Theoretical uncertainties for cosmological phase transitions

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Why cosmological phase transitions?

▶ Why not?
Are there really no phase transitions in particle cosmology?
From $T \ll \text{GeV}$, all the way up to inflation ($10^{10}\text{GeV}$)?

▶ Observable remnants
Such as $(n_B - n_{\overline{B}})/s$, topological defects, magnetic fields, gravitational waves, ... ⇒ new probe of particle physics
1\textsuperscript{st}-order phase transitions

Figure: Cutting et al. arXiv:1906.00480.

- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows (sound waves)
- Which in turn create gravitational waves
Gravitational waves from phase transitions

- **Stochastic GW background**
  Bubble collisions produce a stochastic GW background.
  So too do any topological defects created.

**Figure:** $SU(N)$ confinement transitions, Huang et al. arXiv:2012.11614

- **Probing uncharted GW $(f, \Omega)$-space**
  LIGO, Virgo, KAGRA, NANOGrav, Taiji (2033), LISA (2034),
  DECIGO (B-DECIGO late 2020s), Einstein Telescope (2035?),
  BBO, Cosmic Explorer, ...
The pipeline for phase transitions

Figure: The LISA pipeline $\mathcal{L} \rightarrow \text{SNR}(f)$, Caprini et al. arXiv:1910.13125
The inverse pipeline for phase transitions

Figure: The inverse LISA pipeline $\text{SNR}(f) \rightarrow \mathcal{L}$, see e.g. Croon et al. arXiv:1806.02332
Theoretical uncertainties

- How large are they?
- Where do they come from?
- How to overcome them?
Perturbative sensitivity

GW spectra of first-order phase transitions in any given specific model are very sensitive to details of calculation.

Figure: Theoretical uncertainty at one single parameter point in SMEFT, Croon et al. arXiv:2009.10080.

Figure: Scan of $Z_2$-xSM at $O(g^2)$ versus $O(g^3)$, Carena et al. arXiv:1911.10206. Difference is $\Delta\Omega_{GW}/\Omega_{GW} \sim 10^{10}$!
Unwrapping perturbative sensitivity

- $\Omega_{GW}$ depends very strongly on the temperature of the transition,

$$\Omega_{GW} \propto \frac{(\Delta V_*)^2}{T_*^8},$$

so an apparently innocuous uncertainty in $T_*$ can still result in a huge uncertainty in $\Omega_{GW}$.

- Uncertainties in thermodynamic parameters are themselves quite large.

**Figure:** Theoretical uncertainties for $T_c$ at one benchmark point in the 2HDM, Niemi et al. arXiv:1904.01329
The perturbative expansion

- At zero temperature the loop and coupling expansions line up
  \[ V_{\text{eff}}^{T=0} = A_2 g^2 + A_4 g^4 + A_6 g^6 + \ldots \]
  - tree-level
  - one-loop
  - two-loop

- At high temperature the two expansions are misaligned
  \[ V_{\text{eff}}^{\text{high-}T} = a_2 g^2 + a_3 g^3 + a_4 g^4 + a_5 g^5 + a_6 g^6 + \ldots \]
  - tree-level
  - resummed one-loop
  - resummed one and two-loop

So you have to work harder to achieve the same accuracy.
Comparing orders

Dramatic improvements at $O(g^4)$

![Graph showing critical temperature dependence]

**Figure**: Unphysical renormalisation scale dependence of critical temperature at benchmark points in xSM, OG and T. Tenkanen (forthcoming).

This is great for $T_c$ but not known how to extend this to bubble nucleation.
Comparing uncertainties

- Renormalisation scale dependence appears to be the largest source of theoretical uncertainty, $\Delta \Omega_{GW}/\Omega_{GW} \sim 10^{2-3}$ in the SMEFT, and can be as large as $\sim 10^{10}$ in e.g. $x$SM.
- Some sources (e.g. inconsistencies) are hard to estimate.

Figure: Sources of theoretical uncertainty in $\Omega_{GW}$ for the SMEFT, Croon et al. arXiv:2009.10080. See also Guo et al. arXiv:2103.06933.
Conclusions

- Cosmological phase transitions may be observable by GW detectors
- Typically several orders of magnitude uncertainty in $\Omega_{GW}$
- Dramatic improvements apparent at $O(g^4)$
  $\Rightarrow$ Necessary for LISA inverse problem $\Omega_{GW} \rightarrow \mathcal{L}$
- Can test perturbation theory versus lattice simulations
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Thanks for listening!