{*-1}(\lambda, \Gamma)$$

defined by declaring that $\langle \delta_J \alpha, \beta \rangle$ is a $\mathbb{Z}/2\mathbb{Z}$ count of J-holomorphic curves on asymptotic to α at its positive ends, to β at its negative ends, with ECH index 1.

If $\alpha = \{(\alpha_i, m_i)\}$, $\beta = \{(\beta_j, n_j)\}$ and Z is a 2-chain satisfying $\partial Z = \sum_i m_i \alpha_i - \sum_j n_j \beta_j$ then

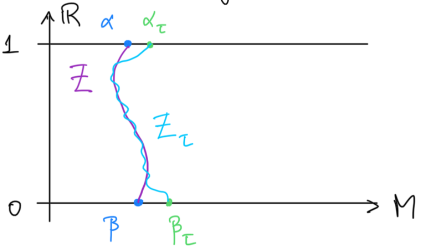
$$I(\alpha, \beta, Z) = c_\tau(Z) + Q_\tau(Z) + \mathrm{CZ}^I(\alpha) - \mathrm{CZ}^I(\beta)$$

where

The structure of the proof

$c_\tau(Z)$ = winding # of τ w.r.t. global triv. of $\xi|_Z$

$Q_\tau(Z) = \mathrm{int}(Z, Z_\tau)$



$\alpha = \{(\alpha_i, m_i)\}$

$$CZ^I_\tau(\alpha) = \sum_i \sum_{\ell=1}^{m_i} CZ_\tau(\alpha_i^\ell)$$

The structure of the proof

$$\delta_J \alpha = \sum_{\beta} \#_2\{C \in \mathcal{M}(\alpha,\beta) \mid I(\alpha,\beta,C) = 1\}\ \beta$$

where $\mathcal{M}(\alpha,\beta)$ is a space whose elements are certain weighted collections of holomorphic curves, asymptotic to α/β at positive/negative ends.

There is a degree -2 map in homology defined by a chain map

$$U_J \alpha = \sum_{\beta} \#_2\{C \in \mathcal{M}(\alpha,\beta) \text{ through } \mathrm{pt} \mid I(\alpha,\beta,C) = 2\}\ \beta$$

where pt is a point not in a closed Reeb orbit. It is called the U-map.

The structure of the proof

$\sigma \in ECH(\xi, \Gamma)$

λ non-degenerate contact form, $\xi = \ker \lambda$

$$c(\sigma, \lambda) = \inf \left\{ a > 0 \;\middle|\; \begin{array}{l} \sigma \text{ can be represented by} \\ \text{a cycle made of orbit sets} \\ \text{with action} \leq a \end{array} \right\}$$

If λ is degenerate then

$$c(\sigma, \lambda) = \lim_{\substack{\lambda' \to \lambda \\ \lambda' \text{ non-deg.}}} c(\sigma, \lambda')$$

The structure of the proof

Theorem (Cristofaro-Gardiner, Hutchings, Ramos)

Let $M =$ closed 3-manifold, λ contact form on M, $\xi = \ker \lambda$. If $c_1(\xi) + 2\mathrm{PD}(\Gamma)$ is torsion for some $\Gamma \in H_1(M)$ then $\exists \; \{\sigma_k\}_{k\in\mathbb{N}} \subset ECH(\xi, \Gamma)$ such that

$$U\sigma_{k+1} = \sigma_k \qquad\qquad \frac{c(\sigma_k, \lambda)^2}{2k} \to \mathrm{vol}(\lambda)\,.$$

Theorem (Cristofaro-Gardiner, Mazzucchelli)

If $c(\sigma_k, \lambda) = c(\sigma_{k+1}, \lambda)$ for some k, then λ is BESSE.

Corollary

If λ has exactly two periodic Reeb orbits then their periods are incommensurable.

The structure of the proof

From now on λ is a contact form on a closed 3-manifold M with exactly two periodic Reeb orbits γ_1, γ_2 with periods T_1, T_2. The contact structure is $\xi = \ker \lambda$.

Lemma

$c_1(\xi)$ is torsion in $H^2(M)$, γ_1, γ_2 are torsion in $H_1(M)$.

Proof. Choose $\Gamma \in H_1(M)$ such that $c_1(\xi) + 2\mathrm{PD}(\Gamma)$ is torsion. Let $\{\sigma_k\}$ be a U-sequence in $ECH(\xi, \Gamma)$. Then $c(\sigma_k, \lambda) = m_{1,k} T_1 + m_{2,k} T_2$ and $\Gamma = m_{1,k}[\gamma_1] + m_{2,k}[\gamma_2]$. Hence the kernel of $(m_1, m_2) \mapsto m_1[\gamma_1] + m_2[\gamma_2]$ has rank at least equal to 1. If this rank is 1 then $c(\sigma_k, \lambda)$ is increasing and contained in an arithmetic sequence, hence grows at least linearly, in contradiction to the ECH-asymptotics. □

Hence we can take $\Gamma = 0$ and still have U-sequences $\{\sigma_k\}$ satisfying the ECH-asymptotics, since $c_1(\xi) + 0 = c_1(\xi)$ is torsion.

The advantage is that there is a "simple" absolute grading on the chain complex $ECC_*(\lambda', 0)$, λ' non-degenerate, given by

$$I(\alpha') = I(\alpha', \emptyset, Z) \qquad\qquad (\partial Z = \alpha') \,.$$

The structure of the proof

Even if $\hat{\lambda}$ is a degenerate contact form,

if $\hat{\alpha} = \{(\hat{\alpha}_i, \hat{m}_i)\}$ are orbit sets for $\hat{\lambda}$

$\hat{\beta} = \{(\hat{\beta}_j, \hat{n}_j)\}$ satisfying $\sum_i \hat{m}_i [\hat{\alpha}_i] = \sum_j \hat{n}_j [\hat{\beta}_j]$ in $H_1(M)$

then define

$$I(\hat{\alpha}, \hat{\beta}, Z) = c_\tau(Z) + Q_\tau(Z) + \sum_i \sum_{\ell=1}^{\hat{m}_i} CZ_\tau(\hat{\alpha}_i^\ell) - \sum_j \sum_{\ell=1}^{\hat{n}_j} CZ_\tau(\hat{\beta}_j^\ell)$$

Choose a preferred definition of CZ such that

$$|CZ(\text{deg. orbit}) - CZ(\text{non-deg. pert.})| \leq \text{univ. cte.}$$

The structure of the proof

Main new technical statement

$\lambda' \longrightarrow \hat{\lambda}$, λ' non-degenerate

$\alpha' \longrightarrow \hat{\alpha}$ orbit sets on a

$\beta' \longrightarrow \hat{\beta}$ fixed class Γ

$$\hat{\alpha} = \{(\hat{\alpha}_i, \hat{m}_i)\} \qquad \hat{\beta} = \{(\hat{\beta}_j, \hat{n}_j)\}$$

$$\left| I(\alpha',\beta',Z') - I(\alpha,\beta,Z) \right| \leq C \left(\sum_i \hat{m}_i + \sum_j \hat{n}_j \right)$$

C: universal constant

$$\partial W_+ = \alpha' - \hat{\alpha}$$
$$\partial W_- = \hat{\beta} - \beta'$$

$$Z' = Z + W_+ + W_-$$

The structure of the proof

$$\alpha = \{(\gamma_1, m_1), (\gamma_2, m_2)\}$$

$$I(\alpha) = c_\tau(Z) + Q_\tau(Z) + CZ_\tau^I(\alpha)$$

Z 2-cycle

$\partial Z = m_1\gamma_1 + m_2\gamma_2$

γ_1, γ_2 torsion $\Rightarrow \exists\; x \in \mathbb{N} \mid x\gamma_1 = x\gamma_2 = 0$ in $H_1(M)$

D_1, D_2 2-chains $\mid \partial D_1 = x\gamma_1 \quad \partial D_2 = x\gamma_2$

$$x^2 Q_\tau(Z) = Q_\tau(xZ) = Q_\tau(m_1 D_1 + m_2 D_2)$$

$$= m_1^2 Q_\tau(D_1) + m_2^2 Q_\tau(D_2) + 2m_1m_2 Q_\tau(D_1, D_2)$$

$$= m_1^2 \underbrace{sl^\tau(x\gamma_1)}_{x^2 sl_Q^\tau(\gamma_1)} + m_2^2 \underbrace{sl^\tau(x\gamma_2)}_{x^2 sl_Q^\tau(\gamma_2)} + 2m_1m_2 \underbrace{link_Z(x\gamma_1, x\gamma_2)}_{x^2 link_Q(\gamma_1, \gamma_2)}$$

$$\Rightarrow Q_\tau(Z) = m_1^2 sl_Q^\tau(\gamma_1) + m_2^2 sl_Q^\tau(\gamma_2) + 2m_1m_2\, link_Q(\gamma_1, \gamma_2)$$

The structure of the proof

$$CZ^{T}_{\tau}(\alpha) = \sum_{j=1}^{2} \sum_{\ell=1}^{m_j} \underbrace{CZ_\tau(\gamma_j^\ell)}_{=2\ell\theta_{j,\tau} + \underbrace{O(1)}_{\in\{-1,0,1\}}} = \sum_{j=1}^{2} (m_j^2 + m_j)\,\theta_{j,\tau} + O(m_1+m_2)$$

$$I(\alpha) = \left(c_\tau(Z) + \sum_{j=1}^{2} m_j \theta_{j,\tau} \right) = \underbrace{O}_{\text{cte. depending only on } \lambda}(m_1+m_2)$$

$$+\, m_1^2 \underbrace{\left(\theta_{1,\tau} + s\ell^{\tau}_{Q}(\gamma_1)\right)}_{\phi_1} + m_2^2 \underbrace{\left(\theta_{2,\tau} + s\ell^{\tau}_{Q}(\gamma_2)\right)}_{\phi_2}$$

$$+\, 2m_1m_2\, \mathrm{link}_{Q}(\gamma_1,\gamma_2)$$

$$+\, \underbrace{O}_{\text{universal cte.}}(m_1+m_2)$$

The structure of the proof

σ_k U-seq. in $ECH(\xi, 0)$

$c(\sigma_k, \lambda) = m_{1,k} T_1 + m_{2,k} T_2$

$\alpha_k = \{(\gamma_1, m_{1,k}), (\gamma_2, m_{2,k})\}$ $\qquad \lambda' \xrightarrow{C^\infty} \lambda$

$c(\sigma_k, \lambda') = \mathcal{A}(\alpha_k') \longrightarrow m_{1,k} T_1 + m_{2,k} T_2 = \mathcal{A}(\alpha_k)$

$\alpha_k' \longrightarrow \alpha_k$ as 1-currents

$$m_{1k}^2 \phi_1 + 2 m_{1k} m_{2k} \operatorname{link}_{\circledast}(\gamma_1, \gamma_2) + m_{2k}^2 \phi_2 = I(\alpha_k) + O(m_{1,k} + m_{2,k})$$

$$= I(\alpha_k') + O(m_{1,k} + m_{2,k}) = 2k + O(m_{1,k} + m_{2,k})$$

ECH asymptotics: $m_{1,k}^2 T_1^2 + 2 m_{1,k} m_{2,k} T_1 T_2 + m_{2,k}^2 T_2^2$

$$= 2k \operatorname{vol}(\lambda) + o(k)$$

The structure of the proof

Conclusion:

$$m_{1,k}^2\left(\mathrm{vol}(\lambda)\phi_1 - T_1^2\right) + 2m_{1,k}m_{2,k}\left(\mathrm{vol}(\lambda)\,\mathrm{link}_{\mathbb{Q}}(\gamma_1,\gamma_2) - T_1T_2\right) + m_{2,k}^2\left(\mathrm{vol}(\lambda)\phi_2 - T_2^2\right) = O(m_{1,k}+m_{2,k})$$

Quadratic form on (m_1, m_2)

cte. depending only on λ

ECH asymptotics: $[m_{1,k}:m_{2,k}]$ has ∞-many accumulation pts. in $\mathbb{R}P^1$

$\Longrightarrow$ Quadratic form vanishes $\Longrightarrow$

$$\begin{cases} \mathrm{vol}(\lambda)\phi_1 = T_1^2 \\ \mathrm{vol}(\lambda)\phi_2 = T_2^2 \\ \mathrm{vol}(\lambda)\,\mathrm{link}_{\mathbb{Q}}(\gamma_1,\gamma_2) = T_1T_2 \end{cases}$$

The structure of the proof

Conclusion:

$$m_{1,k}^2\left(\mathrm{vol}(\lambda)\phi_1 - T_1^2\right) + 2m_{1,k}m_{2,k}\left(\mathrm{vol}(\lambda)\,\mathrm{link}_{\mathbb{Q}}(\gamma_1,\gamma_2) - T_1T_2\right) + m_{2,k}^2\left(\mathrm{vol}(\lambda)\phi_2 - T_2^2\right) = O(m_{1,k}+m_{2,k})$$

Quadratic form on (m_1, m_2)

cte. depending only on λ

ECH asymptotics: $[m_{1,k} : m_{2,k}]$ has ∞-many accumulation pts. in $\mathbb{RP}^1$

$\Longrightarrow$ Quadratic form vanishes $\Longrightarrow$ $\begin{cases} \mathrm{vol}(\lambda)\phi_1 = T_1^2 \\ \mathrm{vol}(\lambda)\phi_2 = T_2^2 \\ \mathrm{vol}(\lambda)\,\mathrm{link}_{\mathbb{Q}}(\gamma_1,\gamma_2) = T_1T_2 \end{cases}$

$\Longrightarrow \dfrac{\phi_1}{\phi_2} = \left(\dfrac{T_1}{T_2}\right)^2 \notin \mathbb{Q}$, $\quad \phi_1\phi_2 = \dfrac{T_1^2T_2^2}{\mathrm{vol}(\lambda)^2} = \mathrm{link}_{\mathbb{Q}}(\gamma_1,\gamma_2)^2 \in \mathbb{Q}$

$\Longrightarrow \phi_1, \phi_2 \notin \mathbb{Q} \Longrightarrow \gamma_1, \gamma_2$ irrationally elliptic

The structure of the proof

$\hat{\alpha} = \{(\hat{\alpha}_i, \hat{m}_i)\}$ orbit set for ctct. form $\hat{\lambda}$

$\lambda' \xrightarrow{C^2} \hat{\lambda}$, α' orbit set for λ', $\alpha' \longrightarrow \hat{\alpha}$

α' "splits into" α'_i = orbit set on small tub. nbd. $\mathcal{O}_i$ of $\hat{\alpha}_i$

W 2-cycle in $\bigsqcup_i \mathcal{O}_i$ s.th. $\partial W = \alpha' - \hat{\alpha}$

then $I(\alpha', \hat{\alpha}, W) = c_\tau(W) + Q_\tau(W) + CZ^I_\tau(\alpha') - CZ^I_\tau(\hat{\alpha})$

$c_\tau(W) = 0$: Can assume τ comes from a global triv. on $\bigsqcup_i \mathcal{O}_i$

$Q_\tau(W)$ → $\text{writhe}_\tau(\alpha') = \sum_i \text{writhe}_\tau(\alpha'_i)$

$CZ^I_\tau(\alpha') - CZ^I_\tau(\hat{\alpha})$ → $\sum_i CZ^I_\tau(\alpha'_i) - CZ^I_\tau(\hat{\alpha}_i, \hat{m}_i)$

Reduced to: $\text{writhe}_\tau(\alpha'_i) + CZ^I_\tau(\alpha'_i) - CZ^I_\tau(\hat{\alpha}_i, \hat{m}_i) = O(\hat{m}_i)$

The structure of the proof

α_i' in a "weighted braid with $\hat{m}_i$ strands"

$\xi_1, \dots, \xi_L$ braids

$$\alpha_i' = \bigcup_{\ell=1}^{L} \xi_\ell$$

ξ_ℓ with n_ℓ strands and multiplicity N_ℓ.

$$\sum_\ell n_\ell N_\ell = \hat{m}_i$$

$$\mathrm{writhe}_\tau(\alpha_i') = \sum_{\substack{\ell, \hat{\ell}=1 \\ \ell \neq \hat{\ell}}}^{L} N_\ell N_{\hat{\ell}} \, \mathrm{link}_\tau(\xi_\ell, \xi_{\hat{\ell}}) + \sum_{\ell=1}^{L} N_\ell^2 \, \mathrm{writhe}_\tau(\xi_\ell)$$

Work of Victor Bangert $\Longrightarrow$ $\exists\, n \mid n_\ell = n \ \forall \ell$

$\exists\, a_i \in \mathbb{Z} \mid \mathrm{rot}_\tau(\hat{\alpha}_i) = \frac{a_i}{n}$

The structure of the proof

+ Extra work $\Longrightarrow$ all ξ_ℓ are (n,a) torus knots in $\mathcal{O}_i$

[not done in the paper since it can be avoided]

$$\text{writhe}_\tau(\alpha_i') = \sum_{\ell \neq \hat{\ell}} an N_\ell N_{\hat{\ell}} + \sum_\ell N_\ell^2 a(n-1)$$ (+)

$$CZ_\tau^I(\hat{\alpha}_i, \hat{m}_i) = \frac{a}{n}(\hat{m}_i^2 + \hat{m}_i) + O(\hat{m}_i)$$

$$= \dots = an \sum_\ell N_\ell^2 + an \sum_{\ell \neq \hat{\ell}} N_\ell N_{\hat{\ell}} + a \sum_\ell N_\ell + O(\hat{m}_i)$$ (−)

$$CZ_\tau^I(\alpha_i') = a\left(\sum_\ell N_\ell^2 + N_\ell\right) + O\left(\sum_\ell N_\ell\right)$$ (+)

A question

Question: Is a smooth pseudo-rotation with Diophantine boundary rotation number smoothly conjugated to a rigid rotation?

Question (Hofer): If a contact form on S^3 has exactly two periodic Reeb orbits, and one of the rotation numbers (equivalently both) of these orbits is Diophantine, then is it strictly contactomorphic to the boundary of an irrational ellipsoid?

Thank you!