Metric reconstruction with gravitational waves and shadows

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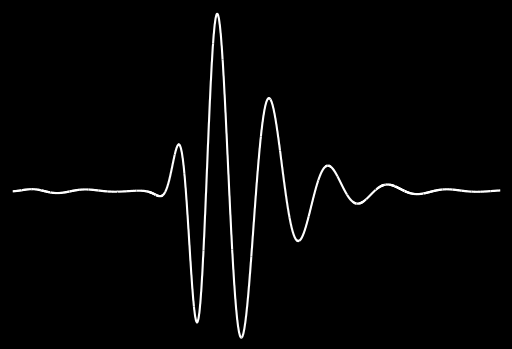
SHV, Enrico Barausse, PRD 102 084025, `arXiv:2007.02986`
SHV, Enrico Barausse, Nicola Franchini, Avery E. Broderick, `arXiv:2011.06812`
Arthur G. Suvorov, **SHV**, PRD 103 044027, `arXiv:2101.09697`

Meeting of the National Research Group on Gravitational Waves
Institut Henri Poincaré, Paris, (online)

01.04.2021

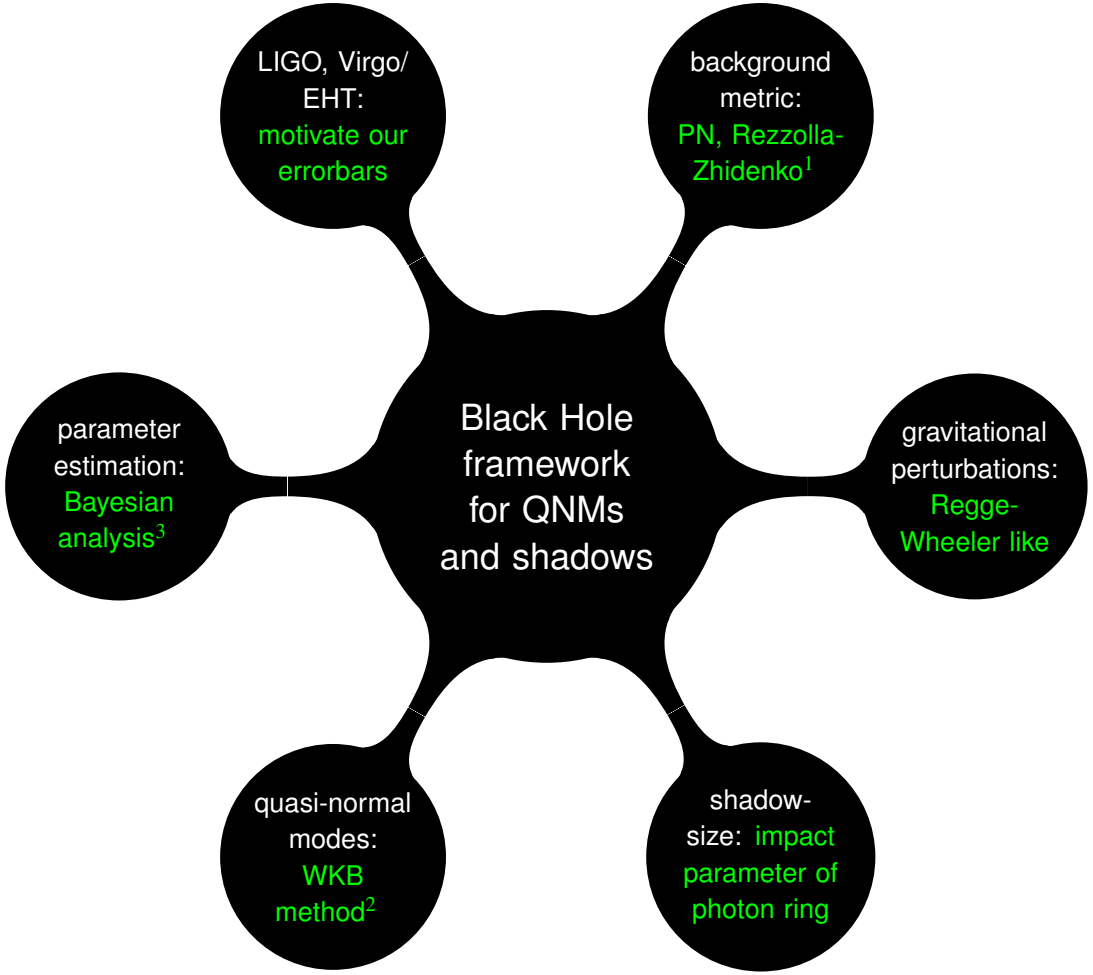
ERC-2018-COG GRAMS 815673

Two complementary ways to test black holes



gravitational waves
(LIGO/Virgo/KAGRA)

BH imaging
(EHT collaboration)



Rezzolla-Zhidenko metric:

$$\mathrm{d}s^2 = -N^2(r)\mathrm{d}t^2 + \frac{B^2(r)}{N^2(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Perturbation potential:

$$V_l(r) = \frac{l(l+1)}{r^2}N^2(r) - \frac{K}{r}\frac{\mathrm{d}}{\mathrm{d}r^*}\frac{N^2(r)}{B(r)}$$

QNM computation:

$$\frac{iQ_0}{\sqrt{2Q_0''}} - \sum_i \Lambda_i = n + \frac{1}{2}$$

Shadow size:

$$b_{\mathrm{ph}} = \frac{r_{\mathrm{ph}}}{\sqrt{-g_{tt}(r_{\mathrm{ph}})}}$$

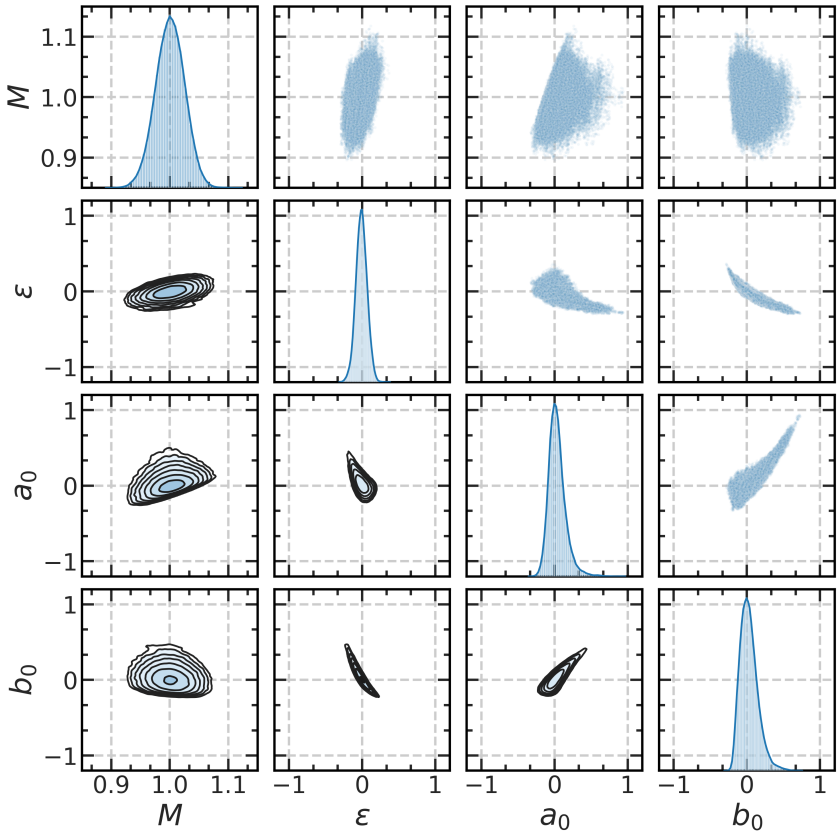
[1] Rezzolla, Zhidenko, Phys. Rev. D 90, 084009 (2014)
[2] Iyer, Will, Phys. Rev. D 35, 3621 (1987); Konoplya, Phys. Rev. D 68, 024018 (2003); ...
[3] MCMC sampling with PyMC3

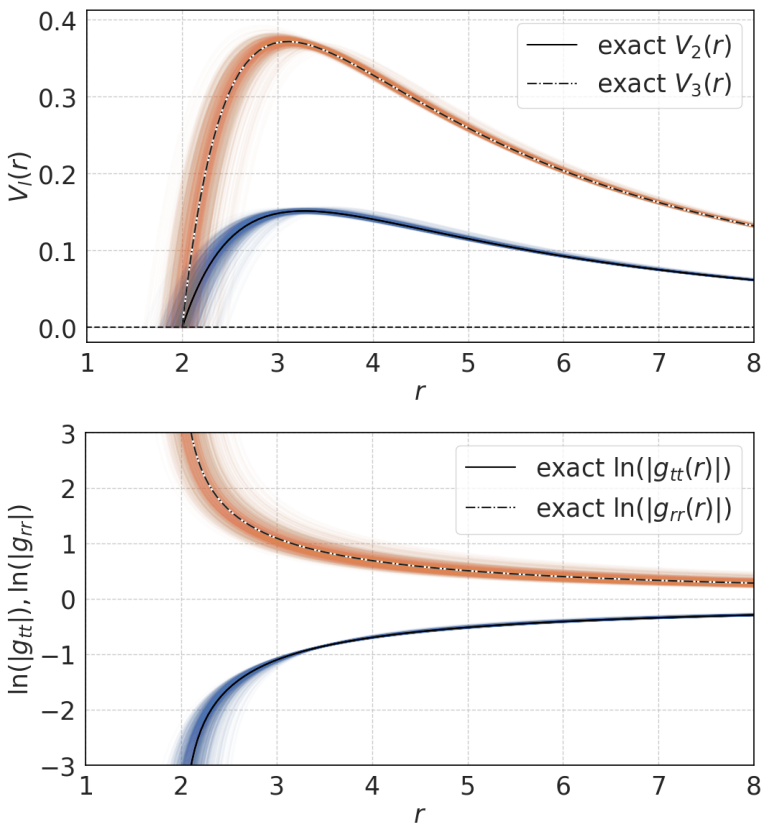
Part I

Bayesian Metric Reconstruction with Gravitational Wave Observations

SHV, Enrico Barausse

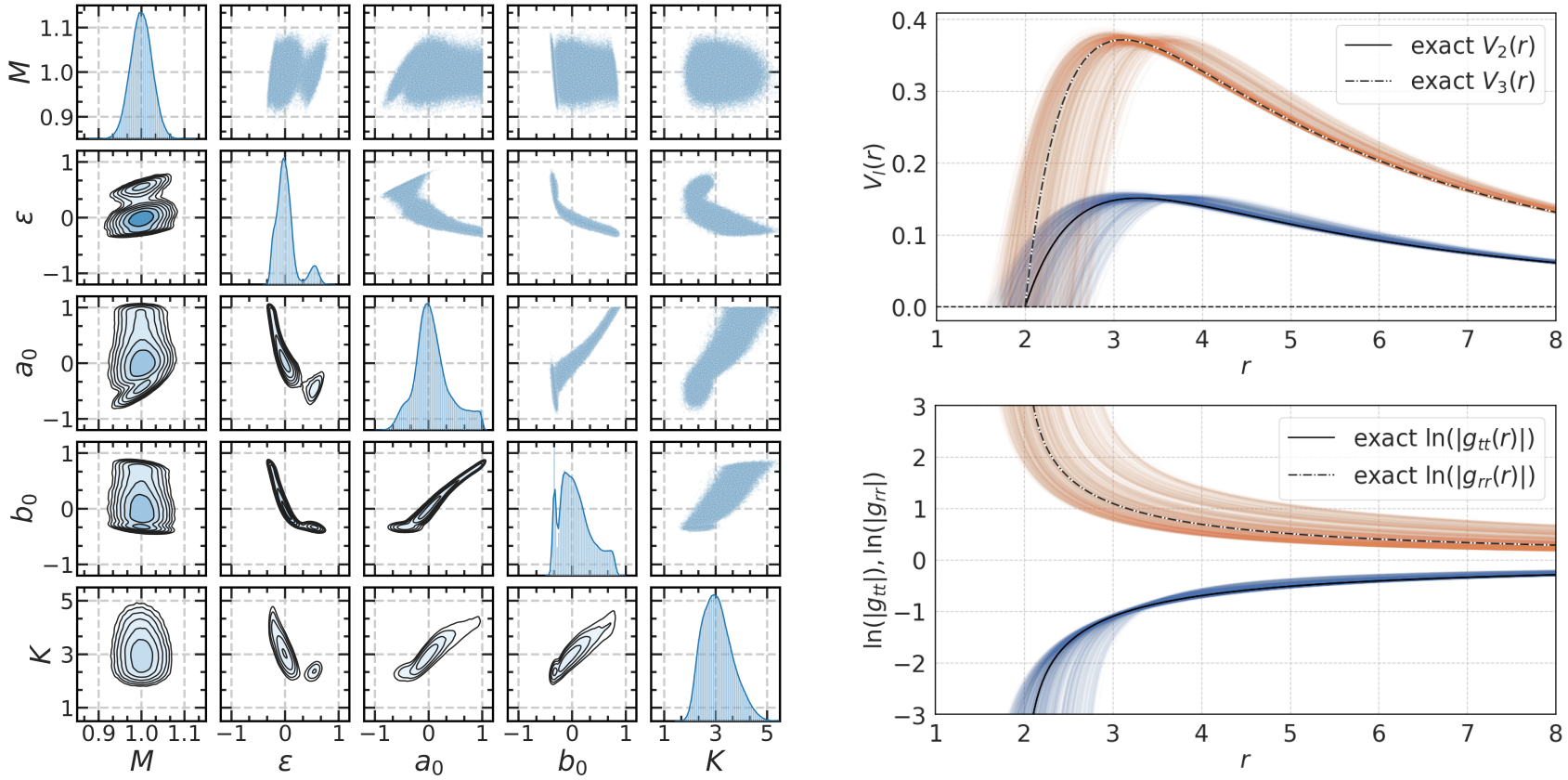
MODEL$_2$ WITH SPECTRUM$_2$ AT 1 %





Results for model$_2$ obtained by using spectrum$_2$ with $\pm 1\,\%$ relative error. Left: MCMC parameter estimation. Right top: Exact (black lines) and reconstructed (color lines) potentials $V_2(r)$ and $V_3(r)$. Right bottom: Exact (black lines) and reconstructed (color lines) metric functions $g_{tt}(r)$ and $g_{rr}(r)$.

MODEL$_{K2}$ WITH SPECTRUM$_2$ AT 1 %



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Part II

EHT tests of the strong-field regime of General Relativity

SHV, Enrico Barausse, Nicola Franchini, Avery E. Broderick

EHT TESTS OF THE STRONG-FIELD REGIME OF GENERAL RELATIVITY

Using the BH image to test GR?[4]

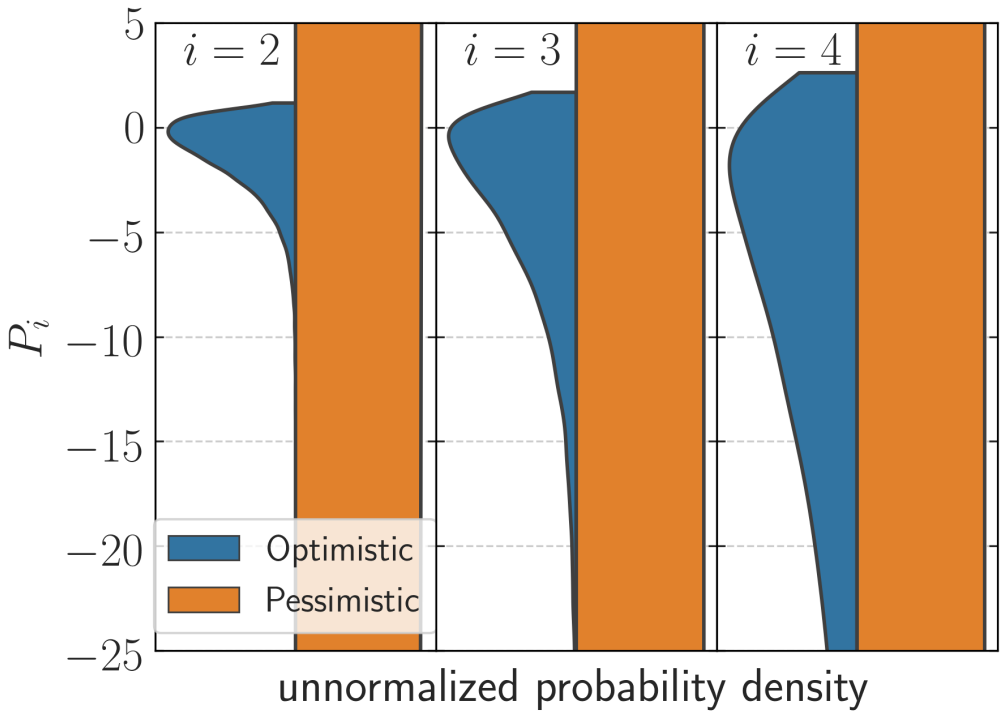
- involved data analysis, GRMHD simulations, feature extraction,...
- EHT: shadow size robust and identified in image as predicted ($\sim$ 17%)[5]
- Recently: claiming gravitational tests beyond first PN order [6]

How robust are shadow-size measurements to test GR?[7]

[4] E.g., Cunha, Herdeiro, & Radu, PRL 123, 011101, (more Refs. in paper!)
[5] The Event Horizon Telescope Collaboration et al 2019 ApJL 875 L1
[6] Psaltis et al. (EHT Collaboration), PRL 125, 141104
[7] SHV, Barausse, Franchini, and Broderick, arXiv:2011.06812

EHT TESTS OF THE STRONG-FIELD REGIME OF GENERAL RELATIVITY

PN metric: $-g_{tt} = 1 - \frac{2M}{r} + \sum_{i=1}^{\infty} P_i \left(\frac{M}{r}\right)^{i+1}$, "shadow-size": $b_{\text{ph}} = \frac{r_{\text{ph}}}{\sqrt{-g_{tt}(r_{\text{ph}})}}$



Optimistic (blue) and conservative (orange) posterior distributions for the PN coefficients P_i using the 17% relative error margin for the observed shadow-size of M87* as observed by the EHT collaboration with the expected shadow-size predicted by the independent measurement of the BH mass coming from stellar dynamics.

Part III

Exact theory for the Rezzolla-Zhidenko metric and self-consistent calculation of quasinormal modes

Arthur G. Suvorov, **SHV**

"SELF CONSISTENT" QNMS BEYOND GR

Building a theory around a given RZ metric [8]

- full details in our paper, basic summary:

$$\mathcal{A} = \kappa \int d^4x \sqrt{-g} f(X),$$

with

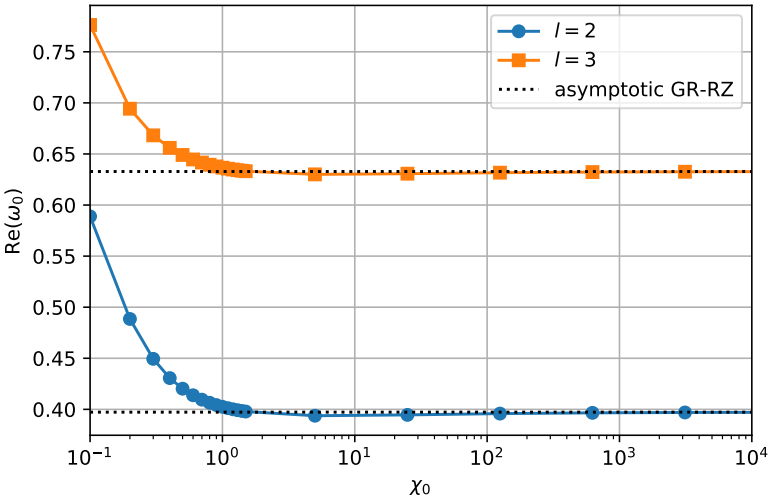
$$f(X) = X^{1+\sigma}, \qquad \text{and} \qquad X \equiv F(\phi)R + \mathcal{V}(\phi) - \chi(\phi)\nabla_\alpha\phi\nabla^\alpha\phi,$$

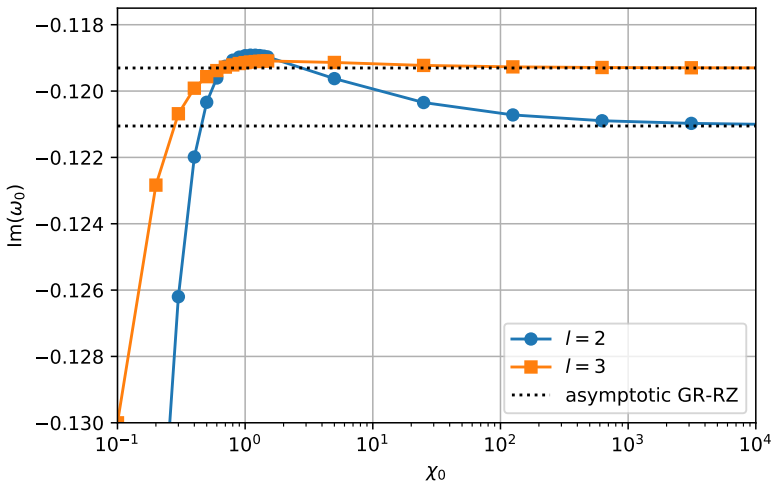
- choosing metric determines $\chi(\phi)$
- axial perturbations still Schrödinger like wave equation

[8] Based on Arthur G. Suvorov, Gen.Rel.Grav. 53 (2021) 1, 6, https://arxiv.org/abs/2008.02510

GR-LIKE VS SPECIFIC THEORY

Fixing RZ metric, varying theory within “family”:





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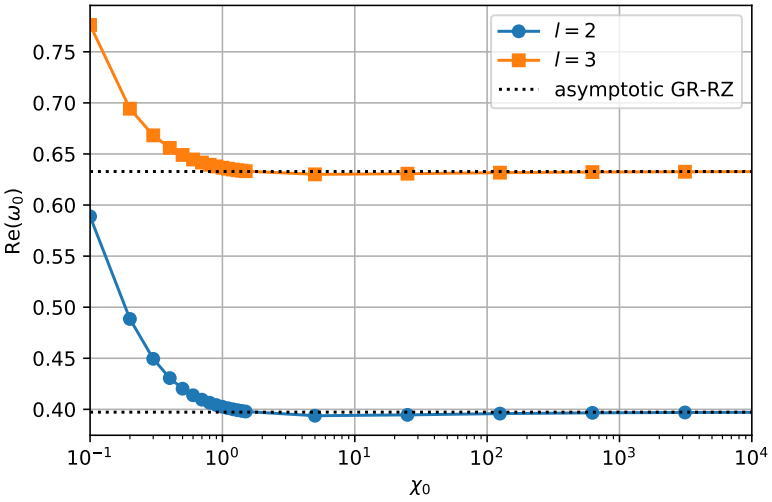
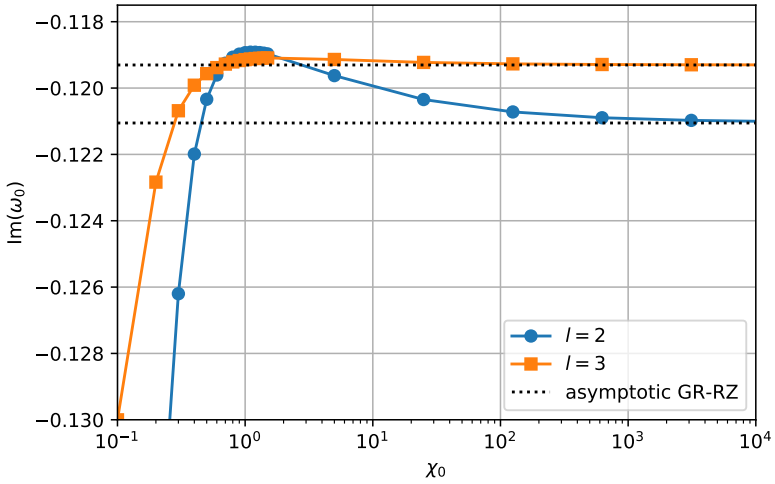
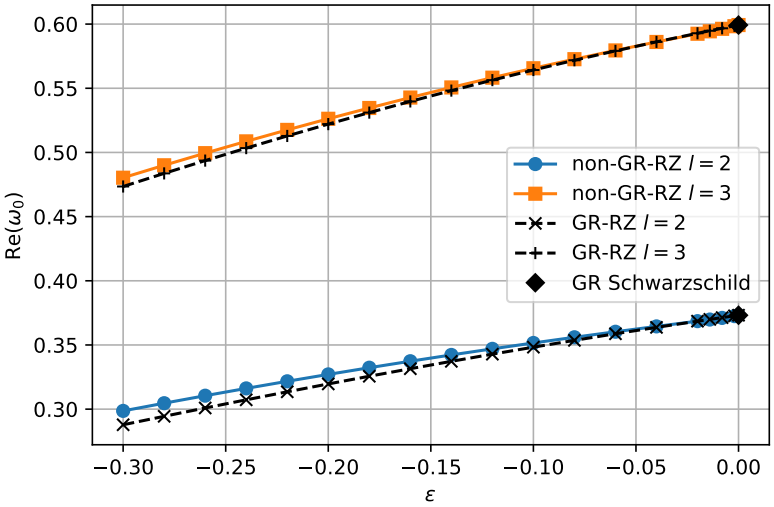
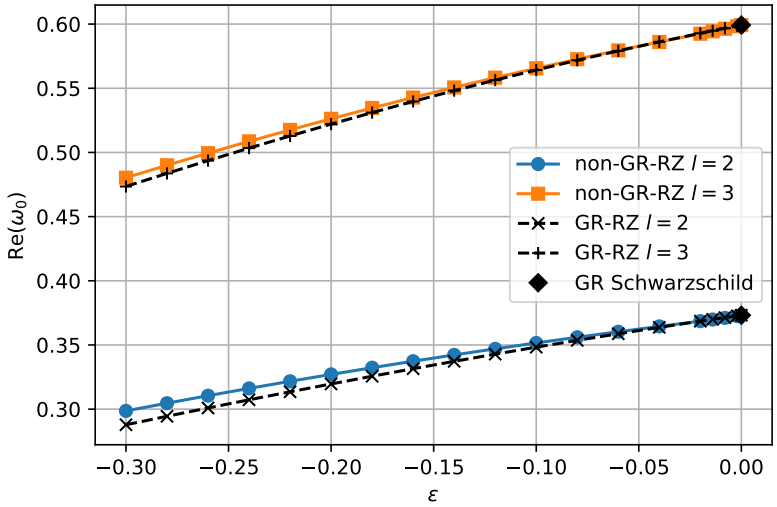
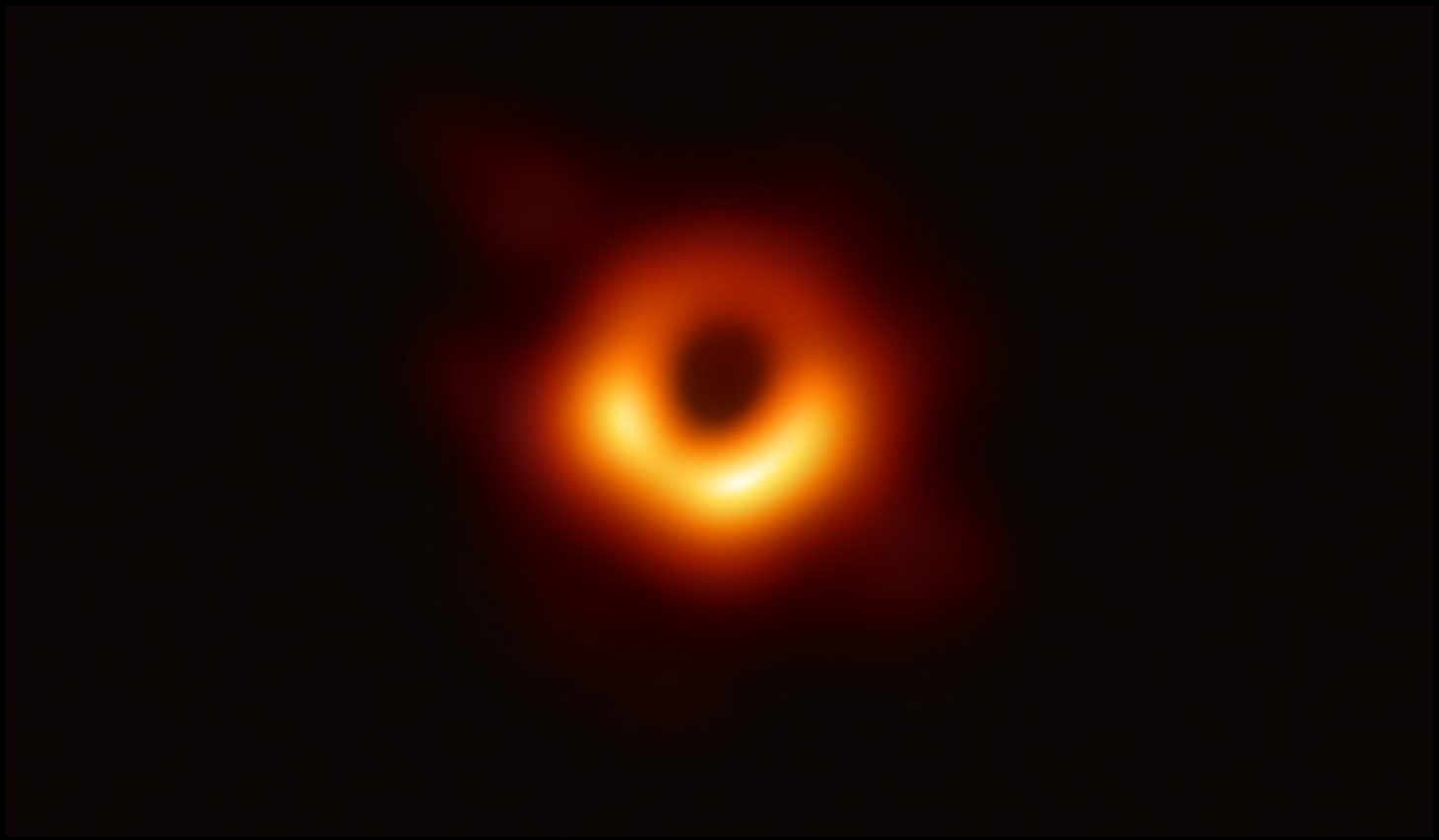




Fixing theory within "family", varying RZ metric:







Hope to see all of you in person next time!

Background Metric

We use the Rezzolla-Zhidenko (RZ) metric[9]

- parametrization for spherically symmetric and static black holes
- continued fraction expansion for $\tilde{A}(x)$ and $\tilde{B}(x)$
- relation to PPN parameters β and γ possible

$$\mathrm{d}s^2 = -N^2(r)\mathrm{d}t^2 + \frac{B^2(r)}{N^2(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \qquad x \equiv 1 - \frac{r_0}{r}, \qquad N^2 = xA(x), \tag{1}$$

$$A(x) = 1 - \varepsilon(1-x) + (a_0 - \varepsilon)(1-x)^2 + \tilde{A}(x)(1-x)^3, \tag{2}$$

$$B(x) = 1 + b_0(1-x) + \tilde{B}(x)(1-x)^2. \tag{3}$$

$$\varepsilon = -\left(1 - \frac{2M}{r_0}\right), \qquad a_0 = \frac{(\beta-\gamma)(1+\varepsilon)^2}{2}, \qquad b_0 = \frac{(\gamma-1)(1+\varepsilon)}{2}. \tag{4}$$

[9] Phys. Rev. D 90, 084009, 2014

Perturbation Equations

We study "theory agnostic" gravitational axial perturbations

- we consider $\delta R_{\mu\nu} = 0$ for the RZ metric
- corresponds to GR, but also holds for some scalar tensor theories

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} Z + \left[\omega^2 - V_l(r)\right] Z = 0\,, \tag{5}$$

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$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} Z + \left[\omega^2 - V_l(r)\right] Z = 0\,, \tag{5}$$

- also include parametrized modification of the potential (K)

$$V_l(r) = \frac{l(l+1)}{r^2} N^2(r) - \frac{K}{r} \frac{\mathrm{d}}{\mathrm{d}r^*} \frac{N^2(r)}{B(r)}\,, \tag{6}$$

Quasi-Normal Modes/ WKB

Quasi-Normal Modes (QNMs) describe ringdown of black holes

- defined by purely outgoing ($r \to \infty$) and ingoing ($r \to r_0$) waves
- computation of QNMs via higher order WKB method

$$\frac{iQ_0}{\sqrt{2Q_0''}} - \Lambda_2 - \Lambda_3 - \Lambda_4 - \Lambda_5 - \Lambda_6 = n + \frac{1}{2}, \tag{7}$$

with $Q(r^*) \equiv \omega_n^2 - V_l(r^*)$ evaluated at the maximum of potential[10].

$$\Lambda_2(n) = \Big((-11(V^{(3)})^2 + 9V^{(2)}V^{(4)} - (30(V^{(3)})^2)n + \tag{8}$$

$$18V^{(2)}V^{(4)}n - (30(V^{(3)})^2)n^2 + 18V^{(2)}V^{(4)}n^2) \Big) / 144(V^{(2)})^2 \tag{9}$$

[10] R. A. Konoplya, Phys. Rev. D 68, 024018, 2003

Bayesian Analysis

Bayes theorem:

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} \tag{10}$$

with

- θ **parameters of a model**
- D **observed data**

and

- **posterior** $P(\theta|D)$: probability of parameters given the data
- **likelihood** $P(D|\theta)$: probability of data given the parameters
- **prior** $P(\theta)$: probability of parameters before looking at data
- **evidence** $P(D)$: probability of Data