Alexandre Toubiana (APC/IAP)

H2020-MSCA-RISE-2015 StronGrHEP-690904

Populations/multi-messenger meeting
10/03/2021

Discriminating between different scenarios for the formation and evolution of massive black holes with LISA

With S. Babak, E. Barausse, E. Berti, J. Gair, N. Pol, S. Taylor, K. Wong

Inferring the MBH population

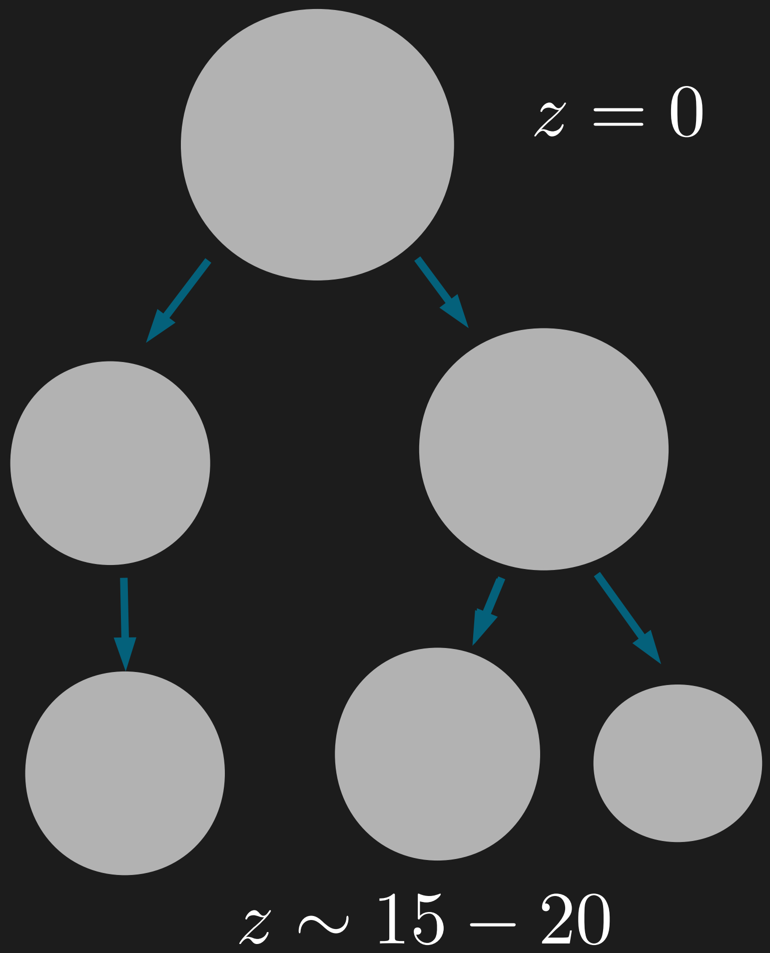
Credits: NASA/ESA/
Hubble Space Telescope

- Massive black hole (MBH) binaries are LISA main target
- Inform us on MBH formation and evolution, e.g. seeding mechanism and “final parsec problem”
- Use MBHs to trace the history of their host galaxies

Semi-analytic models

- Use Press-Schechter formalism to build dark matter merger tree
- Evolve black holes and baryonic components with semi-analytic prescriptions
- Computationally “cheap”

- Bonetti, Sesana, Haardt, Barausse, Colpi 2018:
  - Light (Pop III star remnants, $M_{\rm seed} \sim 300 M_{\odot}$) vs heavy seeds (collapse of protogalactic disks, $M_{\rm seed} \sim 10^5 M_{\odot}$)
  - Binary merger triggered by interaction with gas, stars or triplet interactions



Model mixing

- Pure models:
  - M_1 : Light seed
  - M_2 : Heavy seed

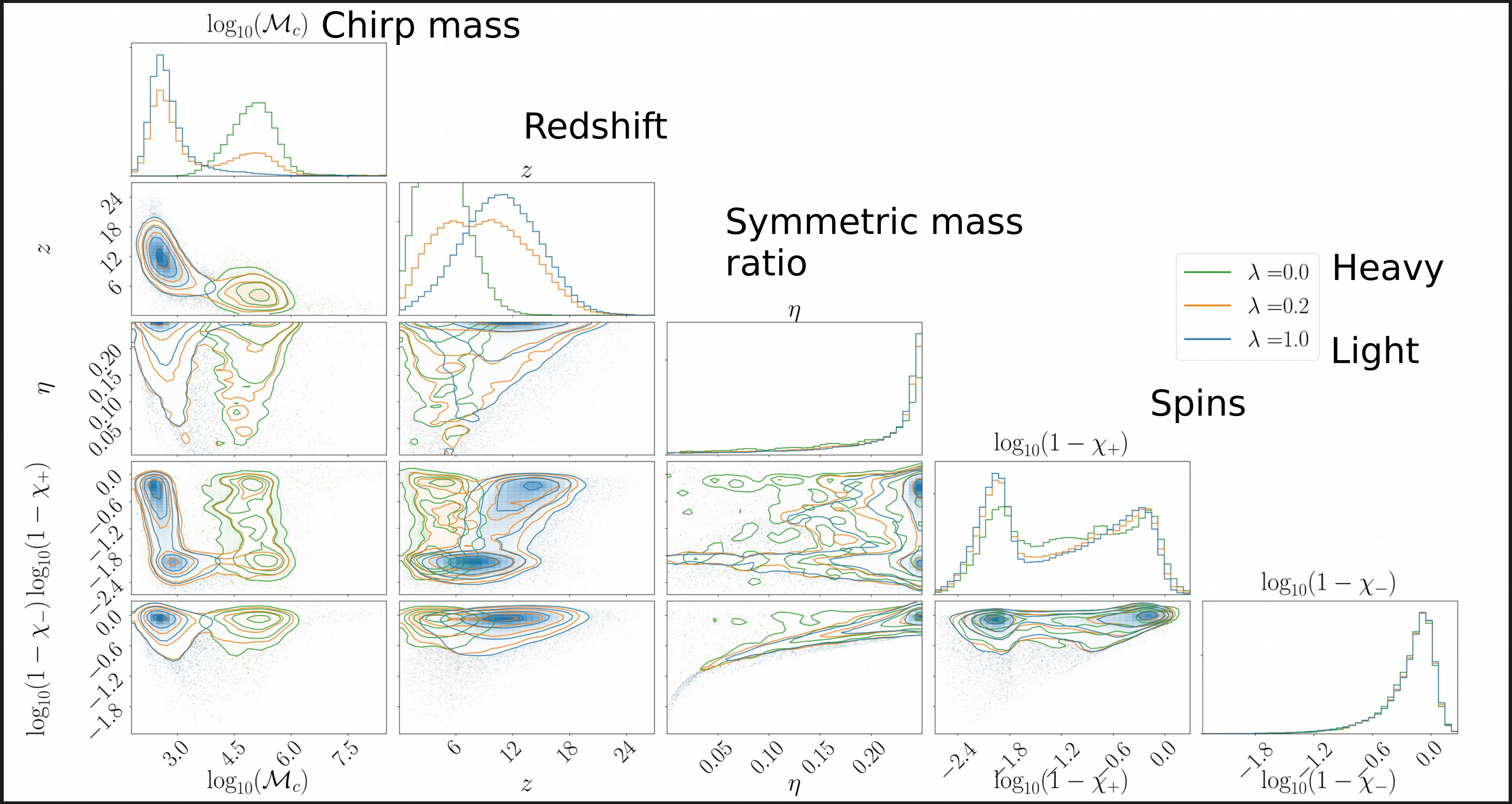
Sesana, Berti
Gair,Volonteri 2010

- Mixing scheme:

$$M = \lambda M_1 + (1 - \lambda) M_2$$

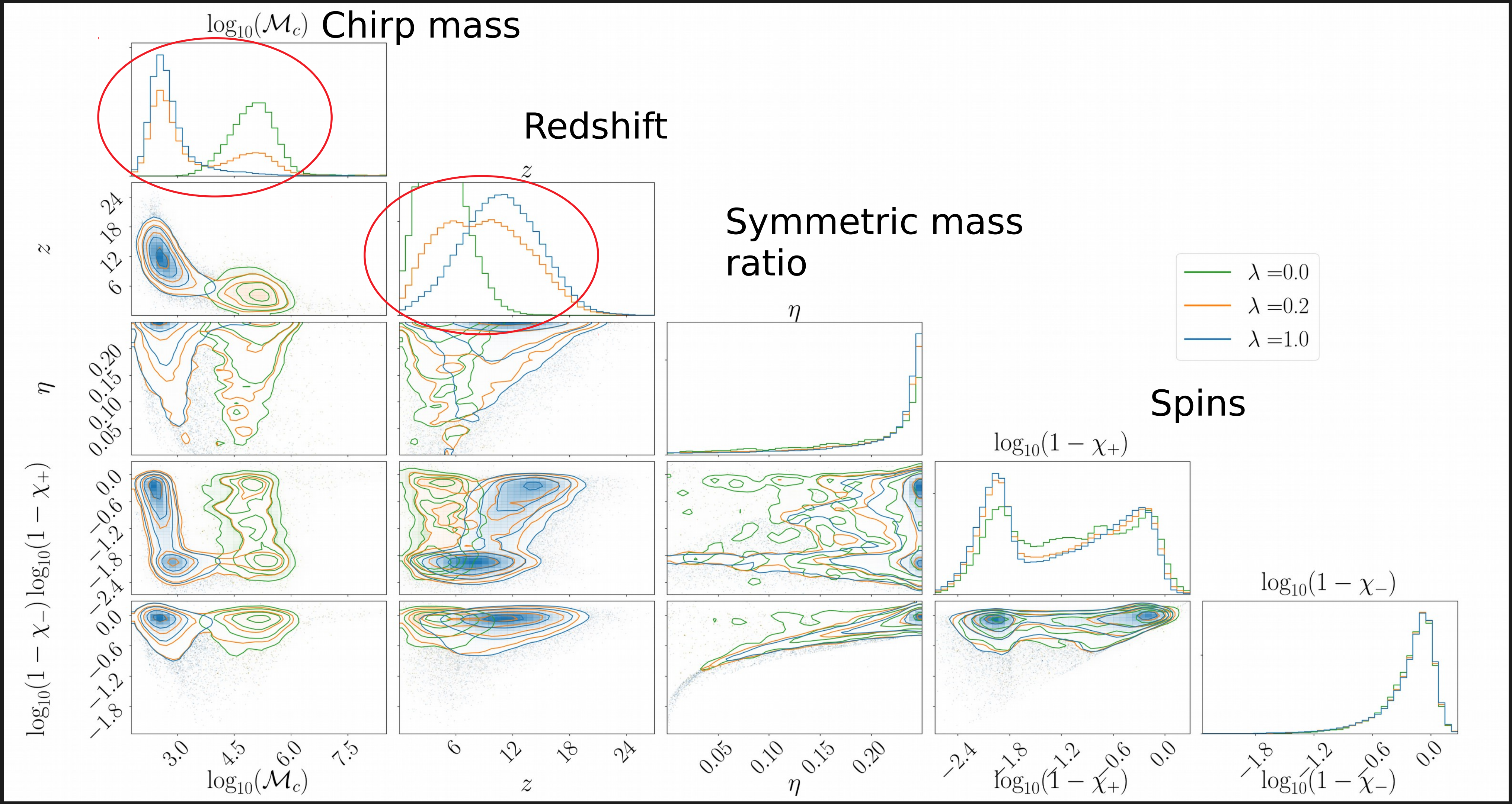
$$R = \lambda R_1 + (1 - \lambda) R_2$$

Distributions



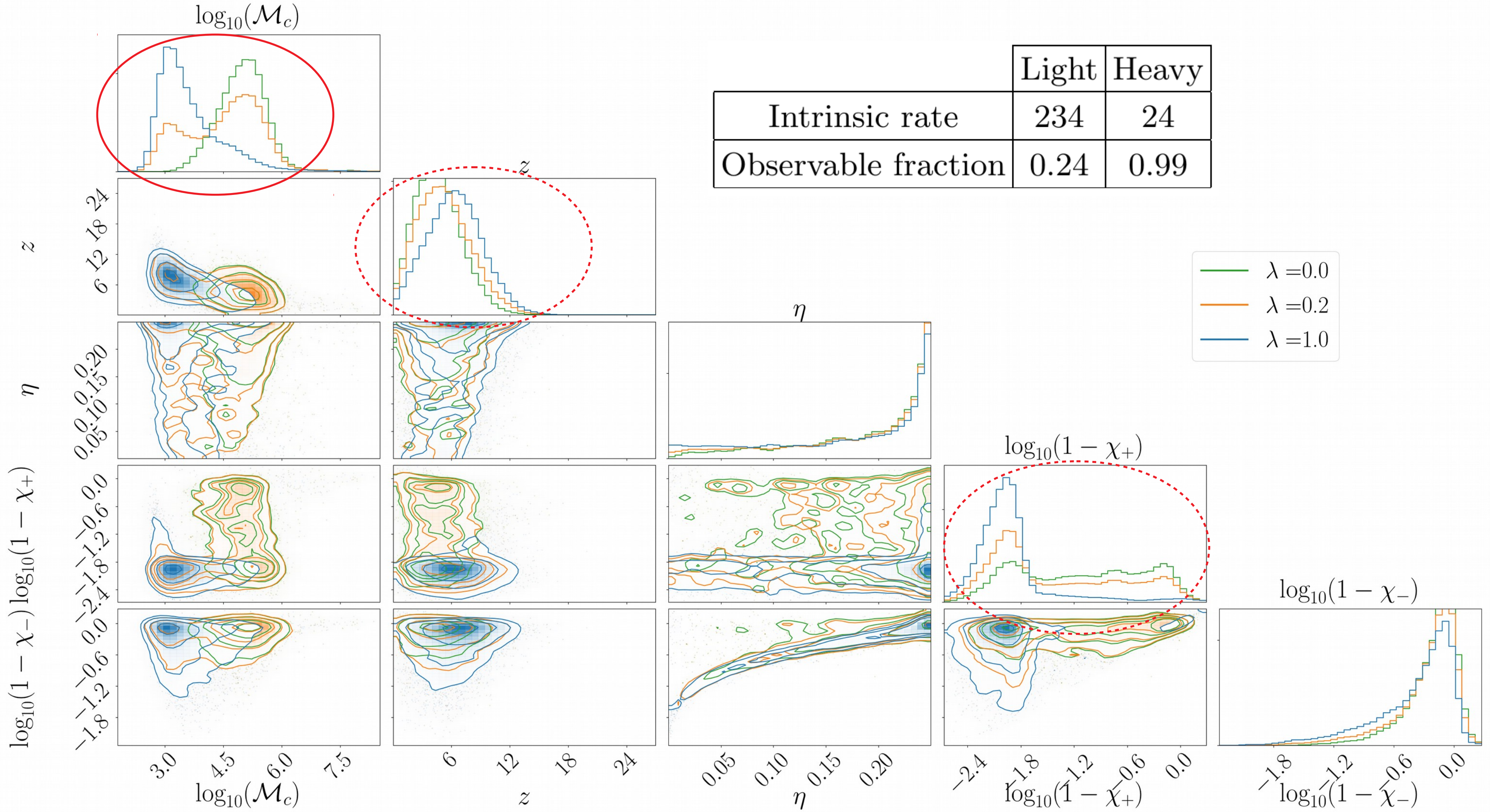
$$\chi_{+,-} = \frac{m_1\chi_1 \pm m_2\chi_2}{m_1 + m_2}$$

Distributions



$$\chi_{+,-} = \frac{m_1\chi_1 \pm m_2\chi_2}{m_1 + m_2}$$

SNR cut, threshold=10



	Light	Heavy
Intrinsic rate	234	24
Observable fraction	0.24	0.99

$$\chi_{+,-} = \frac{m_1\chi_1 \pm m_2\chi_2}{m_1 + m_2}$$

Hierarchical Bayesian analysis

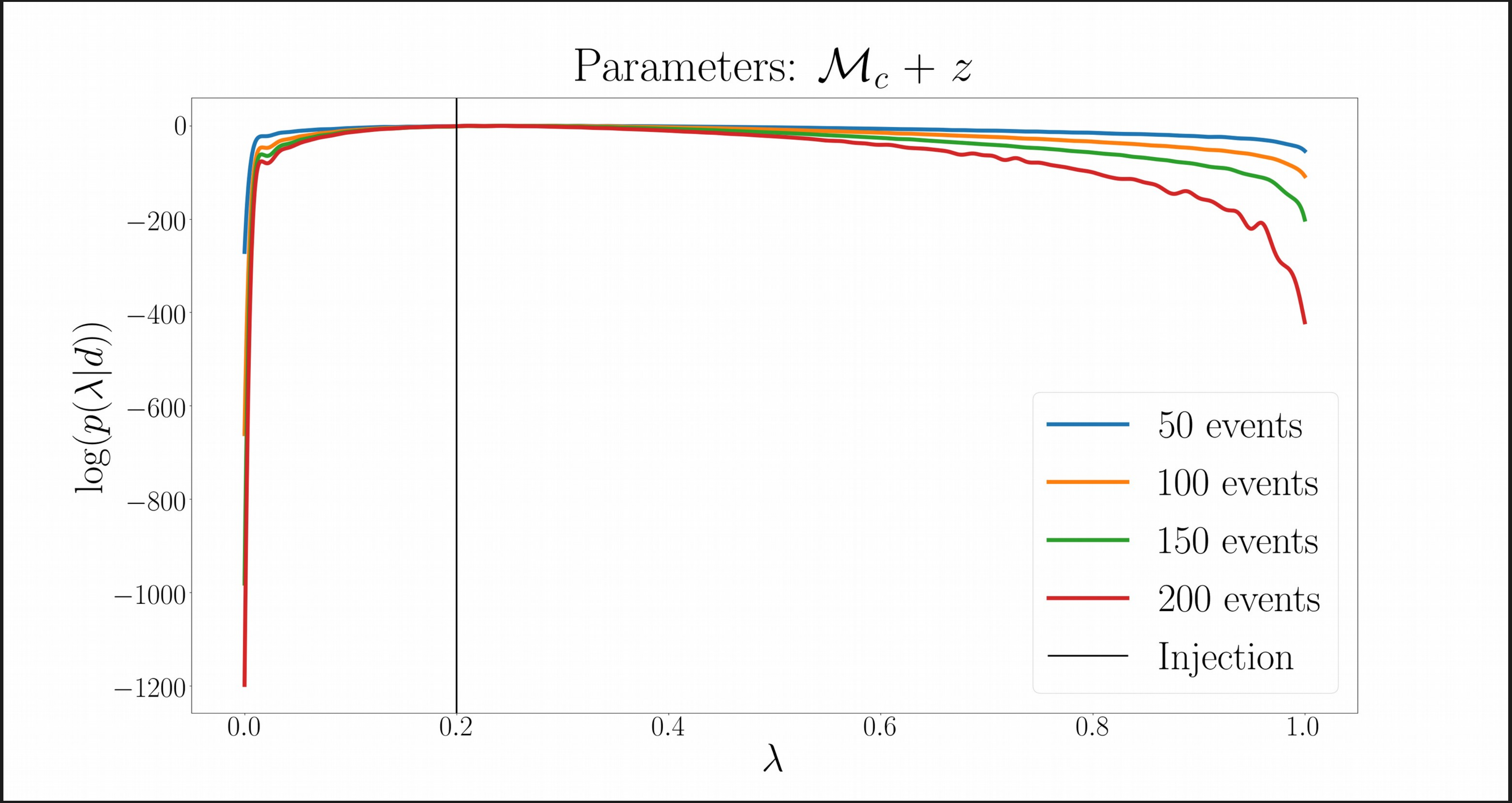
$$p(\lambda|d) = \pi(\lambda)\pi(N_{\mathrm{int}})e^{-N_{\mathrm{det}}(\lambda)}N_{\mathrm{det}}(\lambda)^{N_{\mathrm{obs}}}\prod_{i=1}^{N_{\mathrm{obs}}}\int\frac{p_i(\theta|d)}{\pi_i(\theta)}\frac{p_{\mathrm{pop}}(\theta|\lambda)}{\int p_{\mathrm{pop}}(\theta|\lambda)p_{\mathrm{det}}(\theta)}\mathrm{d}\theta$$

$$\log(p(\lambda|d)) = \mathrm{geometric} - \mathrm{selection} + \mathrm{poisson}$$

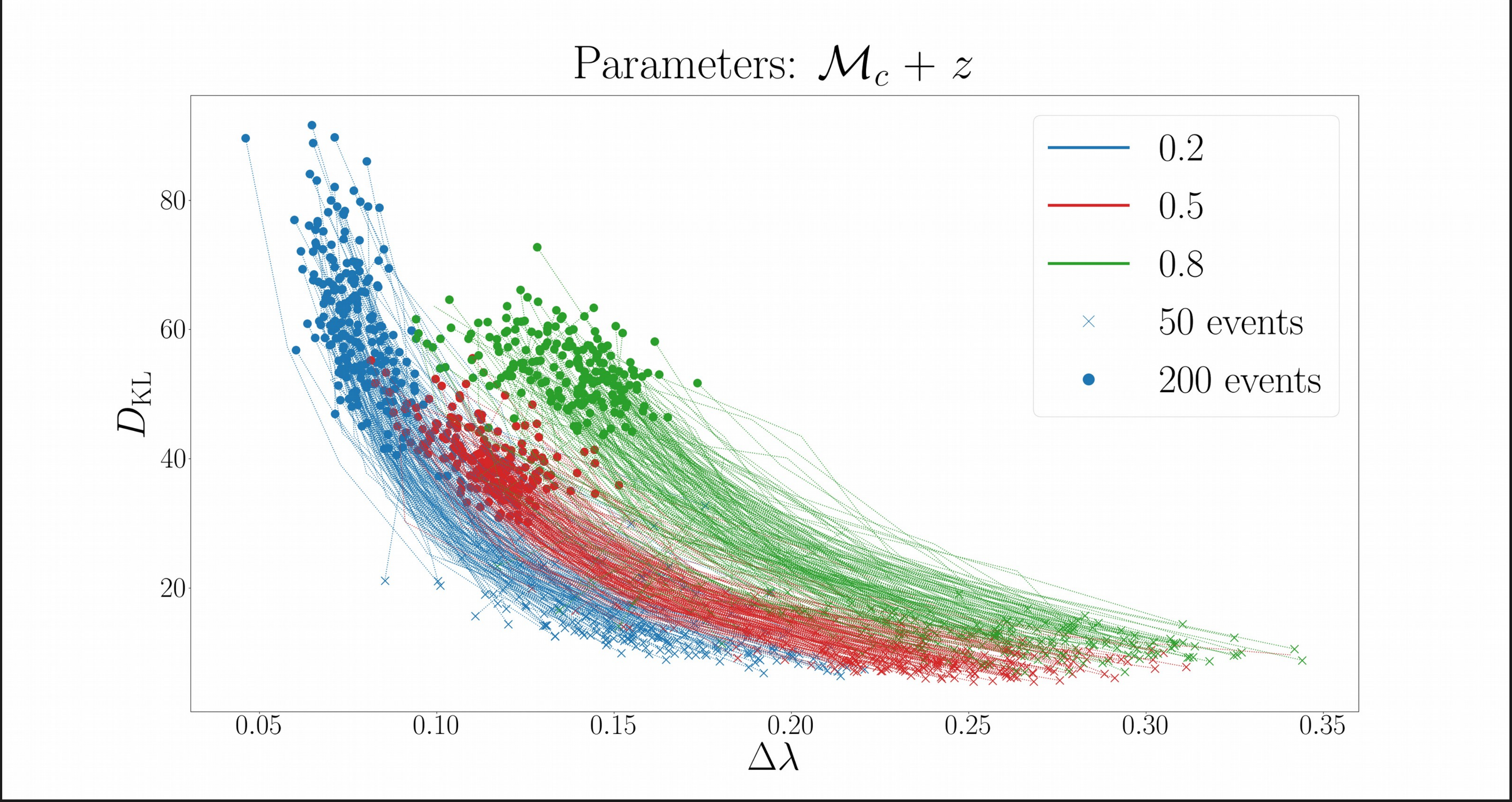
Marginalising over the rate:

$$p(\lambda|d) \propto \pi(\lambda)\prod_{i=1}^{N_{\mathrm{obs}}}\int\frac{p_i(\theta|d)}{\pi_i(\theta)}\frac{p_{\mathrm{pop}}(\theta|\lambda)}{\int p_{\mathrm{pop}}(\theta|\lambda)p_{\mathrm{det}}(\theta)}\mathrm{d}\theta$$

Improvement with number of events

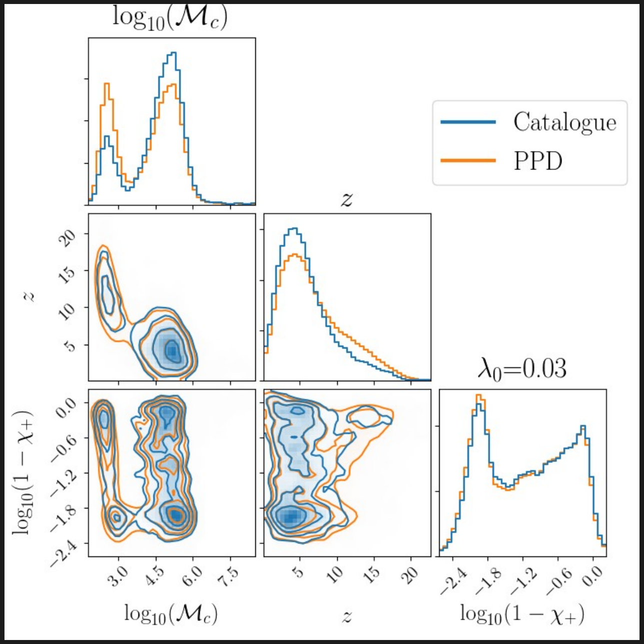


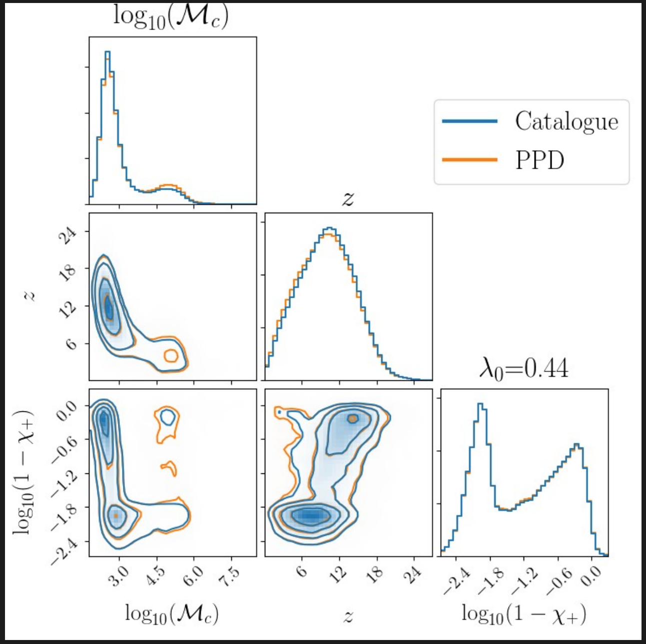
Improvement with number of events

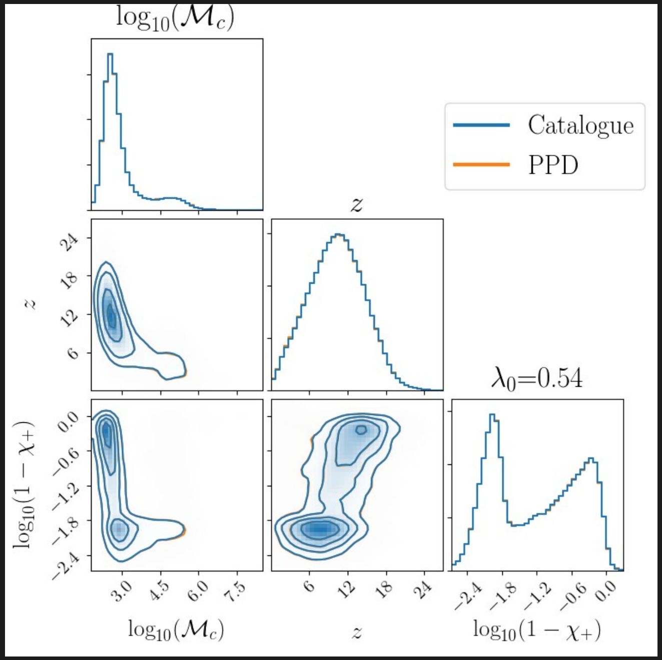


Posterrior predictive distribution

$$\mathrm{PPD}(\theta|d) = \int \mathrm{d}\lambda \; p(\theta|\lambda)p(\lambda|d)$$







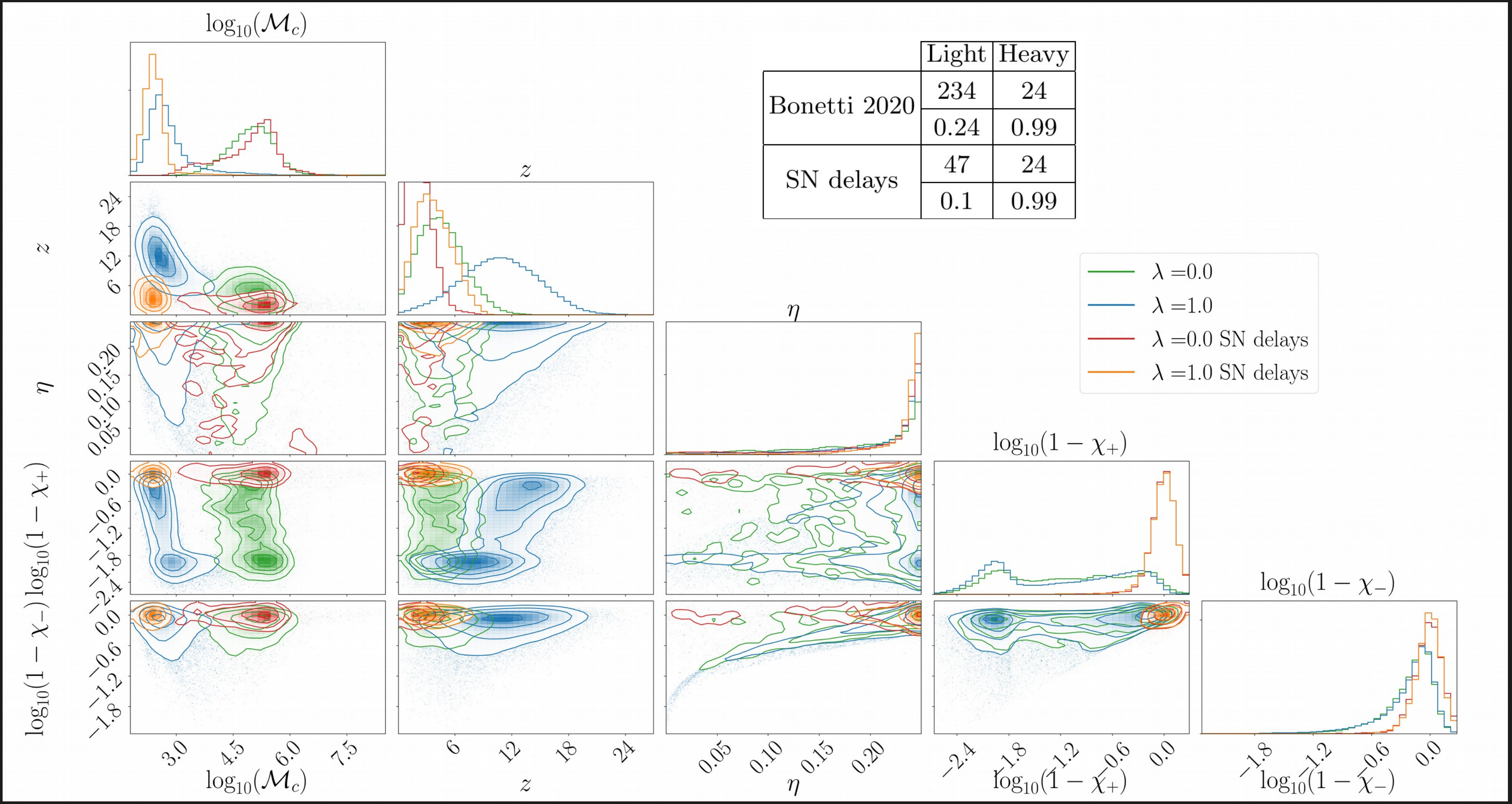
Worse case

Average cases

Robustness

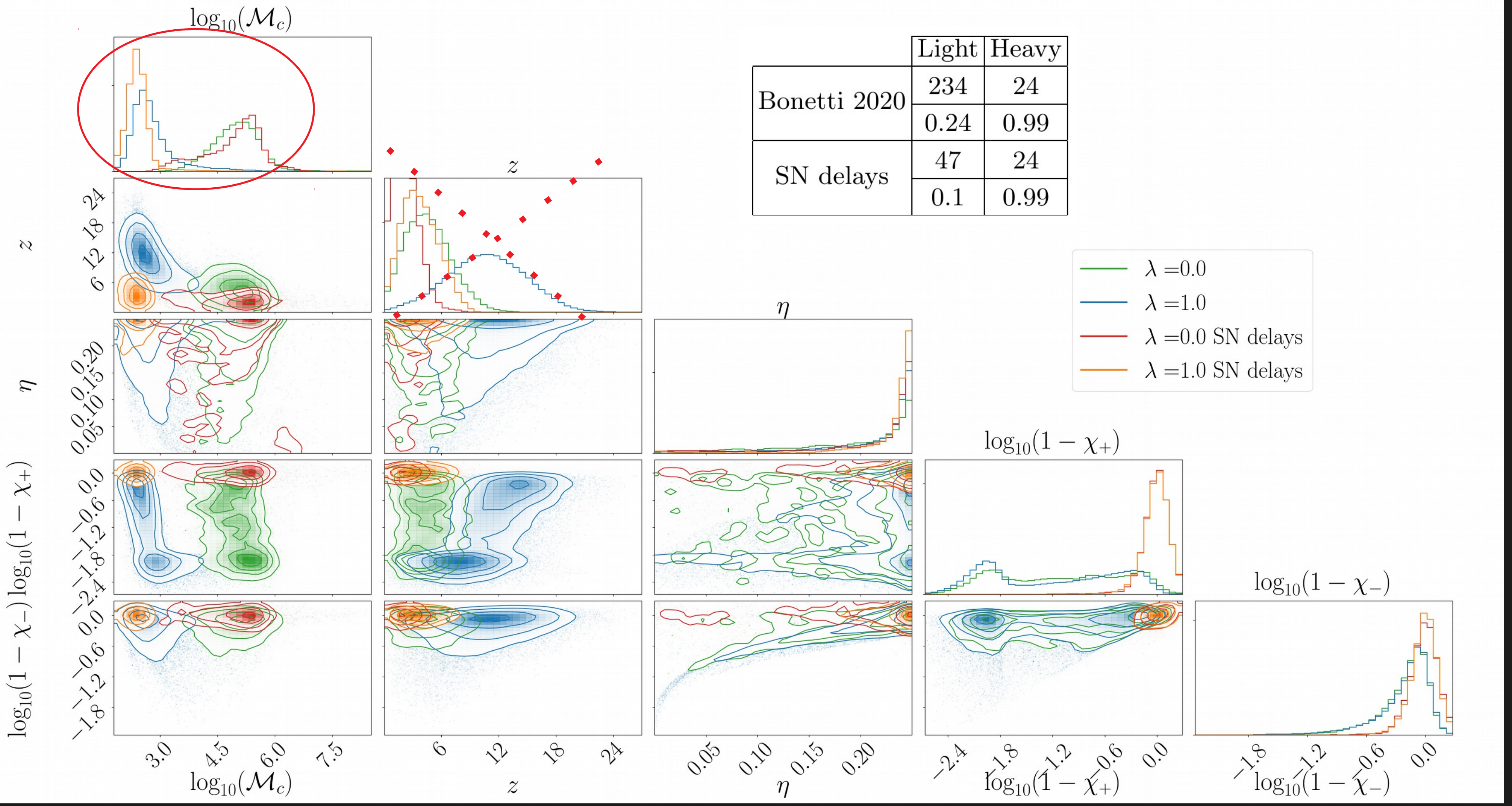
- Use different models for pipeline and data
- E.g. model from Barausse, Dvorkin, Tremmel, Volonteri, Bonetti 2020
- Same seeding mechanisms but consider SN feedback and additional delays

Robustness



	Light	Heavy
Bonetti 2020	234	24
	0.24	0.99
SN delays	47	24
	0.1	0.99

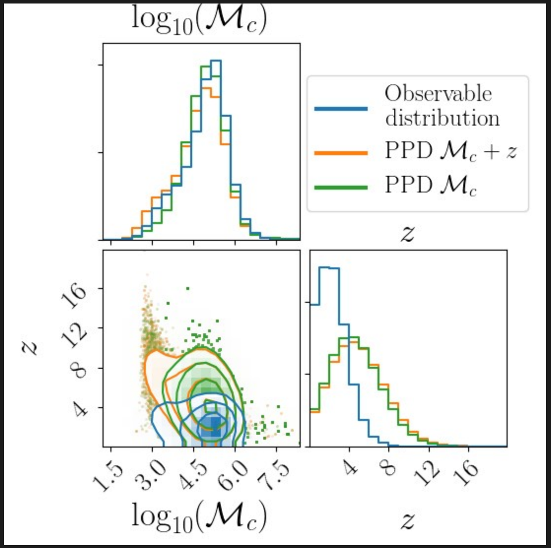
Robustness



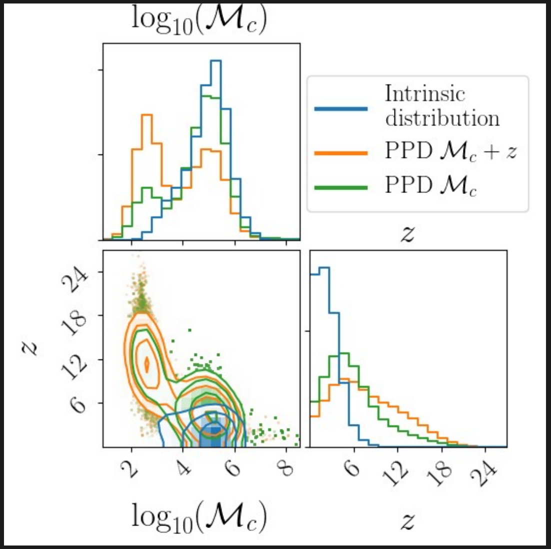
	Light	Heavy
Bonetti 2020	234	24
	0.24	0.99
SN delays	47	24
	0.1	0.99

Robustness (SN delays)

Observable



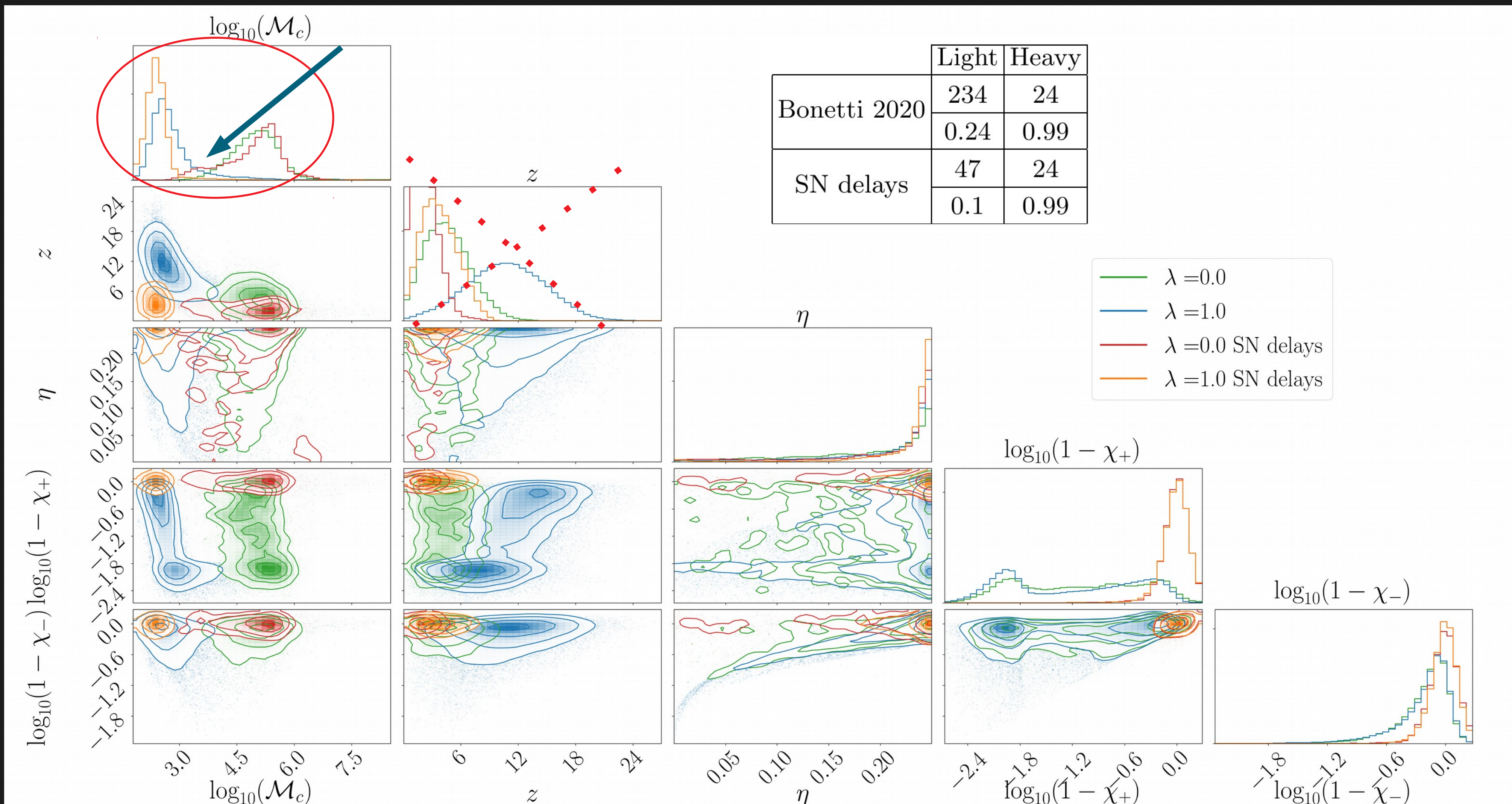
Intrinsic



$$\lambda_0 = 0$$

Data=Heavy
new model

Robustness

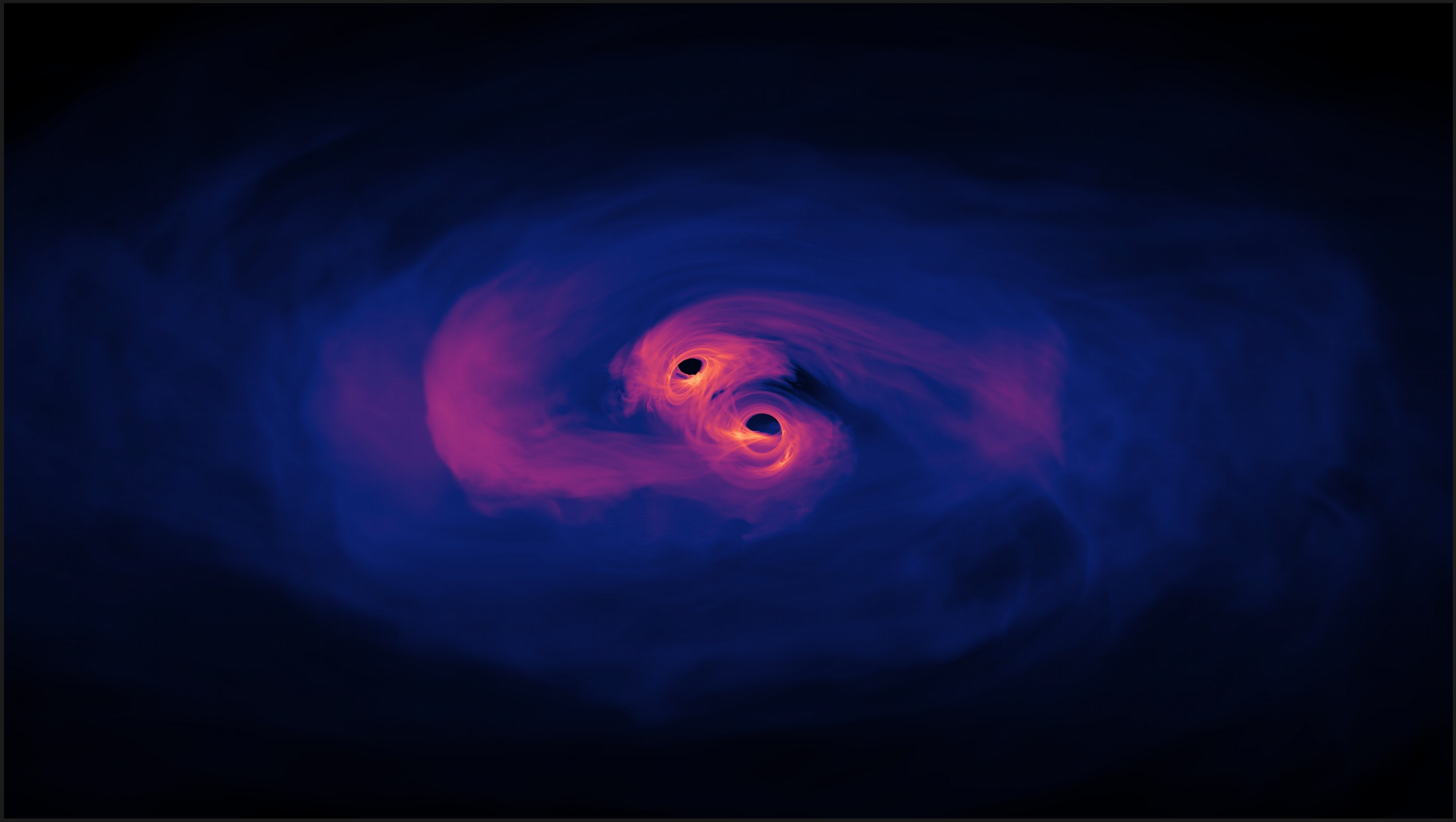


	Light	Heavy
Bonetti 2020	234	24
	0.24	0.99
SN delays	47	24
	0.1	0.99

Conclusions

- We have introduced mixing scheme between "pure" models
- Allow to infer the MBH population self-consistently
- But not robust to the "unknown"
- Currently developing model agnostic approach

Thank you for your attention!



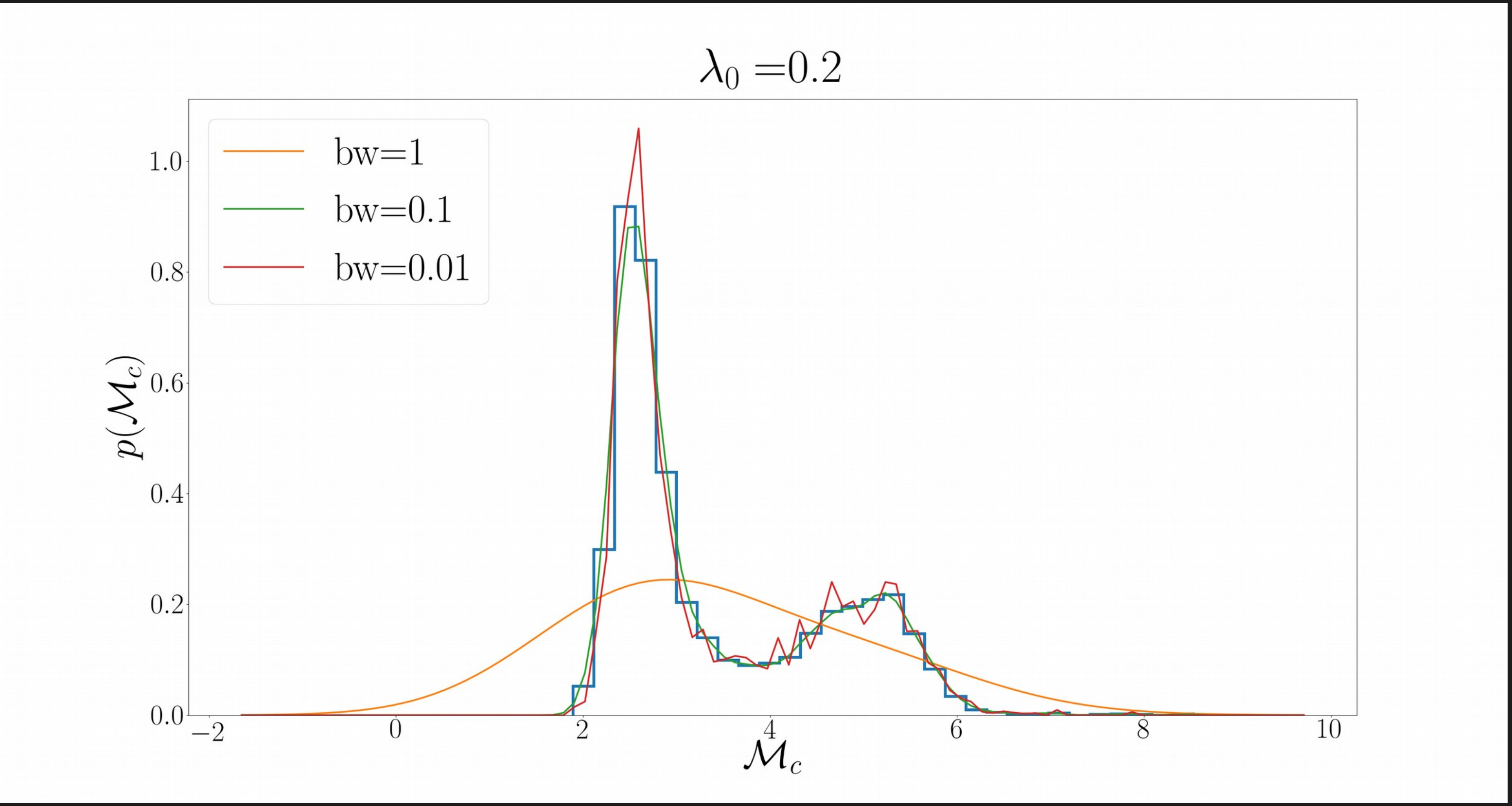
Credits: NASA's Goddard Space Flight Center

KDE

$$\hat{f}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{x_i})$$

$\mathbf{H}$: bandwith

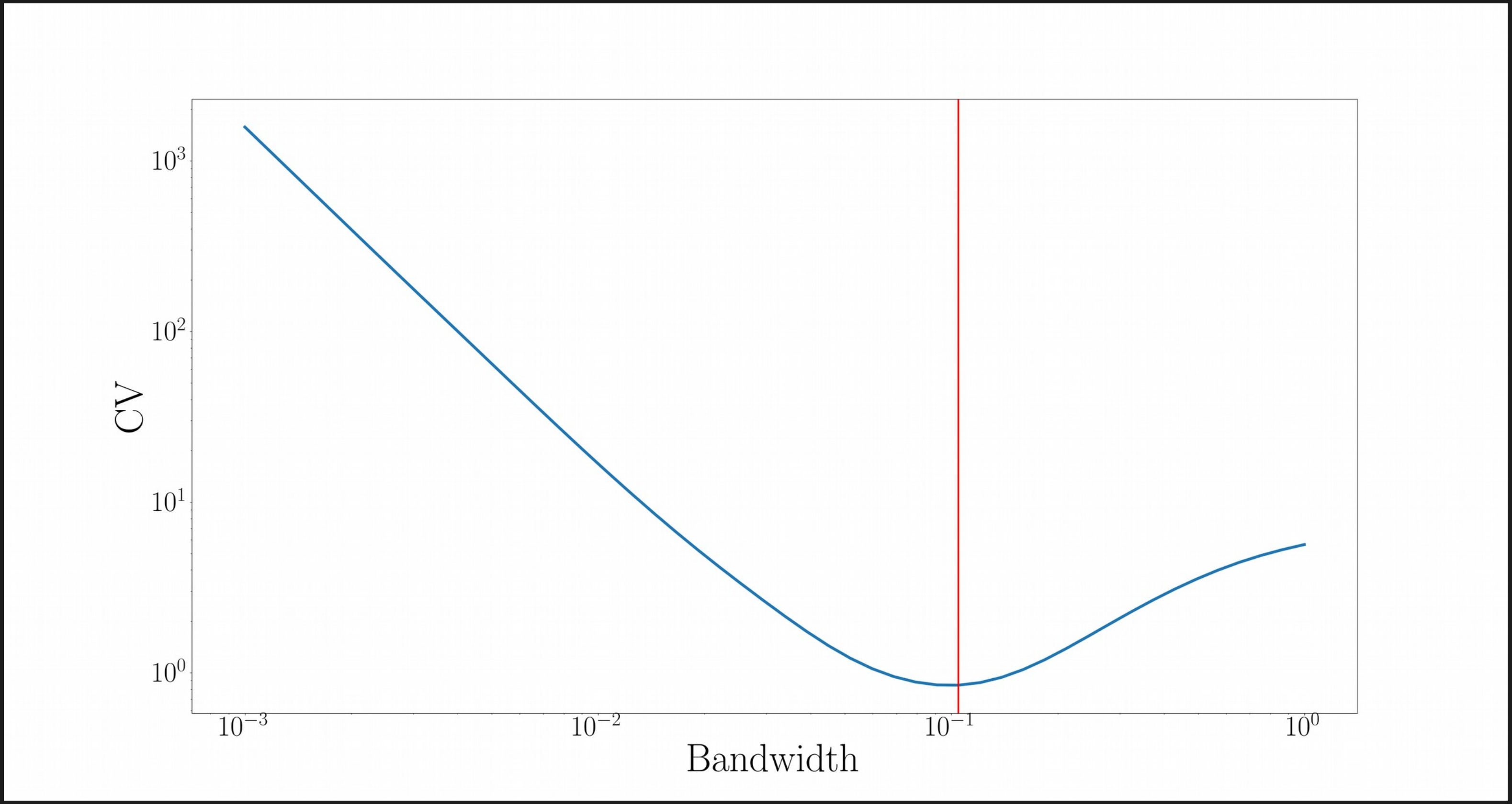
$$K_{\mathbf{H}}(\mathbf{y}) = \frac{\mathbf{1}}{(\mathbf{2}\pi)^{-\mathbf{d}/\mathbf{2}}} |\mathbf{H}|^{-\mathbf{1}/\mathbf{2}} \mathbf{e}^{-\frac{1}{2}\mathbf{y}^{\mathbf{T}}\mathbf{H}^{-\mathbf{1}}\mathbf{y}}$$



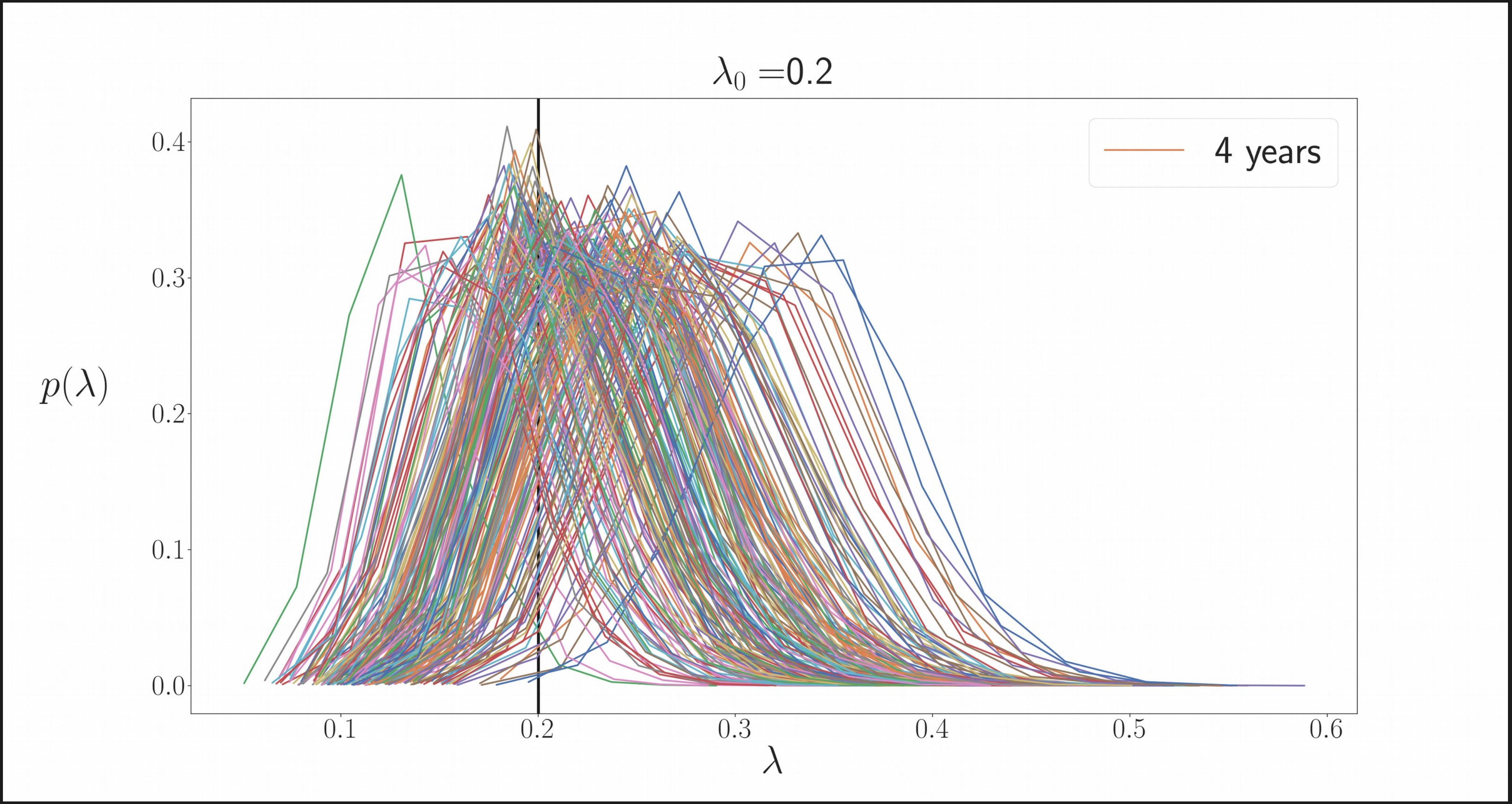
KDE

Integrated square error: $$\int \mathrm{d}\theta (f(\theta) - f(\hat{\theta}))^2$$

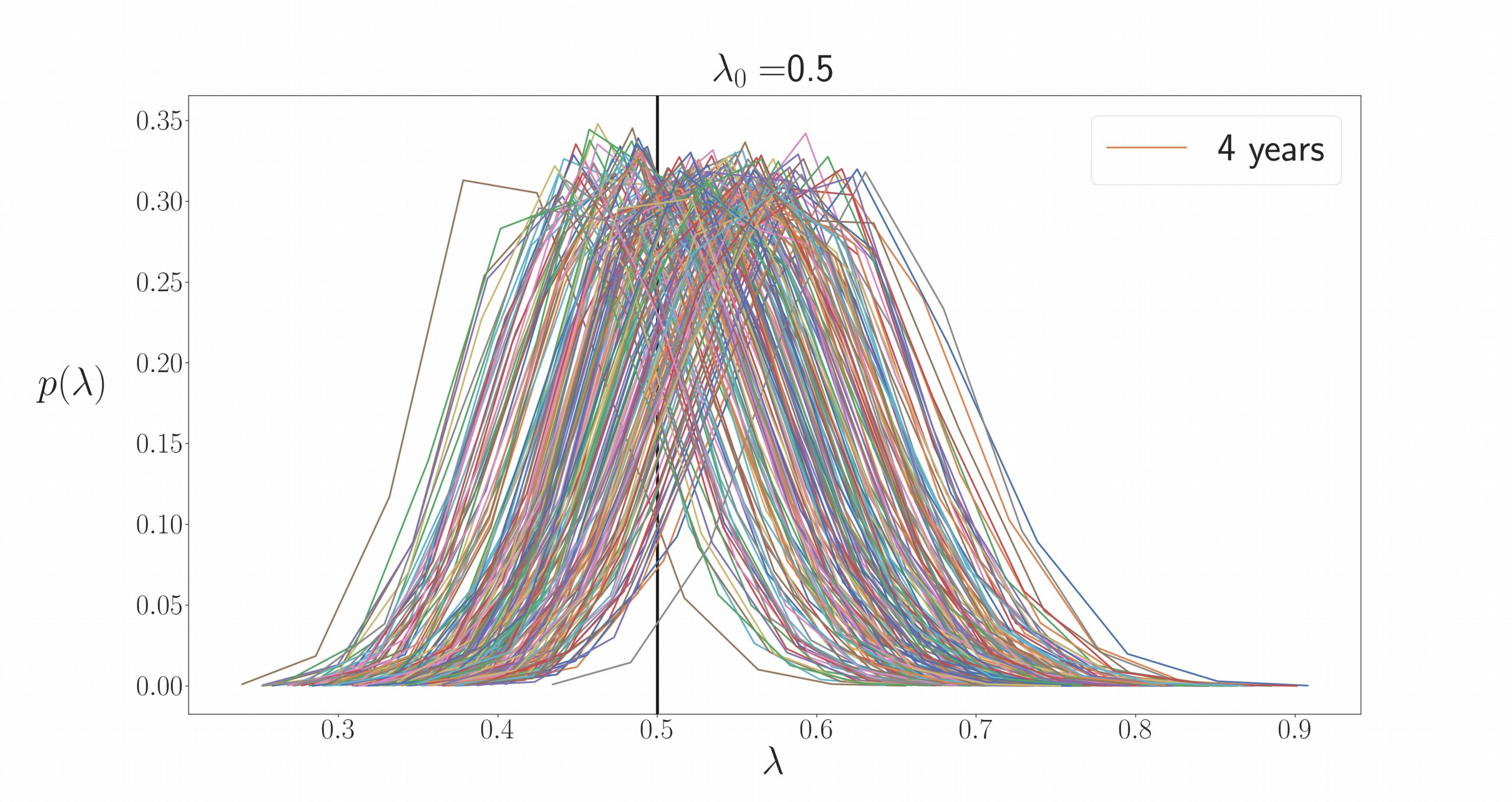
Minimize: $$\mathrm{CV} = \int \mathrm{d}\theta \hat{f}(\theta)^2 - \frac{2}{n}\sum_{i=1}^{n} \hat{f}_{-i}(\theta_i)$$



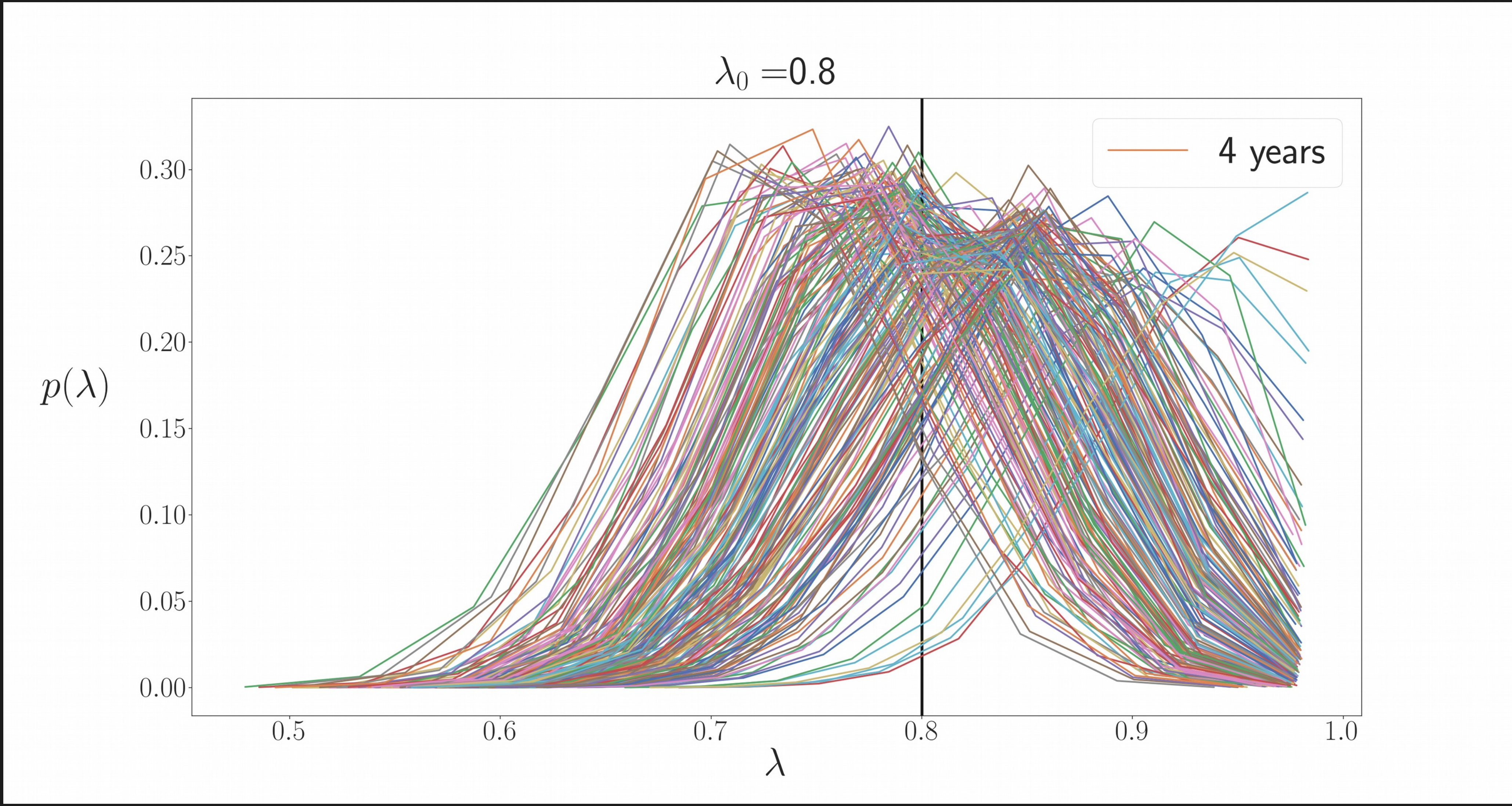
Results


λ_0 =0.2
4 years
$p(\lambda)$
0.0
0.1
0.2
0.3
0.4
0.1
0.2
0.3
0.4
0.5
0.6
λ

Results


λ_0 =0.5
4 years
$p(\lambda)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.3
0.4
0.5
0.6
0.7
0.8
0.9
λ

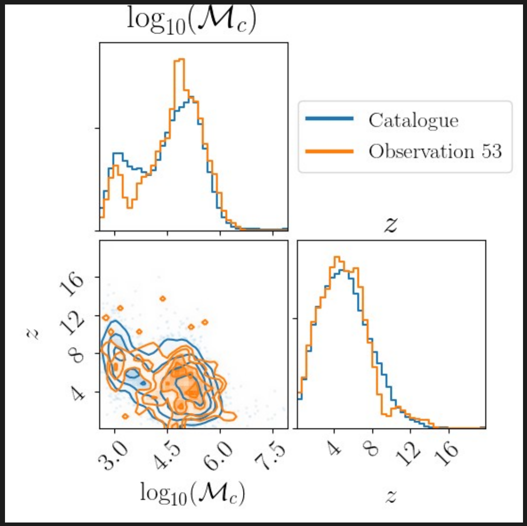
Results

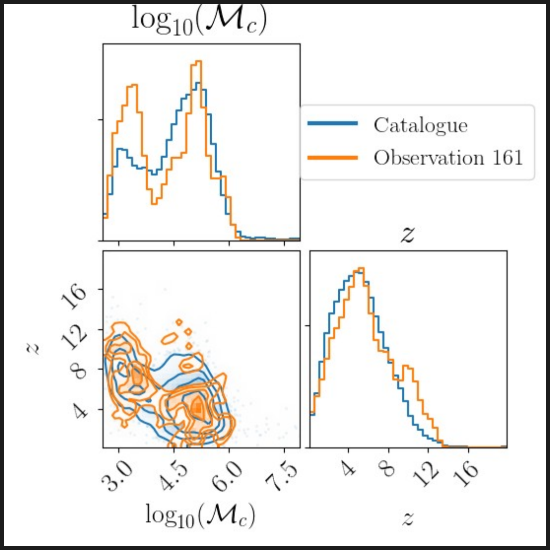

λ_0 =0.8
4 years
$p(\lambda)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.5
0.6
0.7
0.8
0.9
1.0
λ

Results

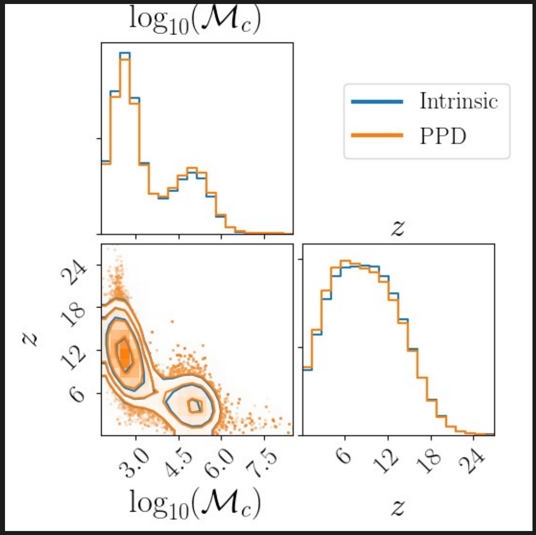
$\lambda_0 = 0.2$

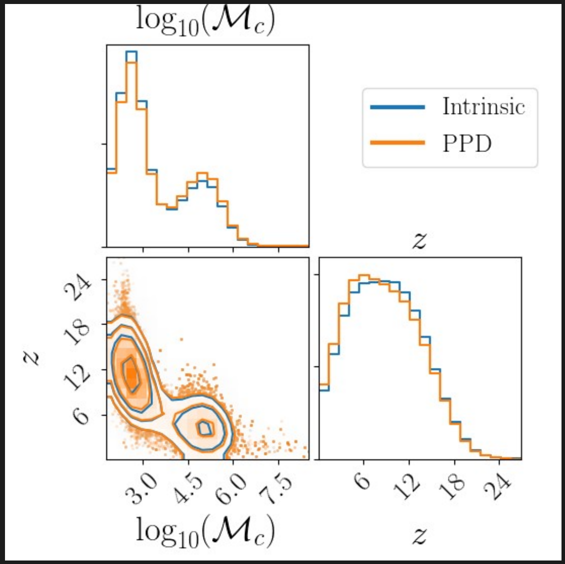
Obs





PPD





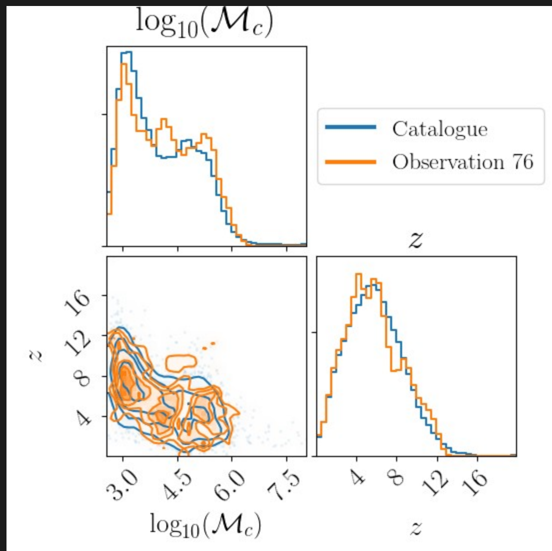
Biased to the left

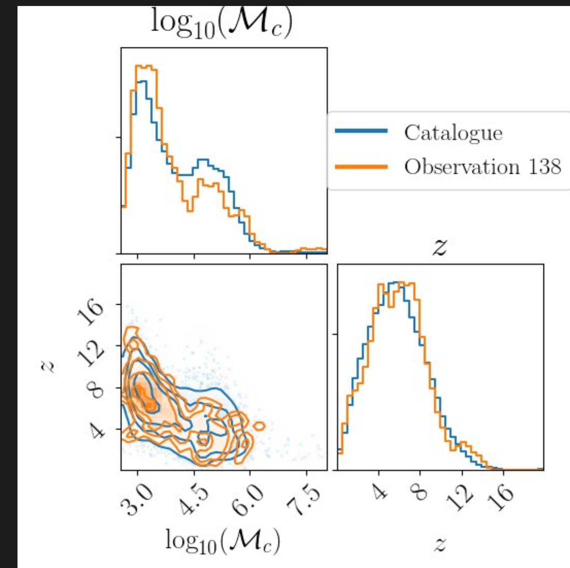
Biased to the right

Results

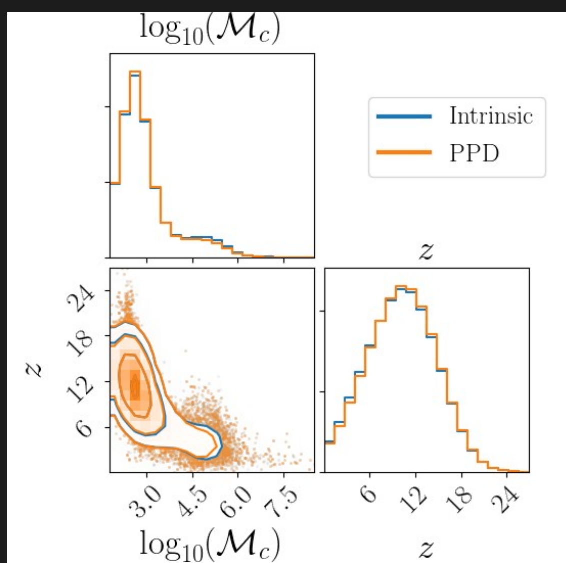
$$\lambda_0 = 0.5$$

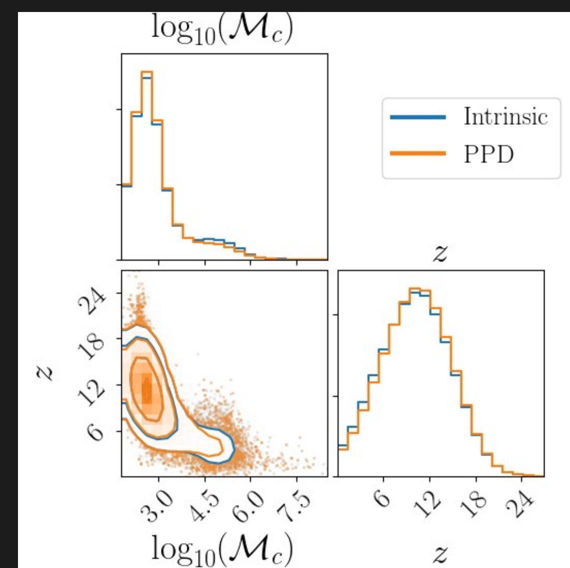
Obs





PPD





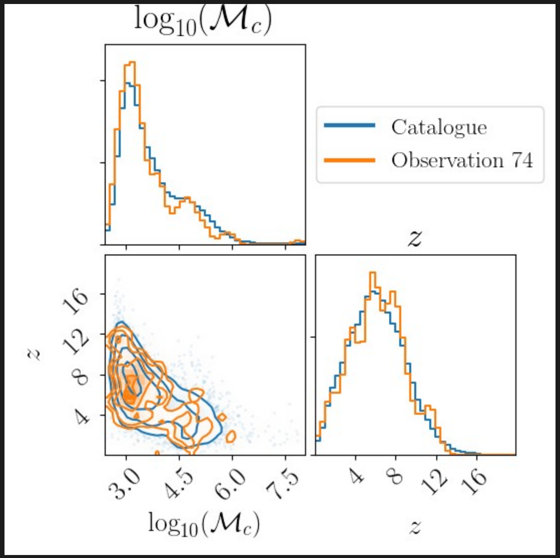
Biased to the left

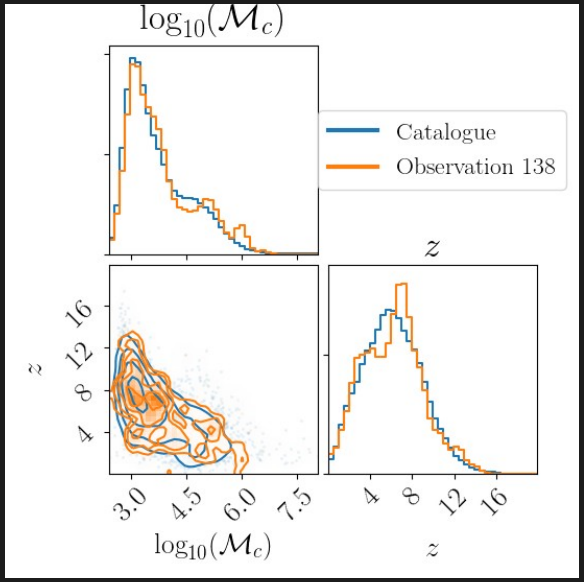
Biased to the right

Results

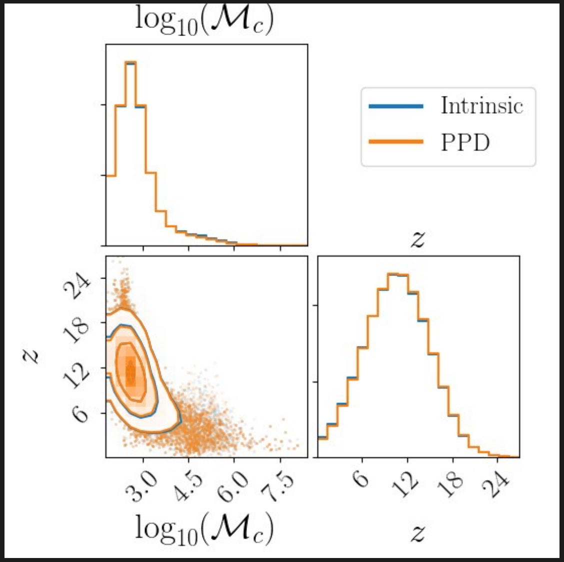
$$\lambda_0 = 0.8$$

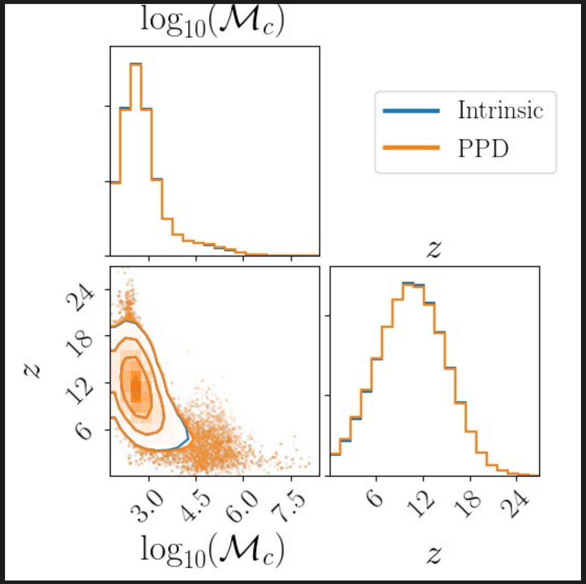
Obs





PPD

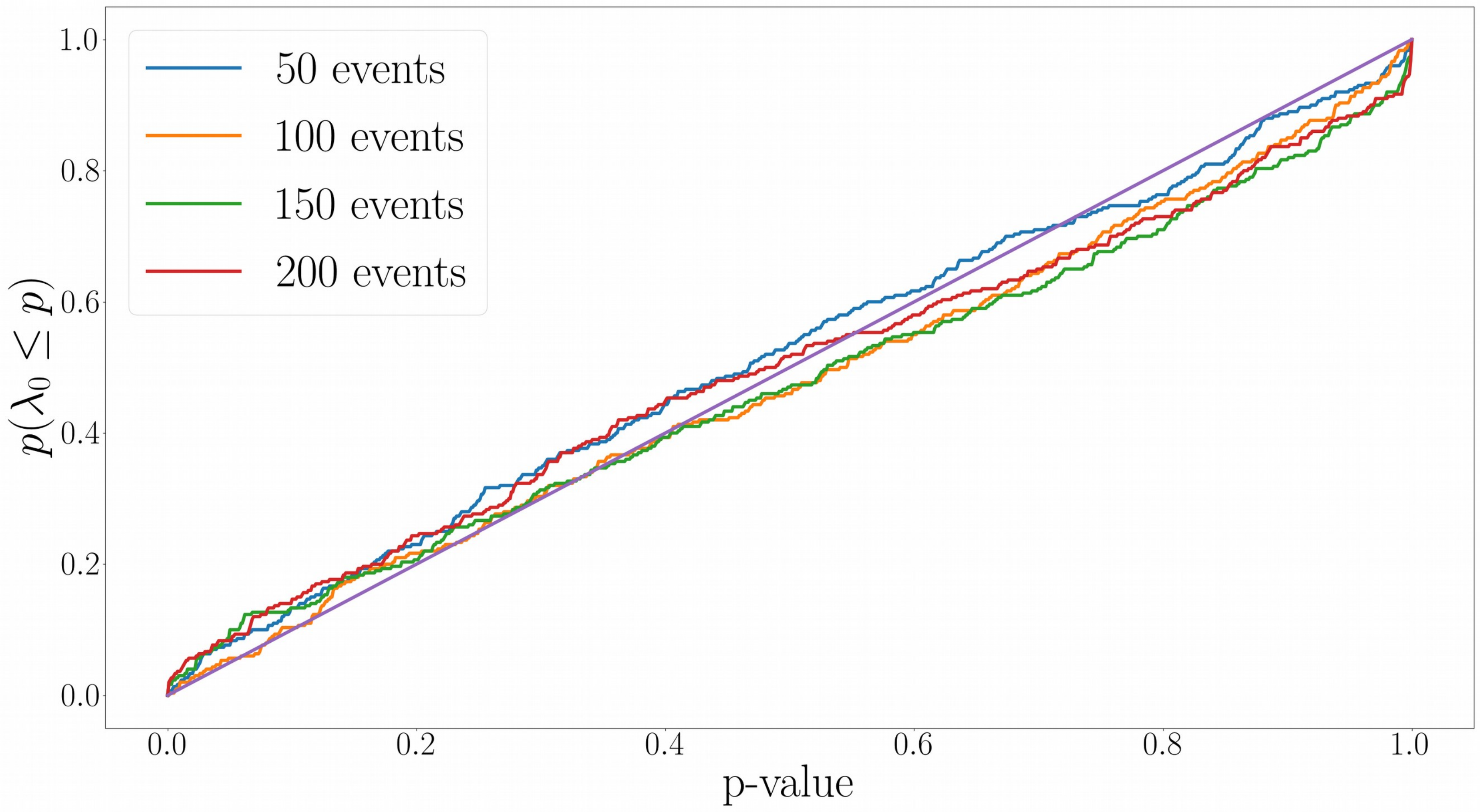




Biased to the left

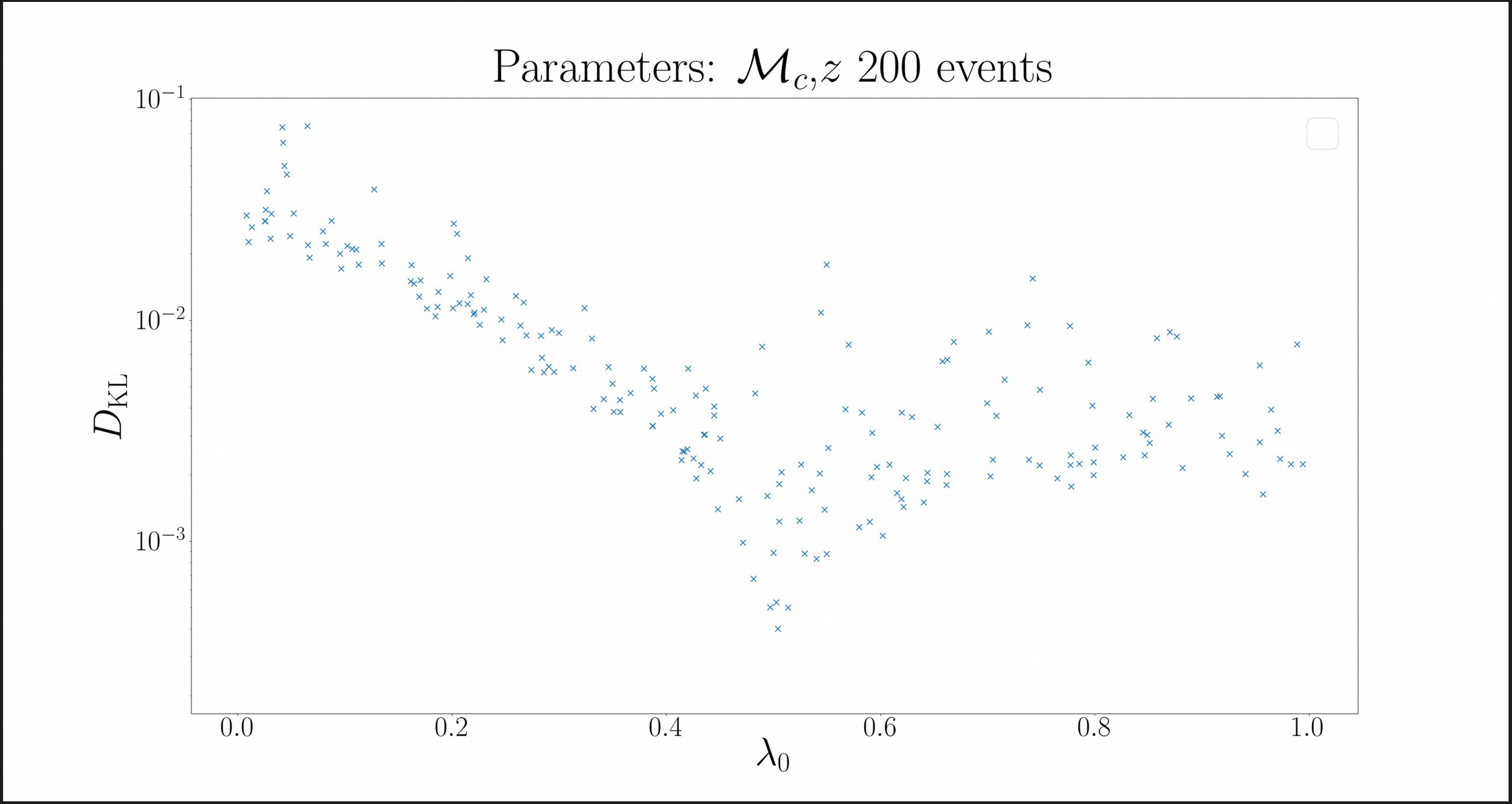
Biased to the right

PP plot



50 events
100 events
150 events
200 events
$p(\lambda_0 \leq p)$
p-value
0.0
0.2
0.4
0.6
0.8
1.0

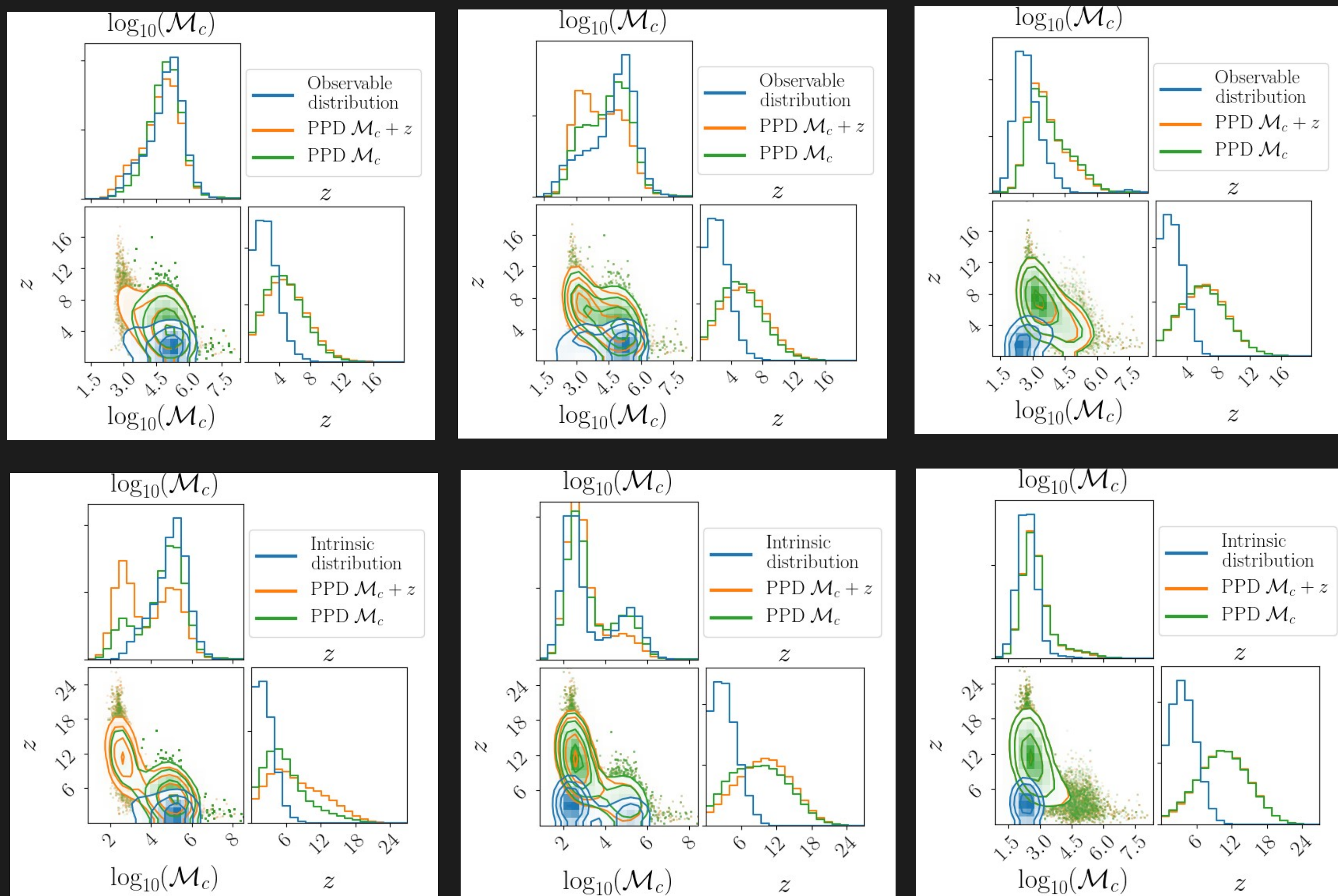
PPD


Parameters: $\mathcal{M}_c, z$ 200 events
D_{KL}
10^{-1}
10^{-2}
10^{-3}
0.0
0.2
0.4
0.6
0.8
1.0
λ_0

Robustness (SN delays)

Observable

Intrinsic



$\lambda_0 = 0$ $\lambda_0 = 0.5$ $\lambda_0 = 1$