

Gravitational wave signature of proto-neutron star convection

Raphaël Raynaud (CEA),
Pablo Cerdá-Durán (Universitat de València),
& Jérôme Guilet (CEA)

Meeting of the National Research Group on Gravitational Waves
Institut Henri Poincaré – March 30th, 2020



Groupement de recherche
Ondes gravitationnelles



Table of contents

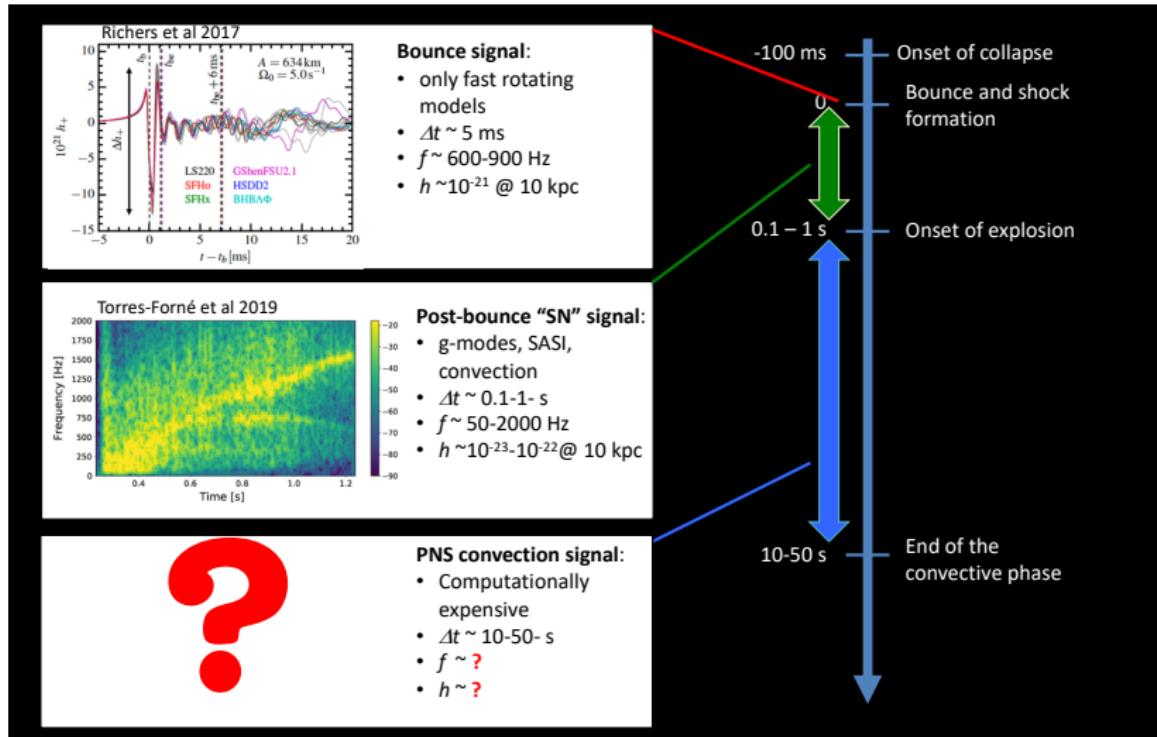
1 Introduction

2 Model

3 Results

4 Conclusion

GW signal in CCSNe



Protoneutron star structure ... and evolution

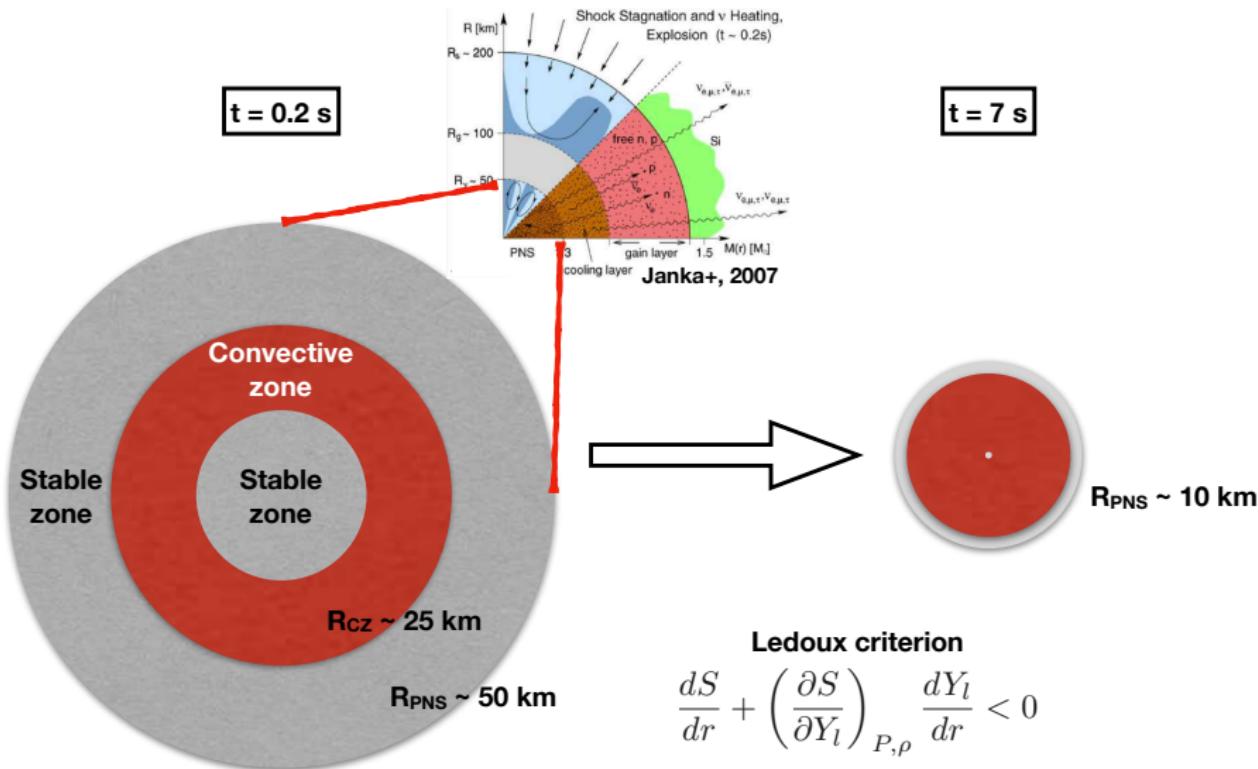


Table of contents

1 Introduction

2 Model

3 Results

4 Conclusion

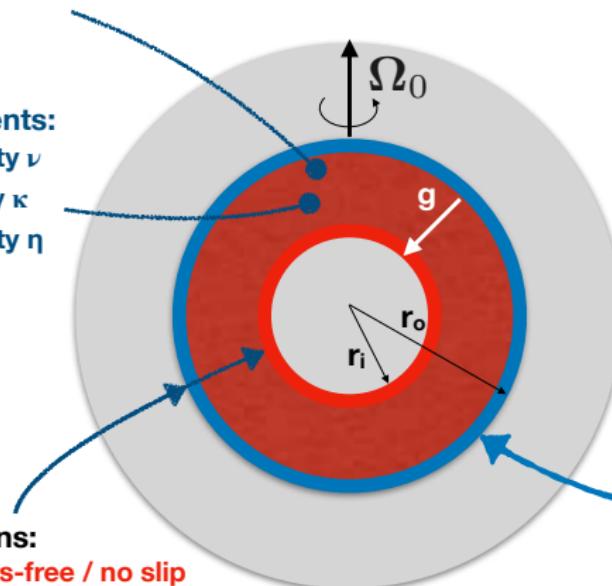
Requirements and simplifying assumptions

Input:

- Temperature profile
- Density profile

Transport coefficients:

- Kinematic viscosity ν
- Thermal diffusivity κ
- Magnetic diffusivity η



Boundary conditions:

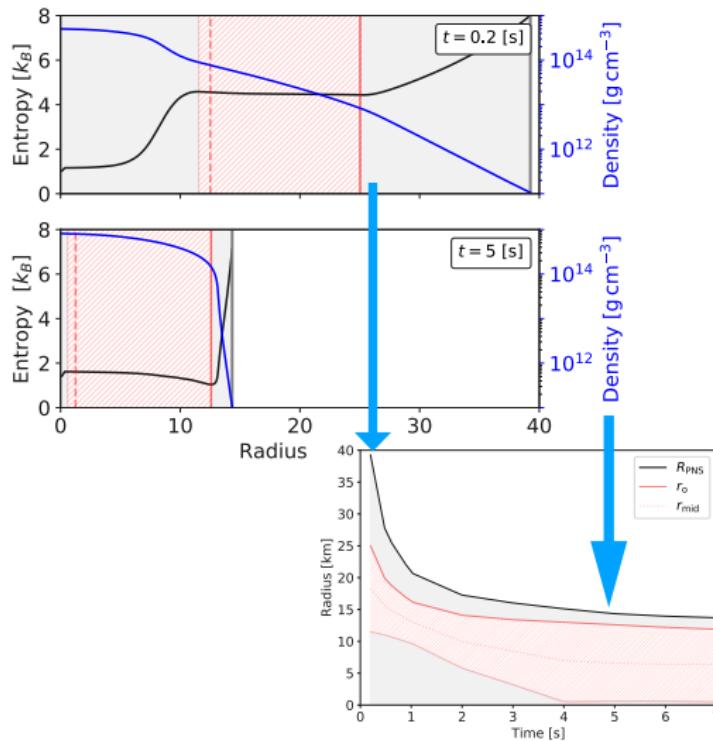
- Mechanical: stress-free / no slip
- Thermal: fixed entropy flux
- Magnetic: perfect conductor ($B_{||}$) / pseudo-vacuum (B_{\perp})

Hypothesis:

- Spherical geometry
- Adiabatic stratification
- Low Mach convection
- 2nd order diffusion approximation for the neutrino transport
- Electrical conductivity of degenerate, relativistic electrons
- Orders of magnitude

$$\left\{ \begin{array}{l} \Phi_o \sim 10^{52} \text{ erg/s} \\ r_o \sim 25 \text{ km} \\ T_o \sim 10^{11} \text{ K} \\ \varrho_o \sim 10^{13} \text{ g/cm}^3 \\ \nu_o \sim 10^{10} \text{ cm}^2/\text{s} \\ \kappa_o \sim 10^{12} \text{ cm}^2/\text{s} \\ \eta_o \sim 10^{-3} \text{ cm}^2/\text{s} \end{array} \right.$$

Early and late time background models



Source

Lorenz Hüdepohl's PhD thesis
 Prometheus-Vertex code
 1D model + MLT
 LS220 EoS
 $27 M_\odot$ progenitor
 PNS baryonic mass $1.78 M_\odot$

Method

1. stability determined according to the Schwarzschild criterion
2. deduce the shell geometry
3. fit the background profile $(\tilde{\varrho}, \tilde{T})$

The MHD anelastic equations

Braginsky+95, Lantz+99, Jones+[11,14]

$$[d] = r_o - r_i, \quad [t] = d^2/\nu_o, \quad [S] = d \partial S / \partial r|_{r_o}, \quad [p] = \Omega \varrho_o \nu_o, \quad [B] = \sqrt{\Omega \varrho_o \mu_0 \eta_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction}} - \underbrace{\frac{1}{Pm} \nabla \times (\eta \nabla \times \mathbf{B})}_{\text{Dissipation}}$$

$$0 = \nabla \cdot (\tilde{\varrho} \mathbf{u})$$

$$\frac{D\mathbf{u}}{Dt} = - \underbrace{\nabla \left(\frac{p}{E \tilde{\varrho}} \right)}_{\text{Pressure}} - \underbrace{\frac{2}{E} \mathbf{e}_z \times \mathbf{u}}_{\text{Coriolis}} - \underbrace{\frac{Ra}{Pr} \frac{d \tilde{T}}{dr} S \mathbf{e}_r}_{\text{Buoyancy}} + \underbrace{\mathbf{F}_v}_{\text{Viscosity}} + \underbrace{\frac{1}{EPm} \frac{1}{\tilde{\varrho}} (\nabla \times \mathbf{B}) \times \mathbf{B}}_{\text{Lorentz}}$$

$$\frac{DS}{Dt} = \frac{1}{Pr \tilde{\varrho} \tilde{T}} \underbrace{\nabla \cdot (\kappa \tilde{\varrho} \tilde{T} \nabla S)}_{\text{Heat flux}} + \frac{Pr}{Ra \tilde{\varrho} \tilde{T}} \left(\underbrace{\frac{\eta}{Pm^2 E} (\nabla \times \mathbf{B})^2}_{\text{Ohmic heating}} + \underbrace{Q_v}_{\text{Viscous heating}} \right)$$

3D MHD direct numerical simulations

Control parameters

Prandtl number $Pr = \nu_o / \kappa_o$

magnetic Prandtl number $Pm = \nu_o / \eta_o$

Ekman number $E = \frac{\nu_o}{\Omega d^2}$

Rayleigh number $Ra = \frac{\tilde{T}_o d^3 \left. \frac{\partial S}{\partial r} \right|_{r_o}}{\nu_o \kappa_o}$

Input

$Ra/Ra_c \sim 10$

$Pr = 0.1$

$Pm \sim 5 \quad (\ll 10^{14})$

$E \equiv P_{\text{rot}} \in [1 \text{ ms}, 10^2 \text{ ms}]$



Output

Gravitational signal
computed with the quadrupole
approximation

Table of contents

1 Introduction

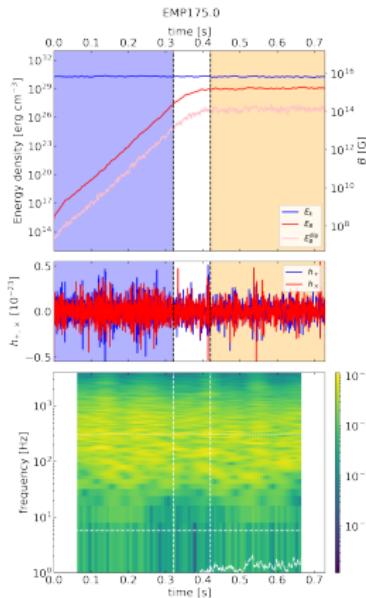
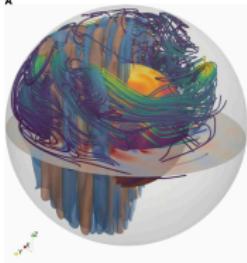
2 Model

3 Results

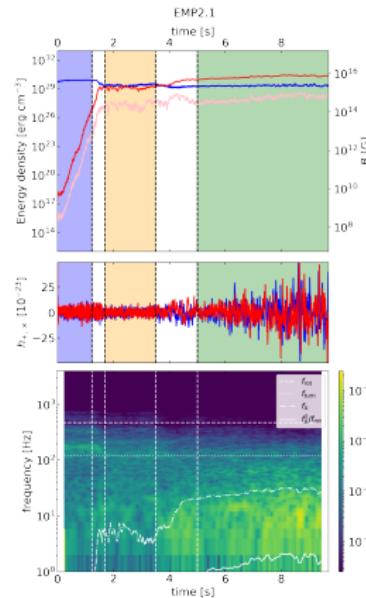
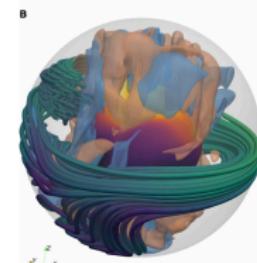
4 Conclusion

Typical cases: slow versus fast rotation

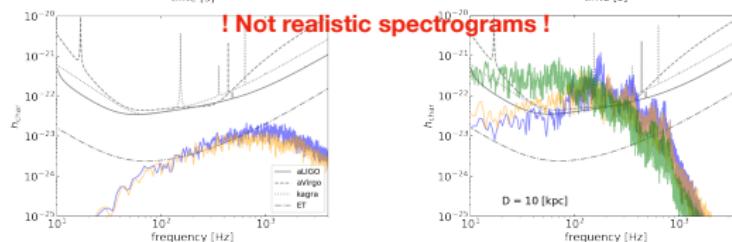
P=175 ms



P=2.1 ms



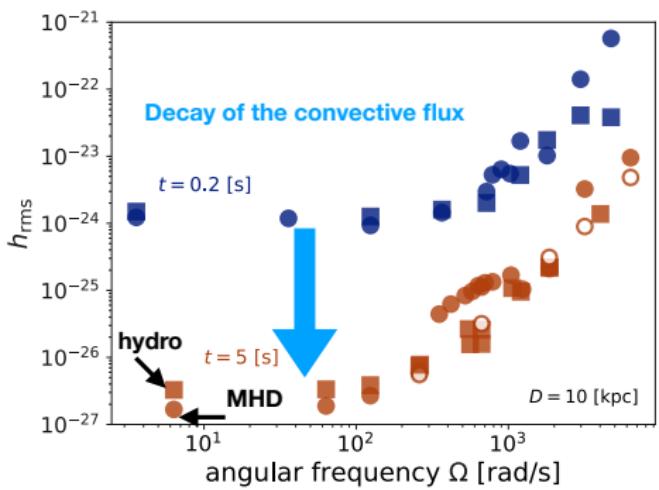
Magnetar formation



(Raynaud et al . 2020)

NB: fixed background !

Amplitude scaling

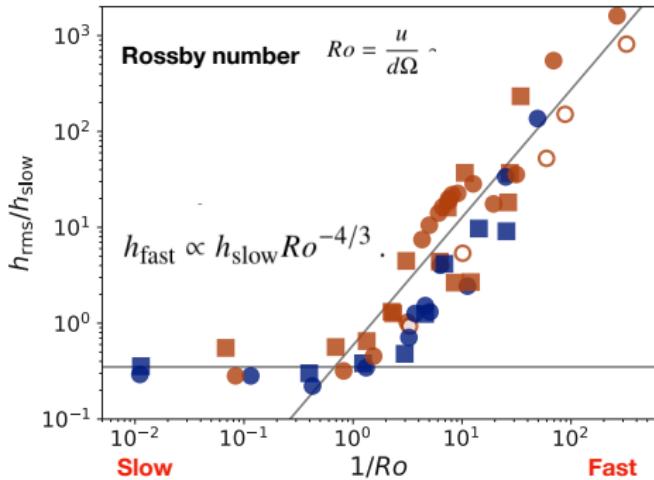
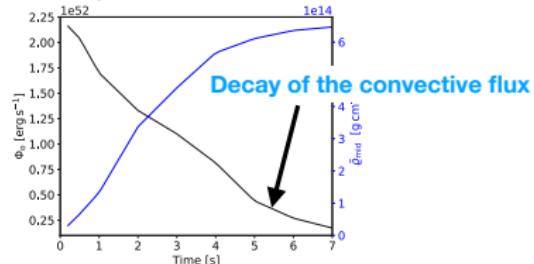


Scaling relation for slow/fast regimes

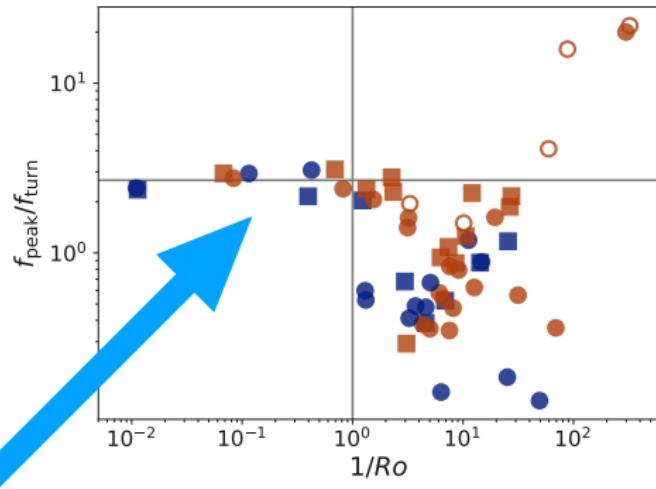
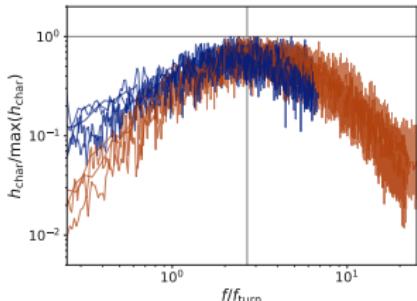
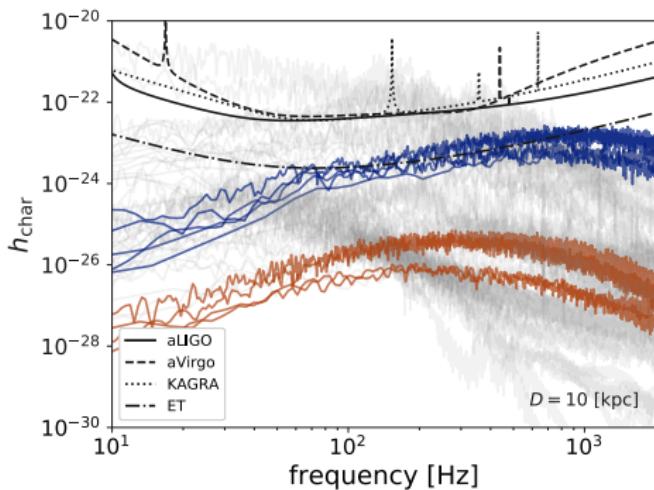
$$h_{\text{slow}} \propto \frac{2G}{Dc^4} r_{\text{mid}}^2 M_{\text{conv}} \left(\frac{\gamma}{2\alpha_{\text{mlt}}} + 1 \right) \frac{d^{-2/3}}{c_s^2} \left(|\partial_r \ln \tilde{T}| \frac{\Phi_0}{4\pi r_{\text{mid}}^2 \tilde{\varrho}} \right)^{4/3}, \quad (33)$$

$$h_{\text{fast}} \propto \frac{2G}{Dc^4} r_{\text{mid}}^2 M_{\text{conv}} \left(\frac{\gamma}{2\alpha_{\text{mlt}}} + 1 \right) \frac{d^{2/5}}{c_s^2} \left(|\partial_r \ln \tilde{T}| \frac{\Phi_0}{4\pi r_{\text{mid}}^2 \tilde{\varrho}} \right)^{4/5} \Omega^{8/5}.$$

Background time evolution



Frequency scaling: slow rotation



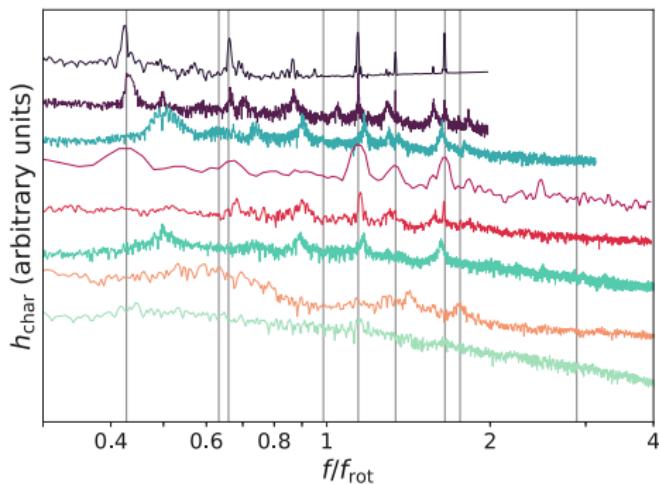
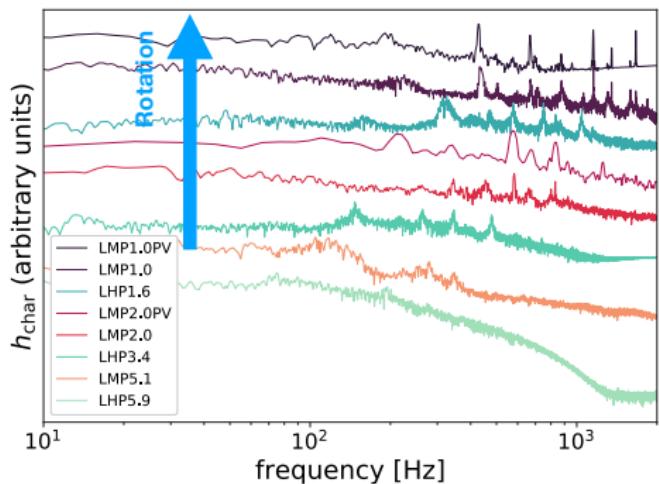
Peak frequency scales with the turnover frequency U/d

Broad spectrum due to convection

Frequency scaling: fast rotation

$t = 5 \text{ s post bounce}$

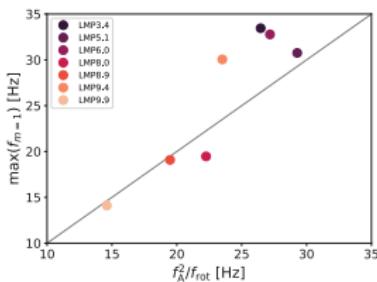
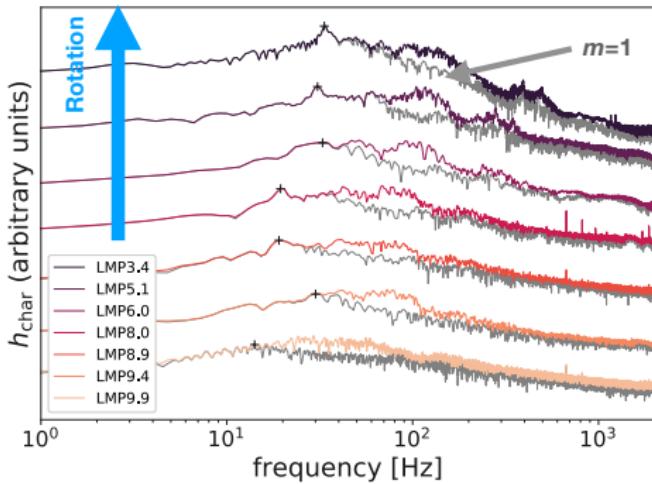
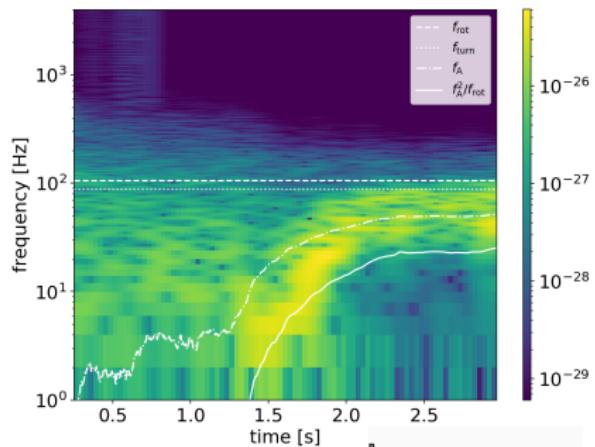
$1 \text{ ms} < \text{Period} < 6 \text{ ms}$



Peaks scale with the PNS rotation frequency

Signature of inertial modes

Strong field dynamo signature ?



Hypothesis: $m=1$ Rossby mode modified by toroidal magnetic field ?

Table of contents

1 Introduction

2 Model

3 Results

4 Conclusion

Conclusions

Raynaud et al. (arXiv:2103.12445)

Slow rotation ($Ro \gg 1$)

- broad spectrum
- peak scales with f_{turn}
- weak impact of B field

Fast rotation ($Ro \ll 1$)

- h_{rms} strongly increases
- complex spectra
- peaks scale with f_{rot}
- inertial modes
- low frequency signature of strong field dynamo

Limitations

- consider only one background model
- no continuous evolution of the PNS cooling (no realistic GW template)
- convective zone only (no g -modes)

Perspectives: detectability

- use amplitude/frequency scalings to rescale the signal as a function of the background evolution