Theoretical uncertainties for cosmological phase transitions

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Why cosmological phase transitions?

- Why not?
  Are there really no phase transitions in particle cosmology?
  From $T \ll$ GeV, all the way up to inflation (10^{lots} GeV)?

- Observable remnants
  Such as $(n_B - n_{\bar{B}})/s$, topological defects, magnetic fields, gravitational waves, . . . $\Rightarrow$ new probe of particle physics

1^{st}-order phase transitions

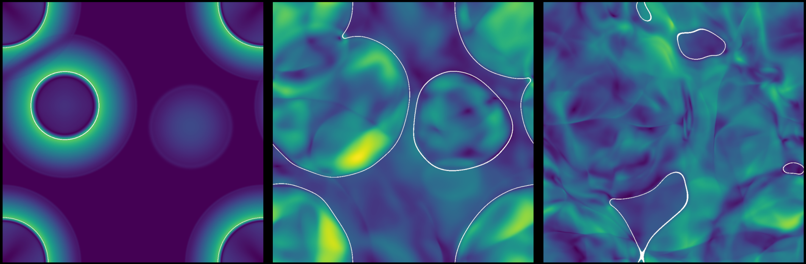


Figure: Cutting et al. arXiv:1906.00480.

- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows (sound waves)
- Which in turn create gravitational waves

Gravitational waves from phase transitions

- **Stochastic GW background**

  Bubble collisions produce a stochastic GW background.

  So too do any topological defects created.

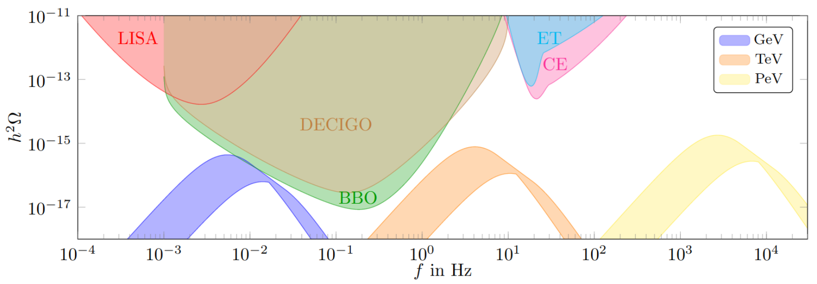
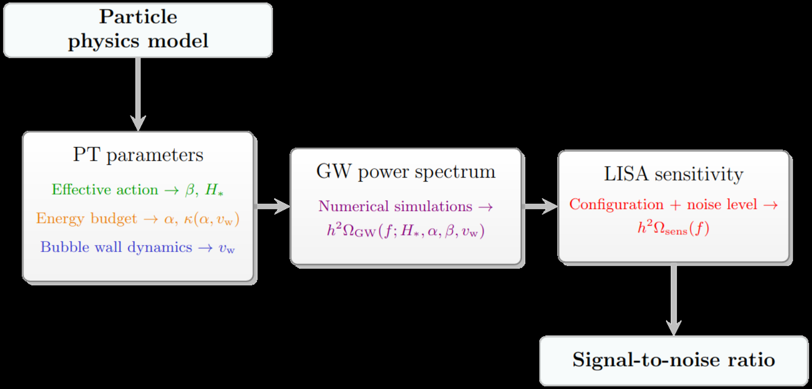


Figure: $SU(N)$ confinement transitions, Huang et al. arXiv:2012.11614

- **Probing uncharted GW (f, Ω)-space**

  LIGO, Virgo, KAGRA, NANOGrav, Taiji (2033), LISA (2034), DECIGO (B-DECIGO late 2020s), Einstein Telescope (2035?), BBO, Cosmic Explorer, . . .

The pipeline for phase transitions



Particle physics model

PT parameters

Effective action → β, H_*

Energy budget → α, $\kappa(\alpha, v_\mathrm{w})$

Bubble wall dynamics → v_w

GW power spectrum

Numerical simulations → $h^2\Omega_\mathrm{GW}(f; H_*, \alpha, \beta, v_\mathrm{w})$

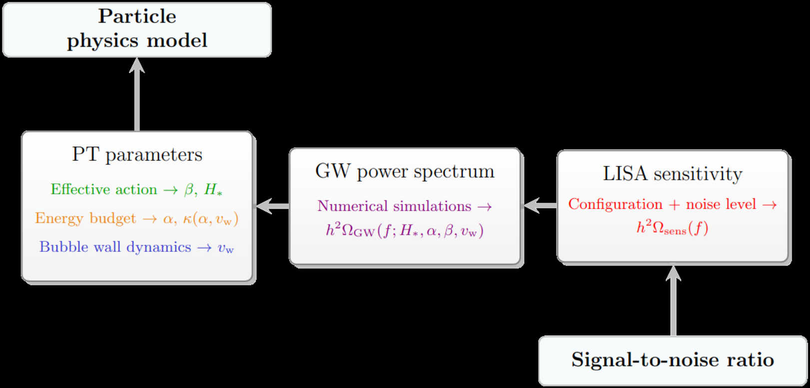
LISA sensitivity

Configuration + noise level → $h^2\Omega_\mathrm{sens}(f)$

Signal-to-noise ratio

Figure: The LISA pipeline $\mathscr{L} \to \mathrm{SNR}(f)$, Caprini et al. arXiv:1910.13125

The inverse pipeline for phase transitions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LISA sensitivity

Configuration + noise level → $h^2\Omega_\mathrm{sens}(f)$

Signal-to-noise ratio

Figure: The inverse LISA pipeline $\mathrm{SNR}(f) \to \mathscr{L}$, see e.g. Croon et al. arXiv:1806.02332

Theoretical uncertainties

- How large are they?
- Where do they come from?
- How to overcome them?

Perturbative sensitivity

- GW spectra of first-order phase transitions in any given specific model are very sensitive to details of calculation.

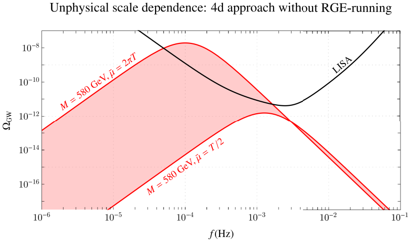

Figure: Theoretical uncertainty at one single parameter point in SMEFT, Croon et al. arXiv:2009.10080.

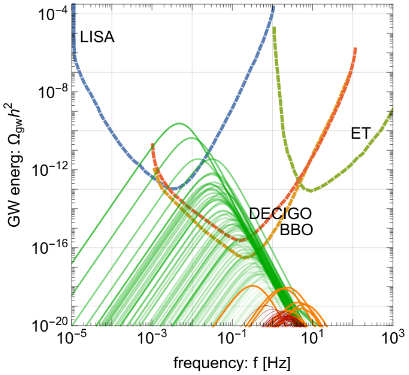

Figure: Scan of Z_2-xSM at $O(g^2)$ versus $O(g^3)$, Carena et al. arXiv:1911.10206. Difference is $\Delta\Omega_{\mathrm{GW}}/\Omega_{\mathrm{GW}} \sim 10^{10}$!

Unwrapping perturbative sensitivity

- Ω_{GW} depends very strongly on the temperature of the transition,

  $$\Omega_{\mathsf{GW}} \propto \frac{(\Delta V_*)^2}{T_*^8},$$

  so an apparently innocuous uncertainty in T_* can still result in a huge uncertainty in Ω_{GW}.
- Uncertainties in thermodynamic parameters are themselves quite large

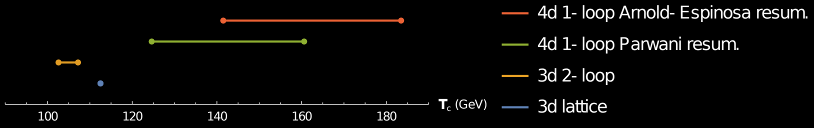

Figure: Theoretical uncertainties for T_c at one benchmark point in the 2HDM, Niemi et al. arXiv:1904.01329

The perturbative expansion

- At zero temperature the loop and coupling expansions line up

$$V_{\text{eff}}^{T=0} = \underbrace{A_2 g^2}_{\text{tree-level}} + \underbrace{A_4 g^4}_{\text{one-loop}} + \underbrace{A_6 g^6}_{\text{two-loop}} + \dots$$

- At high temperature the two expansions are misaligned

$$V_{\text{eff}}^{\text{high-}T} = \underbrace{a_2 g^2}_{\substack{\text{tree-level}\\ \text{and}\\ \text{one-loop}}} + \underbrace{a_3 g^3}_{\substack{\text{resummed}\\ \text{one-loop}}} + \underbrace{a_4 g^4}_{\substack{\text{resummed}\\ \text{one and}\\ \text{two-loop}}} + a_5 g^5 + a_6 g^6 + \dots$$

So you have to work harder to achieve the same accuracy.

Comparing orders

Dramatic improvements at $\mathcal{O}(g^4)$

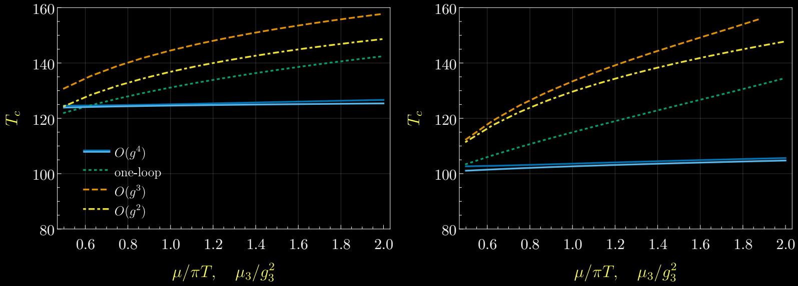

Figure: Unphysical renormalisation scale dependence of critical temperature at benchmark points in xSM, OG and T. Tenkanen (forthcoming).

This is great for T_c but not known how to extend this to bubble nucleation.

Comparing uncertainties

- Renormalisation scale dependence appears to be the largest source of theoretical uncertainty, $\Delta\Omega_{\mathsf{GW}}/\Omega_{\mathsf{GW}} \sim 10^{2-3}$ in the SMEFT, and can be as large as $\sim 10^{10}$ in e.g. xSM.
- Some sources (e.g. inconsistencies) are hard to estimate.

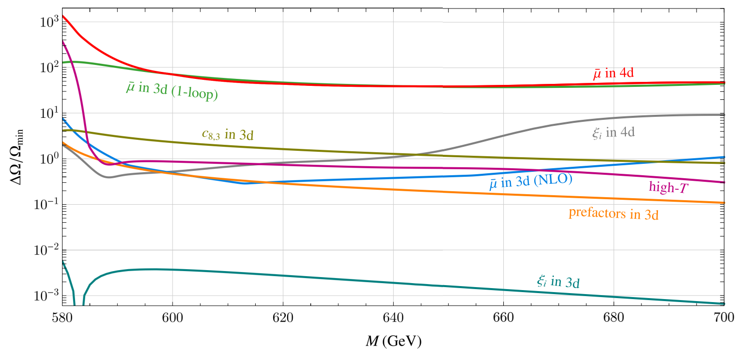

Figure: Sources of theoretical uncertainty in Ω_{GW} for the SMEFT, Croon et al. arXiv:2009.10080. See also Guo et al. arXiv:2103.06933.

Conclusions

- Cosmological phase transitions may be observable by GW detectors
- Typically several orders of magnitude uncertainty in Ω_{GW}
- Dramatic improvements apparent at $\mathcal{O}(g^4)$
  $\Rightarrow$ Necessary for LISA inverse problem $\Omega_{\mathsf{GW}} \rightarrow \mathscr{L}$
- Can test perturbation theory versus lattice simulations
  Niemi et al. arXiv:1904.01329, OG arXiv:2101.05528

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Thanks for listening!