Perturbing binary black holes with effective field theory

Based on 2004.03570

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MEETING OF THE NATIONAL RESEARCH GROUP ON GRAVITATIONAL WAVES

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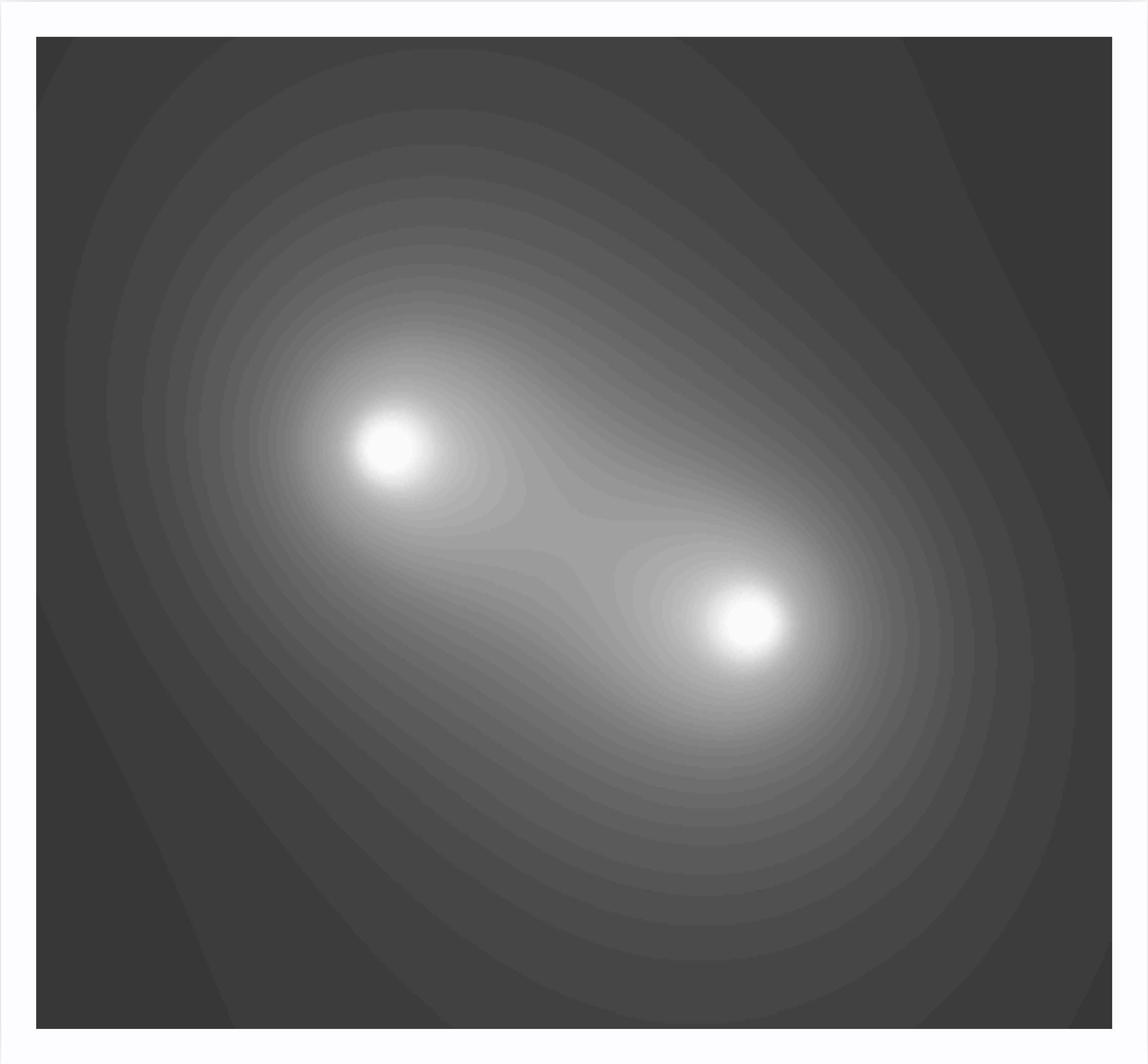
GRAVITATIONAL MOLECULES

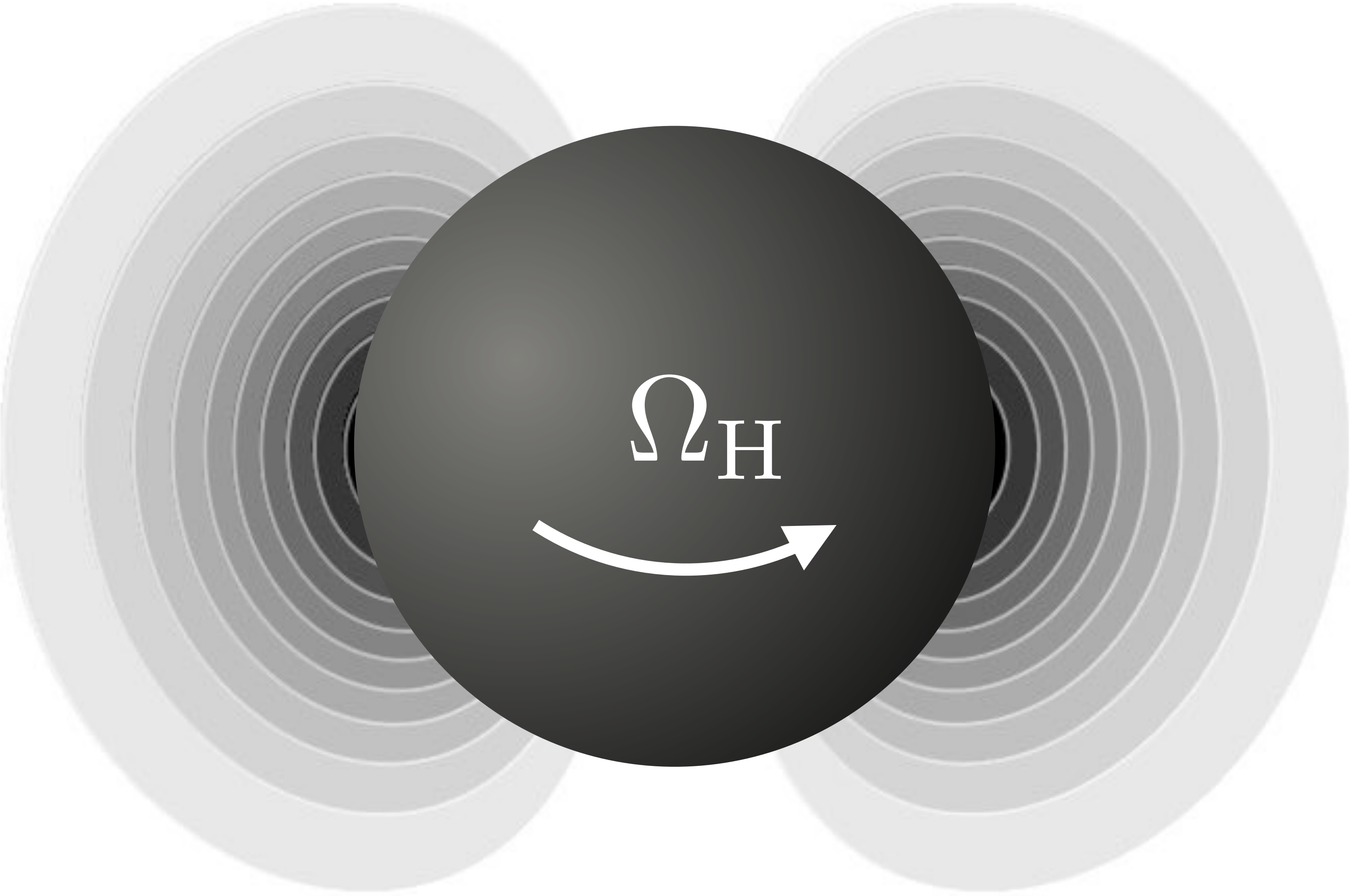
General relativity + ultralight boson



[Ikeda, Bernard, Cardoso, Zilhão 2020]

GRAVITATIONAL ATOM

(n, ℓ, m)



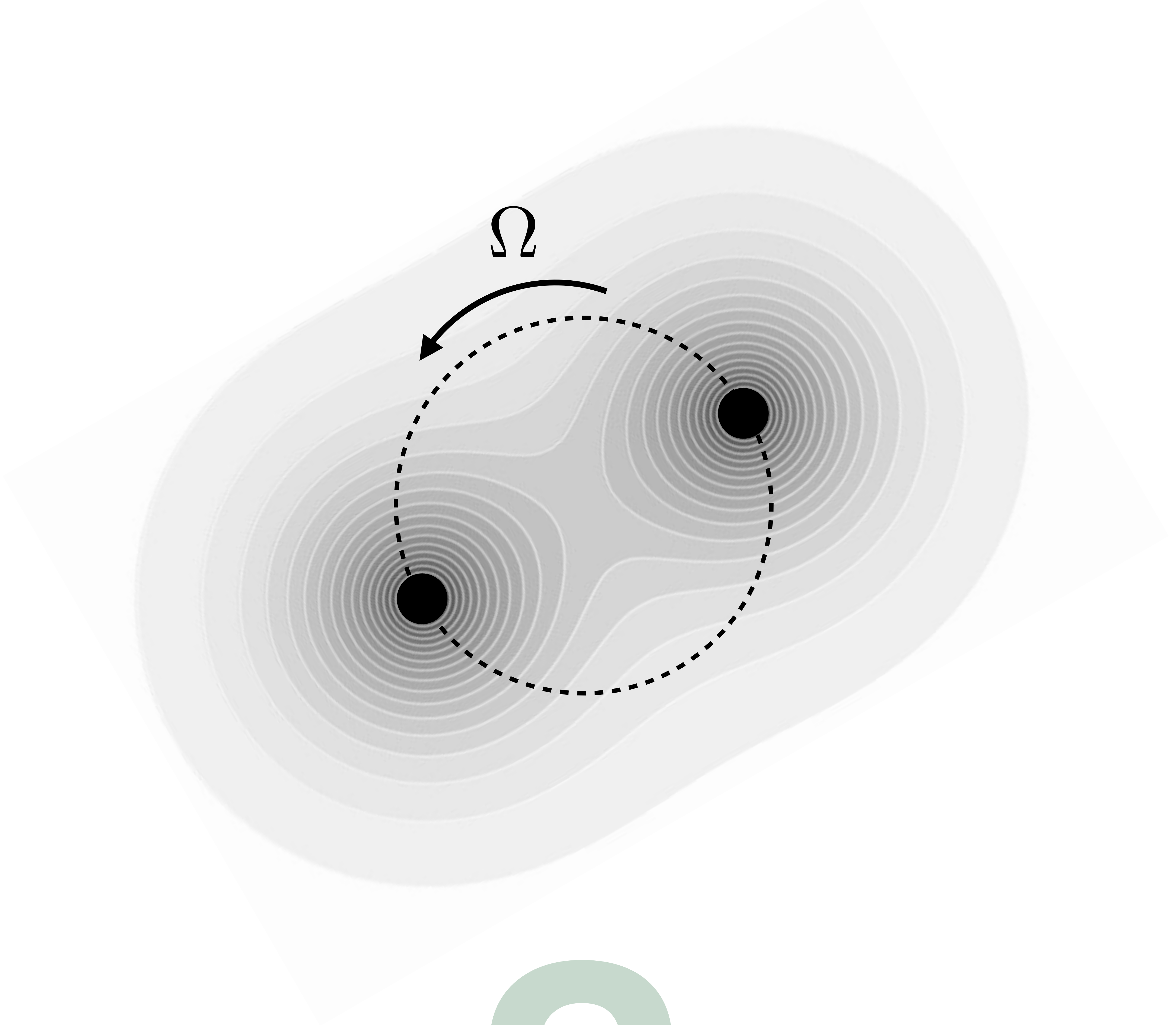
$$0 < E_{n\ell m} < m\Omega_{\rm H}$$

EXPONENTIAL GROWTH

$(E_{n\ell m} = \mu + \text{binding energy})$

[Detweiler 1980; Dolan 2007; etc.]

GRAVITATIONAL MOLECULE



EXPONENTIAL GROWTH ?

SIMPLIFYING ASSUMPTIONS

1. **Low energy density**: Neglect backreaction of the cloud to a first approximation

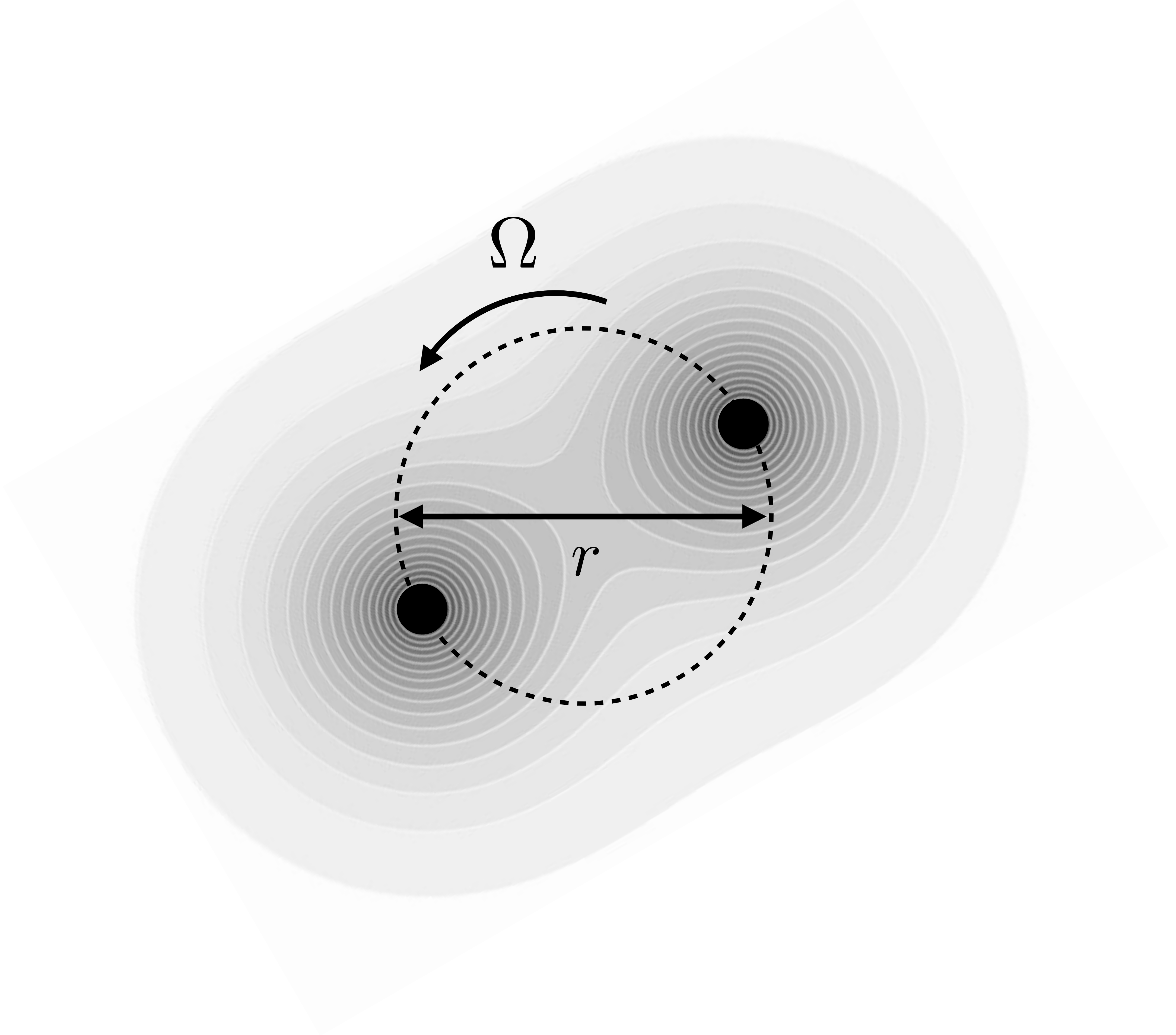
2. **Early inspiral stage**: Post-Newtonian approximation of the (circular) binary

$$v^2 \sim \frac{GM}{r} \ll 1$$

3. **Long-wavelength limit**: Treat the entire binary as an effective point particle

$$r/\lambda \ll 1 \quad \Leftrightarrow \quad \mu \ll \Omega/v^2$$

$$\boxed{\mu \ll 10^{-13}\ \mathrm{eV}\left(\frac{v}{0.1}\right)\left(\frac{100\ M_\odot}{M}\right)}$$



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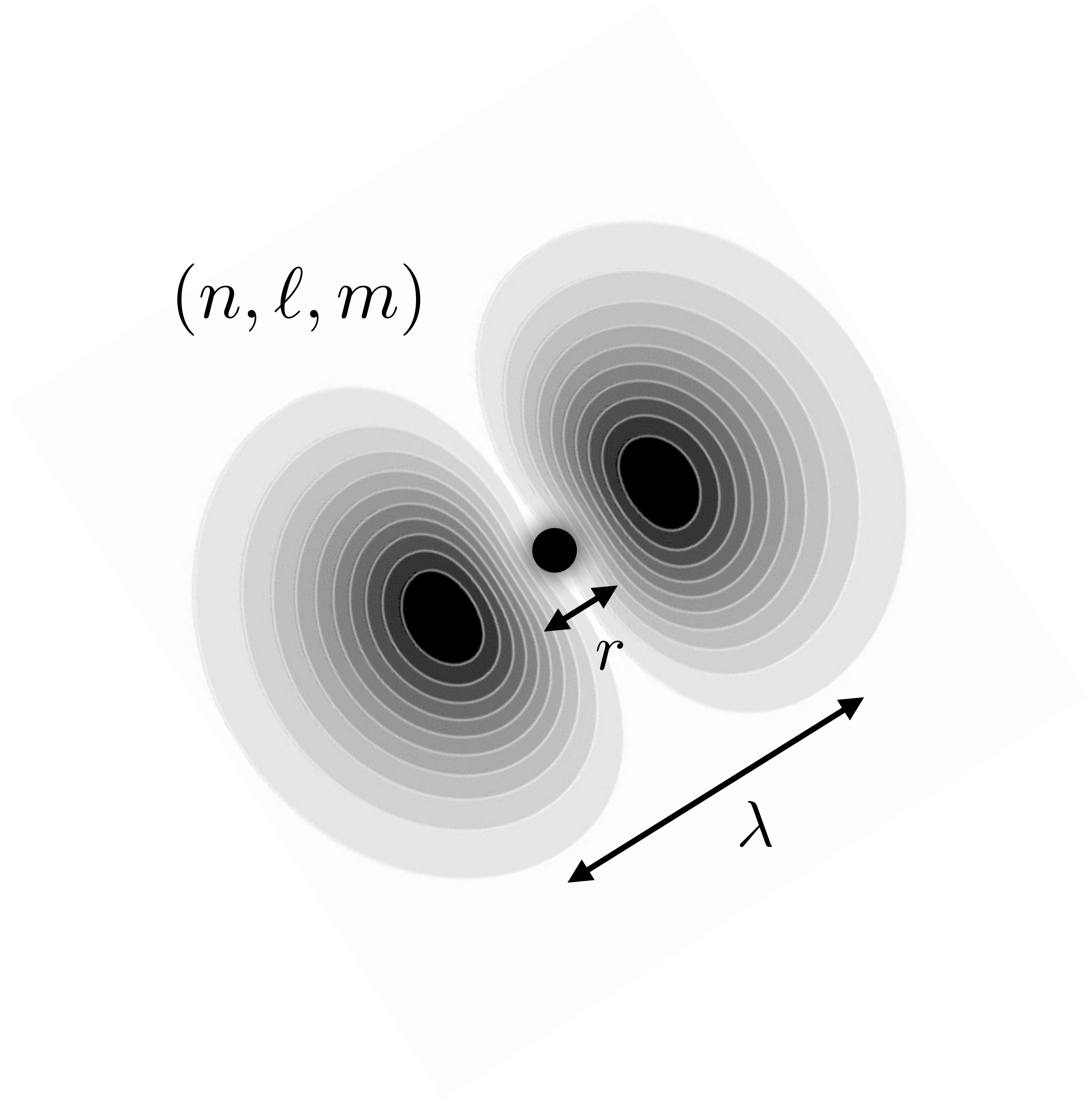


EFFECTIVE EQUATION OF MOTION

The evolution of the cloud is given by solving $\Box_{\text{eff}}\phi = 0$, where

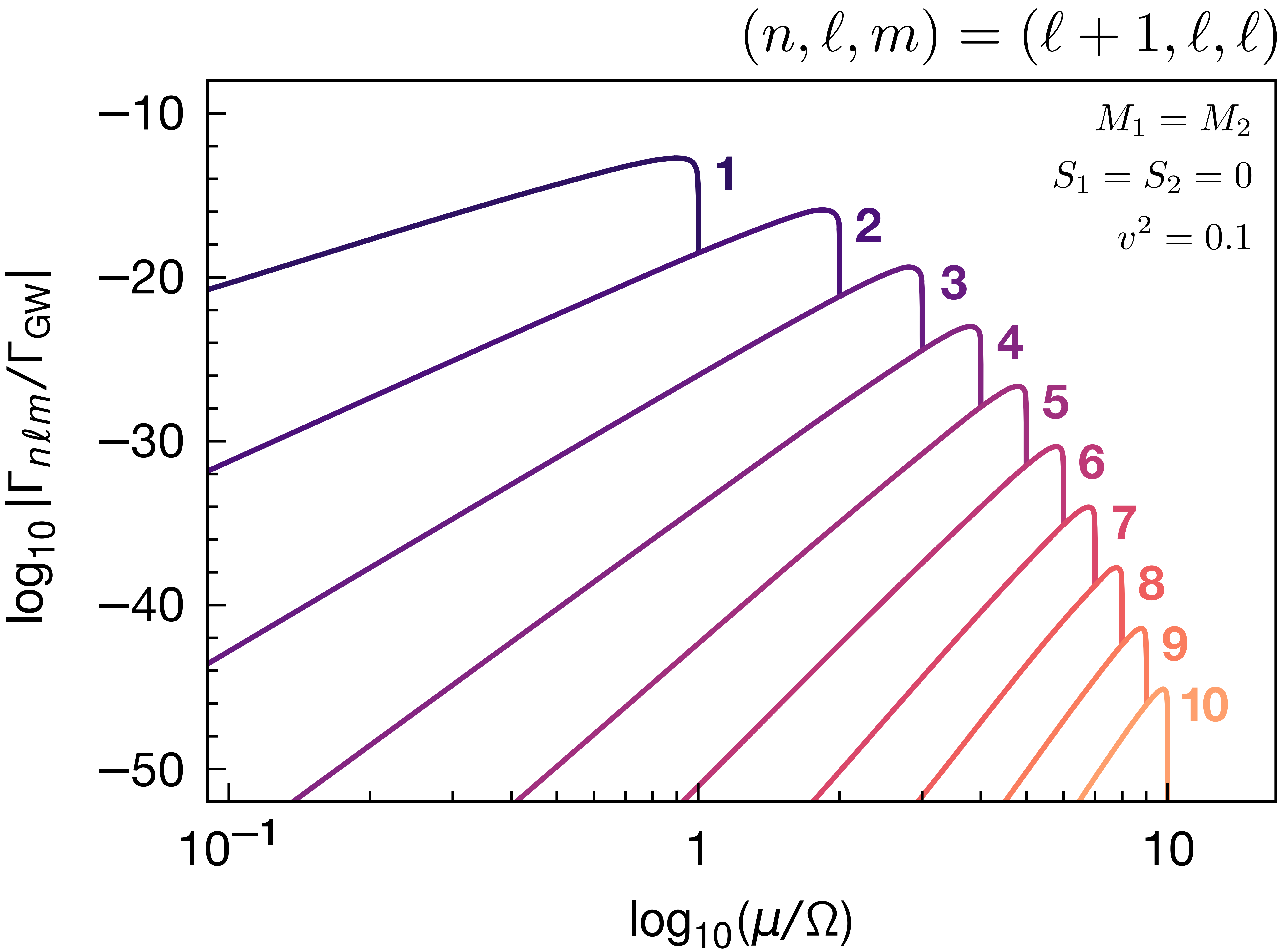
$$\Box_{\text{eff}} = \underbrace{\left(\eta^{\mu\nu}\partial_\mu\partial_\nu - \mu^2 + \frac{2GM\mu^2}{r}\right)}_{\Box_{\text{free}}} + \underbrace{\left(\sum_{\ell=0}^{\infty} \partial_{i_1}\cdots\partial_{i_\ell}\delta^{(3)}(\mathbf{x})\,\mathcal{O}^{i_1\cdots i_\ell}(t)\right)}_{\Box_{\text{int}}} + \cdots$$

$\Box_{\text{free}}$: Leading dynamics of the cloud in the bulk of the spacetime

$\Box_{\text{int}}$: Contact interactions describing absorption by and energy-momentum transfer with the moving black hole horizons

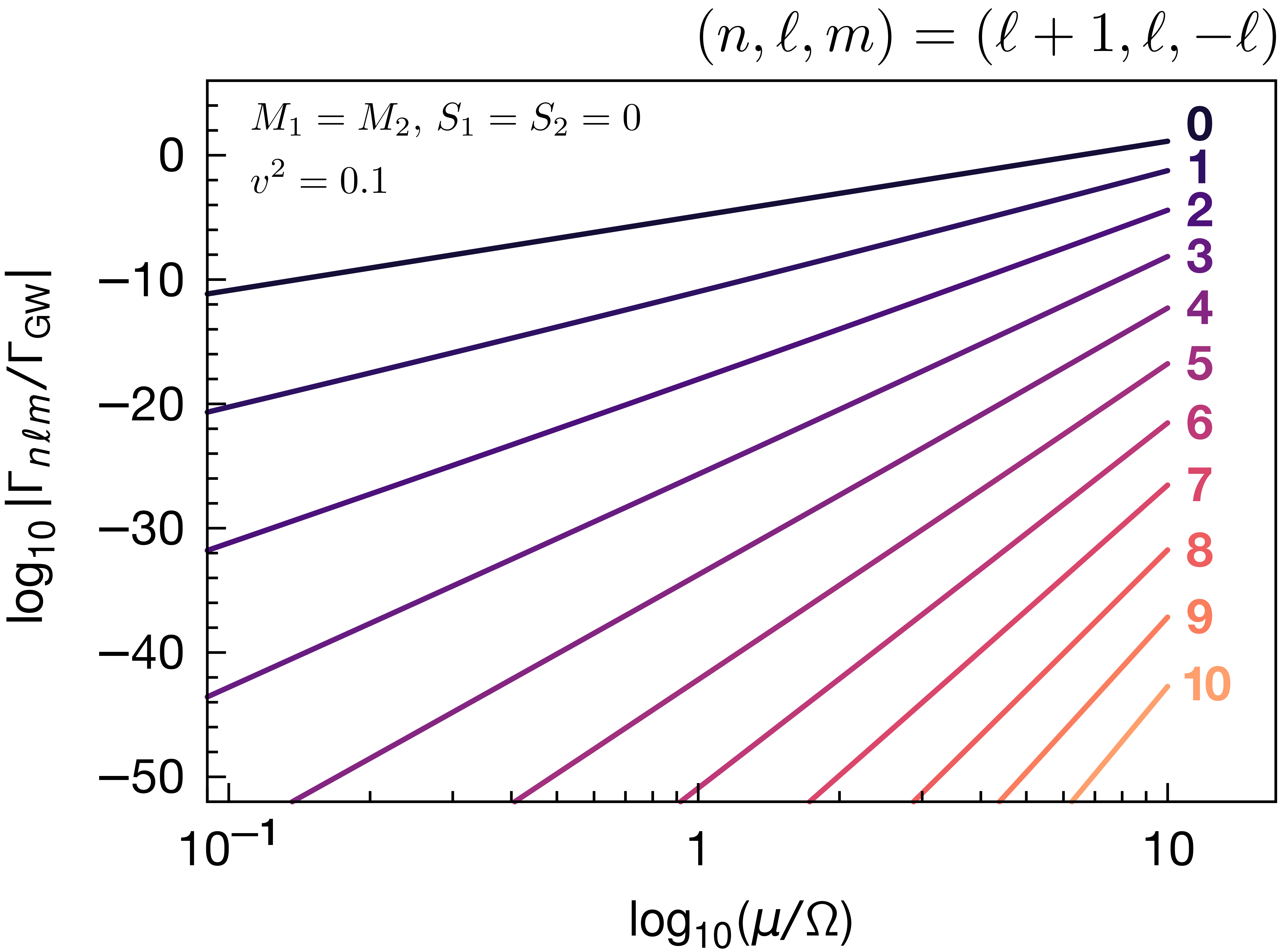
[Wong et al. 1903.07080, 1905.08543, 2004.03570]

$$\mathcal{O}^{i_1\cdots i_\ell}(t) = -\lim_{\mathbf{x}\to 0}\sum_{N=1}^{2}\sum_{\ell'=0}^{\infty}\frac{A_N}{\ell!\ell'!}\mathbf{z}_N^{\langle i_1}\cdots\mathbf{z}_N^{i_\ell\rangle}\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{z}_N^{\langle j_1}\cdots\mathbf{z}_N^{j_{\ell'}\rangle}\,\partial_{j_1}\cdots\partial_{j_{\ell'}}\,\cdot\,\right]$$



GROWTH RATES

- Bound state grows exponentially like $\psi_{n\ell m} \sim e^{\Gamma_{n\ell m} t}$ when $0 < E_{n\ell m} < m\Omega$
- Energy is predominantly extracted from the orbital motion
- But timescale is not observationally relevant

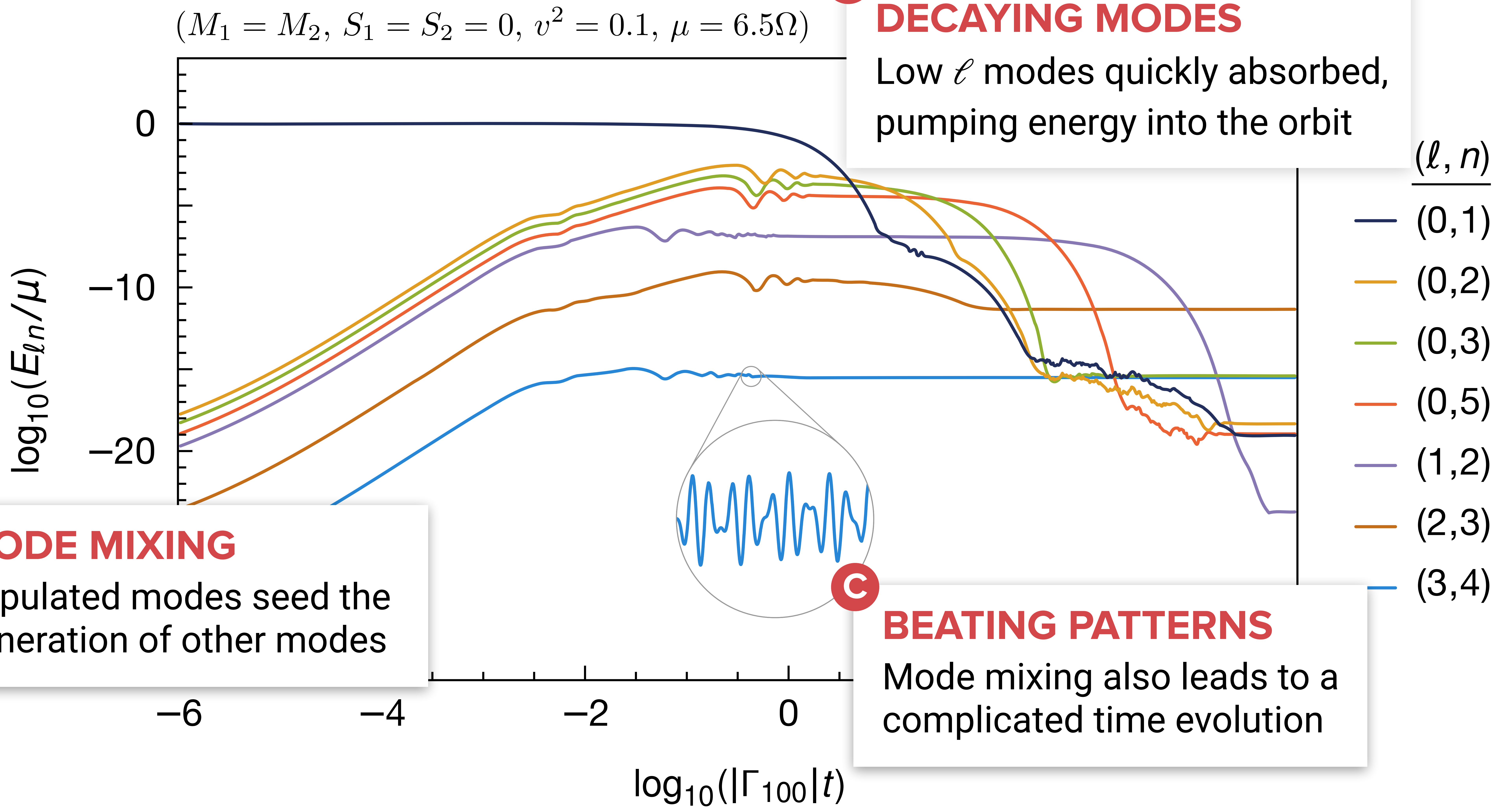


DECAY RATES

- Decaying modes pump energy into the orbit
- Upward trend suggests potential for observational signatures at larger values of μ

EVOLUTION OF THE CLOUD

Energy in the (ℓ, n) levels after a time t, with only the (1,0,0) mode populated at t=0



A DECAYING MODES

Low ℓ modes quickly absorbed, pumping energy into the orbit

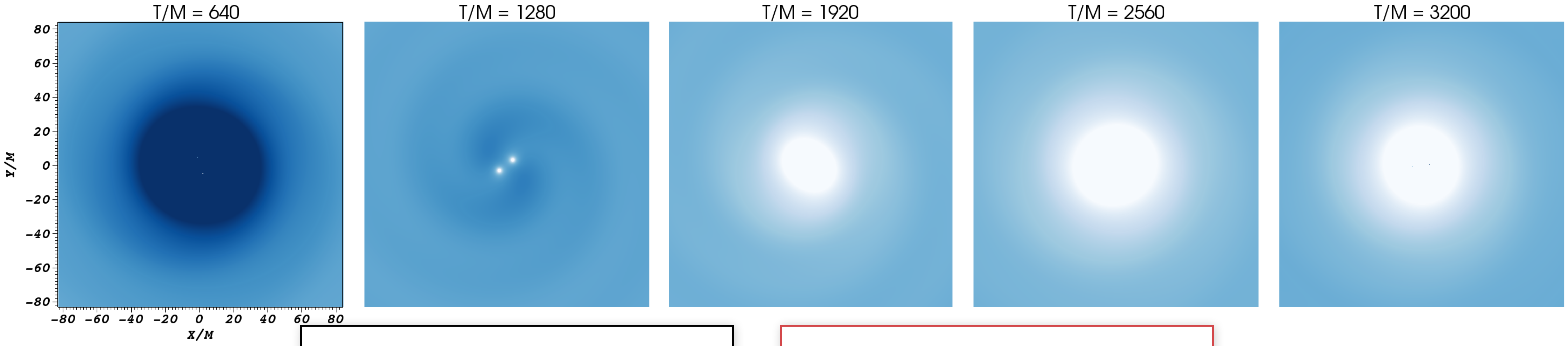
B MODE MIXING

Populated modes seed the generation of other modes

C BEATING PATTERNS

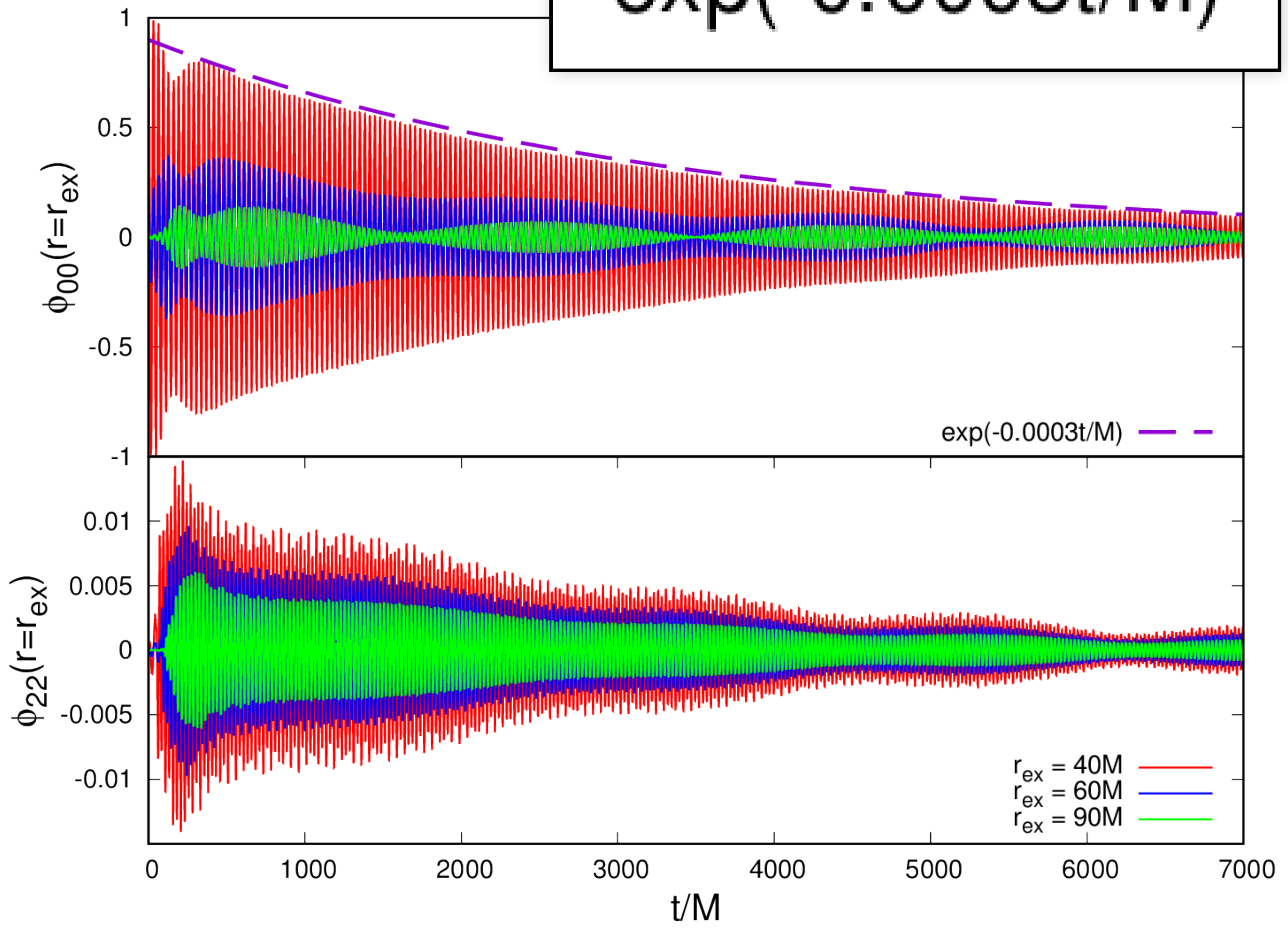
Mode mixing also leads to a complicated time evolution

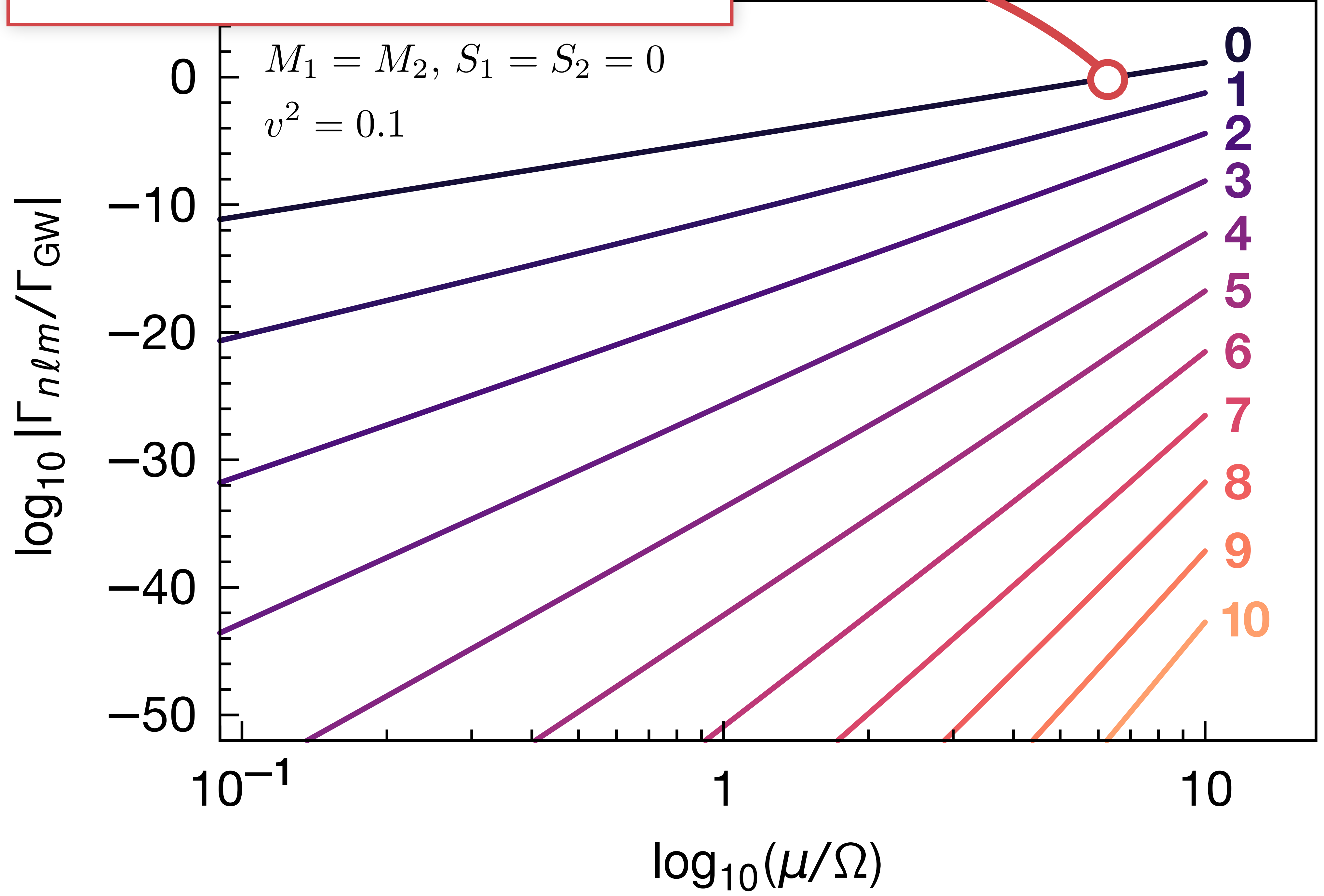
COMPARISON WITH NUMERICAL SIMULATIONS [Ikeda, Bernard, Cardoso, Zilhão 2020]

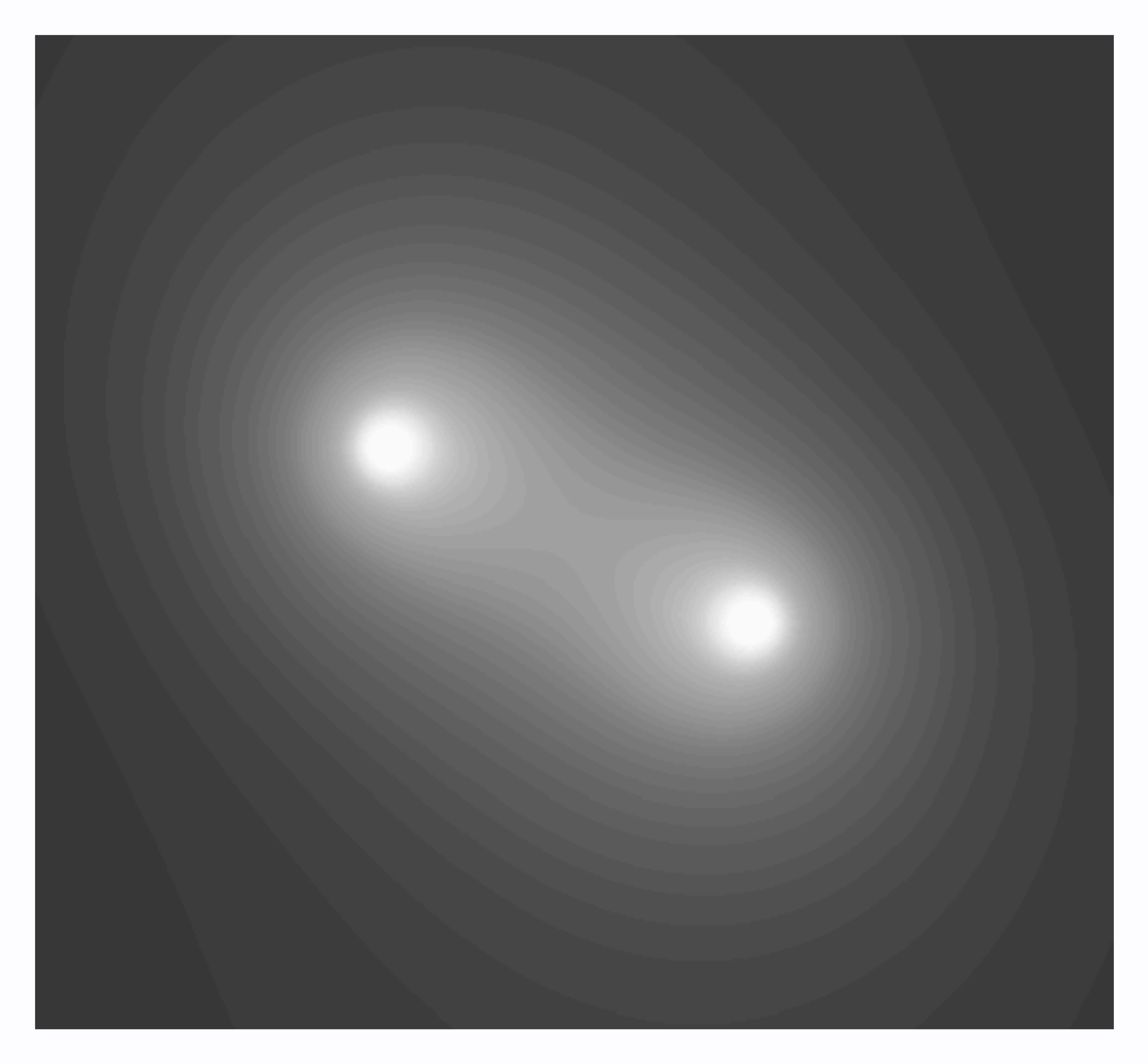


$\exp(-0.0003t/M)$

$0.00026\,(GM)^{-1}$







[Ikeda, Bernard, Cardoso, Zilhão 2020]

OPEN QUESTIONS

- Does the cloud leave a measurable signal for larger μ?
- Viable formation scenarios?

OUTLOOK

- EFT methods facilitate the systematic study of complicated systems
- Analytic results can help us interpret what we see in numerical simulations
- Similar techniques applicable to many other systems of astrophysical interest