Gravitational Bremsstrahlung in the Post-Minkowskian Effective Field Theory

Massimiliano Maria Riva

Based on work with S. Mougiakakos and F. Vernizzi
[arXiv:2102.08339]

Université Paris-Saclay, CNRS, CEA, Institut de physique théorique, 91191, Gif-sur-Yvette, France.

Groupement de Recherche "Ondes Gravitationnelles", IHP, Paris
30 March 2021

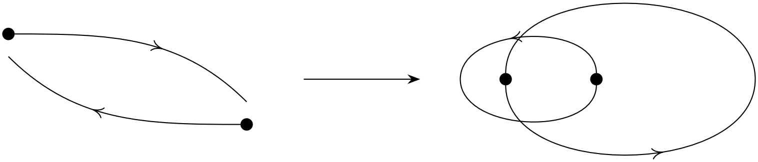
Post-Minkowskian: a complementary approach

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \dots)$	G^1
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ \dots)$	G^2
3PM			$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ \dots)$	G^3
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ \dots)$	G^4
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ \dots)$	G^5
6PM						$(1$	$+\ v^2$	$+\ v^4$	$+\ \dots)$	G^6
							$\vdots$			

Figure: Z. Bern, C. Cheung, R. Roiban, C. H. Shen, M. P. Solon and M. Zeng, JHEP **10** (2019)

- Traditional GR
  S. Kovacs and K. Thorne *Astrophys. J. 200 (1975) - 215, 217 (1977) - 224 (1978)* , K. Westpfahl and M. Goller *Lett. Nuovo Cim. 26 (1979) 573-576* .
- Scattering Amplitude methods
  T. Damour *Phys. Rev. D 94 10 (2016)* , C. Cheung, I. Z. Rothstein, M. P. Solon *Phys.Rev.Lett. 121 25 (2018)* , Z. Bern et al. *JHEP 10 206 (2019)*
- EFT approach
  G. Kälin and R. A. Porto *JHEP 11 106 (2020)* , G. Mogull, J. Plefka, J. Steinhoff *JHEP 02 048 (2021)* , G. U. Jakobsen et al. *[2101.12688]*

PM state of the art



Scattering Amplitude

Conservative

- 4PM hamiltonian recently computed for spinless bodies
  `Z. Bern et al.` *[2101.07254]*
- Adding tidal and spin effects
  `Z. Bern et al.` *[2010.08559]* - *[2005.03071]*

Radiative

- Leading order emission (G^3) for a scattering process
  `D. Kosower, B. Mayee and D. O'Connell` *JHEP 02 137 (2019)* - `E. Herrmann et al.` *[2101.07255]*

EFT Approach

Conservative

- 3 PM dynamics for spinless bodies
  `G. Kälin and R. A. Porto` *Phys. Rev. Lett. 125 26 (2020)*
- Adding tidal and spin effects `G. Kälin, Z. Liu and R. A. Porto` *Phys. Rev. D 102 (2020) 124025* - `Z. Liu, R. A. Porto and Z. Yang` *[2102.10059]*

Radiative

- **Next topic of this presentation**
- Alternative derivation `G. U. Jakobsen et al.` *[2101.12688]*

PM Effective Field Theory

Setting up our EFT

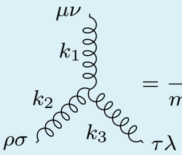
Expansions around Minkowsky $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{\text{Pl}}$

Finite-size effects

$$S = \underbrace{-2m_{\text{Pl}}^2 \int d^4x \sqrt{-g} R} - \sum_{a=1,2} \frac{m_a}{2} \int d\tau_a \left[g_{\mu\nu}(x_a) \mathcal{U}_a^\mu(\tau_a) \mathcal{U}_a^\nu(\tau_a) + 1 \right] + \overbrace{\ldots}^{\text{Finite-size effects}}$$

$$= \frac{i}{(k^0 + i\epsilon)^2 - |\mathbf{k}|^2} P_{\mu\nu;\rho\sigma}$$

$$P_{\mu\nu;\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})$$



$$= \frac{i}{m_{\text{Pl}}} V_{\mu\nu\rho\sigma\lambda\tau}(k_1, k_2, k_3)$$

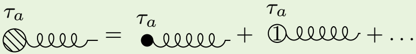
$\hbar = 1\,, \quad c = 1\,, \quad m_{\text{Pl}} = 1/\sqrt{32\pi G}\,, \quad \eta_{\mu\nu} = \text{diag}(+,-,-,-)\,, \quad \int_q \equiv \int \frac{d^4q}{(2\pi)^4}$

PM Effective Field Theory

Setting up our EFT

Expansions around Minkowsky $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{\rm Pl}$

$$S = -2m_{\rm Pl}^2 \int d^4x\sqrt{-g}R \underbrace{- \sum_{a=1,2} \frac{m_a}{2} \int d\tau_a \left[g_{\mu\nu}(x_a)\mathcal{U}_a^\mu(\tau_a)\mathcal{U}_a^\nu(\tau_a) + 1\right]} + \dots$$



Polyakov action reduces the point-particle vertices.

Isolate the powers of G

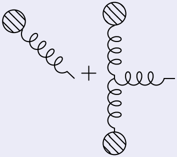
$$x_a^\mu(\tau_a) = b_a^\mu + u_a^\mu \tau_a + \delta^{(1)} x_a^\mu(\tau_a) + \dots$$

$$\mathcal{U}_a^\mu(\tau_a) = u_a^\mu + \delta^{(1)} u_a^\mu(\tau_a) + \dots$$

- $u_a = \lim_{\tau_a \to -\infty} \mathcal{U}_a^\mu(\tau_a)\,, \qquad b_a \cdot u_a = 0$
- $\delta^{(1)} x_a^\mu$, $\delta^{(1)} u_a^\mu$ deviations from the straight motion at order G.

Matching procedure

The pseudo Stress-Energy Tensor



$$+\cdots \equiv -\frac{i}{2m_{\mathrm{Pl}}}\int_k \tilde{T}^{\mu\nu}(-k)\tilde{h}_{\mu\nu}(k)$$

Classical Amplitude and Asymptotic Waveform

$$\mathcal{A}_\lambda(k) = -\frac{1}{2m_{\mathrm{Pl}}}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{k})\tilde{T}^{\mu\nu}(k)\ , \qquad \epsilon^{\lambda}_{0\nu} = 0\,,\ k^\mu\epsilon^{\lambda}_{\mu\nu} = 0\,,\ \eta^{\mu\nu}\epsilon^{\lambda}_{\mu\nu} = 0$$

$$h_{\mu\nu}(x) = -\frac{1}{4\pi r}\sum_{\lambda=\pm 2}\int\frac{dk^0}{2\pi}e^{-ik^0u}\epsilon^{\lambda}_{\mu\nu}(\mathbf{k})\mathcal{A}_\lambda(k)|_{k^\mu = k^0 n^\mu}$$

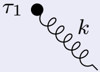
The Amplitude is the only thing we need to compute.

$u \equiv t - r$ is the retarded time.

LO amplitude

$$\gamma \equiv u_1 \cdot u_2 \,, \qquad b \equiv b_1^\mu - b_2^\mu \,, \qquad \omega_a \equiv k \cdot u_a \,, \qquad \hat{\delta}^{(n)}(x) = (2\pi)^n \delta^{(n)}(\omega_a) \,.$$

LO Amplitude



$$\tilde{T}^{\mu\nu}_{(1)}(k) = \sum_a m_a u_a^\mu u_a^\nu e^{ik\cdot b_a} \hat{\delta}(\omega_a)$$

$$\mathcal{A}^{(1)}_\lambda(k) = -\frac{1}{2m_{\mathrm{Pl}}} \sum_a m_a \, \epsilon^{*\lambda}_{\mu\nu}(\mathbf{n}) u_a^\mu u_a^\nu \, e^{ik\cdot b_a} \hat{\delta}(\omega_a)$$

$\hat{\delta}(\omega_a) \to$ non-radiating piece relevant to compute J_{rad} (see later).

NLO Amplitude

$$\mathcal{A}^{(2)}_{\lambda}(k) = -\frac{m_1 m_2}{8 m_{\rm Pl}^3}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\Bigg\{\Bigg[-2\left(\gamma I_{(0)} + \omega_1\omega_2 J_{(0)}\right)u_1^{\mu}u_2^{\nu} \\ + \left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1+i\epsilon)^2} + \frac{2\gamma\omega_2}{\omega_1+i\epsilon}I_{(0)} + 2\omega_2^2 J_{(0)}\right)u_1^{\mu}u_1^{\nu} \\ + \left(\frac{2\gamma^2-1}{\omega_1+i\epsilon}I^{\mu}_{(1)} + 4\gamma\omega_2 J^{\mu}_{(1)}\right)u_1^{\nu} + \frac{2\gamma^2-1}{2}J^{\mu\nu}_{(2)}\Bigg]e^{ik\cdot b_1}\Bigg\} + (1\leftrightarrow 2)$$

Two set of master integrals

$$I^{\mu_1\dots\mu_n}_{(n)} \equiv \int_q \hat{\delta}\left(q\cdot u_1 - \omega_1\right)\hat{\delta}\left(q\cdot u_2\right)\frac{e^{-iq\cdot b}}{q^2}q^{\mu_1}\dots q^{\mu_n}$$

$$J^{\mu_1\dots\mu_n}_{(n)} \equiv \int_q \hat{\delta}\left(q\cdot u_1 - \omega_1\right)\hat{\delta}\left(q\cdot u_2\right)\frac{e^{-iq\cdot b}}{q^2(k-q)^2}q^{\mu_1}\dots q^{\mu_n}$$

- The first set can be solved analytically.
- The second set can be express as a one dimensional integration over a Feynman parameter.

NLO Amplitude

$$
\begin{aligned}
\mathcal{A}_\lambda^{(2)}(k) = -\frac{m_1 m_2}{8 m_{\rm Pl}^3}\epsilon_{\mu\nu}^{*\lambda}(\mathbf{n})\Bigg\{\Bigg[&-2\left(\gamma I_{(0)} + \omega_1\omega_2 J_{(0)}\right)u_1^\mu u_2^\nu \\
&+\left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1+i\epsilon)^2} + \frac{2\gamma\omega_2}{\omega_1+i\epsilon}I_{(0)} + 2\omega_2^2 J_{(0)}\right)u_1^\mu u_1^\nu \\
&+\left(\frac{2\gamma^2-1}{\omega_1+i\epsilon}I_{(1)}^\mu + 4\gamma\omega_2 J_{(1)}^\mu\right)u_1^\nu + \frac{2\gamma^2-1}{2}J_{(2)}^{\mu\nu}\Bigg]e^{ik\cdot b_1}\Bigg\} + (1\leftrightarrow 2)
\end{aligned}
$$

Two set of master integrals

$$
\begin{aligned}
I_{(0)} &= -\frac{1}{2\pi\gamma v}K_0\left(\frac{|\mathbf{b}|\omega_1}{\gamma v}\right) \\
J_{(0)} &= \int_0^1 dy\, e^{-iyk\cdot b}\frac{|\mathbf{b}|}{4\pi s(y)}K_1\left(\frac{|\mathbf{b}|s(y)}{\gamma v}\right) \\
s(y) &= \sqrt{(1-y)^2\omega_1^2 + 2\gamma y(1-y)\omega_1\omega_2 + y\omega_2^2}
\end{aligned}
$$

- Tensorial integral found starting from the scalar results

Ratiated observables

$$h_{\mu\nu}(x) = -\frac{1}{4\pi r}\sum_{\lambda=\pm 2}\int \frac{dk^0}{2\pi} e^{-ik^0 u}\epsilon^{\lambda}_{\mu\nu}(\mathbf{k})\mathcal{A}_\lambda(k)|_{k^\mu = k^0 n^\mu}$$

Linear and Angular momentum fluxes

$$P^\mu_{\rm rad} = \int d\Omega\, du\, r^2\, n^\mu\, \dot{h}_{ij}\dot{h}_{ij}$$

$$J^i_{\rm rad} = \epsilon^{ijk}\int d\Omega\, du\, r^2\left(2h_{jl}\dot{h}_{lk} - x_j\partial_k h_{lm}\dot{h}_{lm}\right)$$

$$\dot{h}_{\mu\nu}(x) \propto \omega\mathcal{A}_\lambda(k) = \omega\mathcal{A}_\lambda(k)_{\rm finite}\,, \qquad \mathcal{A}_\lambda(k)_{\rm finite} = \mathcal{A}_\lambda(k) - \left(\substack{\text{static}\\ \text{contributions}}\right)$$

A time derivative remove all static contributions

Different scaling

$$\mathcal{A}^{(1)}_\lambda(k) \propto \hat{\delta}(\omega)$$

$$P^\mu_{\rm rad} = O\left(G^3\right)\,, \qquad J^i_{\rm rad} = O\left(G^2\right)$$

LO Ratiated Linear Momentum

Momentum in terms of the amplitude

$$P^\mu_{\rm rad} = \sum_\lambda \int_k \hat{\delta}(k^2)\theta(k^0)k^\mu \, |\mathcal{A}_\lambda(k)_{\rm finite}|^2$$
$$= \frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^3} \frac{u_1^\mu + u_2^\mu}{\gamma+1} \mathcal{E}(\gamma) + \mathcal{O}(G^4)$$

- Homogeneous mass dependence, result fixed by the probe limit S. Kovacs and K. Thorne *Astrophys. J. 224 (1978)* .
- $\mathcal{E}(\gamma)$ has been recently found in E. Herrmann et al. *[2101.07255]*

- Analytic result for $\mathcal{E}(\gamma)$ cannot be found due to the involved integration in y.
- The computation is possible at virtually any PN order

$$\frac{\mathcal{E}}{\pi} = \frac{37}{15} v + \frac{2393}{840} v^3 + \frac{61703}{10080} v^5 + \frac{3131839}{354816} v^7 + \mathcal{O}(v^9)$$

  This is in perfect agreement with E. Herrmann et al. *[2101.07255]*
- Agreement with known results at 2PN once written in the CoM frame. L. Blanchet and G. Schaefer *Mon. Not. Roy. Astron. Soc. 239 (1989) 845-867* .

LO Radiated Angular Momentum

$$
\begin{aligned}
J^i_{\text{rad}} &= \epsilon^{ijk} \int d\Omega \, du \, r^2 \left(2 h_{jl} \dot{h}_{lk} - x_j \partial_k h_{lm} \dot{h}_{lm} \right) \\
&= \epsilon^{ijk} \int d\Omega \, r^2 \left(2 h^{(1)}_{jl} \delta_{mk} - x_j \partial_k h^{(1)}_{lm} \right) \int du \, \dot{h}^{(2)}_{lm} + O\left(G^3\right)
\end{aligned}
$$

Gravitational wave memory

Wave memory determined by the Soft Limit

$$
\int du \, \dot{h}_{ij} = \frac{i}{4\pi r} \sum_{\lambda} \int \frac{d\omega}{2\pi} \epsilon^{\lambda}_{ij} \, \bar{\delta}(\omega)\omega \, \mathcal{A}_{\lambda}(k)_{\omega \to 0}
$$

LO Angular Momentum does not depend on gravitational self-interactions.

T. Damour *Phys. Rev. D 102 12 (2020)* .

$$J^i_{\rm rad} = \epsilon^{ijk}\int d\Omega\, r^2 \left(2h^{(1)}_{jl}\delta_{mk} - x_j\partial_k h^{(1)}_{lm}\right)\int du\, \dot{h}^{(2)}_{lm} + O\left(G^3\right)$$

LO Angular momentum

Polar coordinates $\mathbf{e}_\theta = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)\,, \mathbf{e}_\phi = (-\sin\phi, \cos\phi, 0)$, $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.

$$\mathbf{J}_{\rm rad} = \sum_\lambda \int \frac{d\Omega}{(4\pi)^2}\,\omega\mathcal{A}^{(2)*}_\lambda(k)_{\omega\to 0}\ \hat{\mathbf{J}}\ \ a^{(1)}_\lambda + \mathcal{O}(G^3)$$

$$\hat{\mathbf{J}} \equiv \lambda(\mathbf{n} + \cot\theta\,\mathbf{e}_\theta) + \hat{\mathbf{L}}$$

$$a^{(1)}_\lambda = -\frac{m_1}{2m_{\rm Pl}}\,\frac{\gamma v^2}{n\cdot v}\epsilon^{*\lambda}_{ij}\mathbf{e}^i_v\mathbf{e}^j_v$$

$$\hat{L}^x = i(\sin\phi\partial_\theta + \cot\theta\cos\phi\partial_\phi)$$

$$\hat{L}^y = -i(\cos\phi\partial_\theta - \cot\theta\sin\phi\partial_\phi)$$

$$\hat{L}^z = -i\partial_\phi$$

$$\mathbf{J}_{\rm rad} = \frac{2(2\gamma^2-1)}{\gamma v}\,\frac{G^2 m_1 m_2 J}{|\mathbf{b}|^2}\mathcal{I}(v)(\mathbf{e}_b\times\mathbf{e}_v) + \mathcal{O}(G^3)$$

$$\mathcal{I}(v) \equiv -\frac{16}{3} + \frac{2}{v^2} + \frac{2(3v^2-1)}{v^3}\mathrm{arctanh}(v)$$

T. Damour *Phys. Rev. D* 102 12 (2020) .

Conclusions and Future directions

- Interplay between PN and PM scheme is crucial for high precision computations.
- EFT methods prove to be efficient and useful also in the PM study of the binary inspiral problem (unbound case)
- Small number of topologies thanks to the Polyakov action
- First stepping stone for a derivation of P_{rad} and J_{rad} alternative to scattering-amplitude method

- New way of approaching integrals of the form $J_{(n)}^{\mu_1\ldots\mu_n}$
- Include Spin and finite-size effects in the radiative sector
- Build a map to connect unbound and bound quantities

Thank you for your attention!