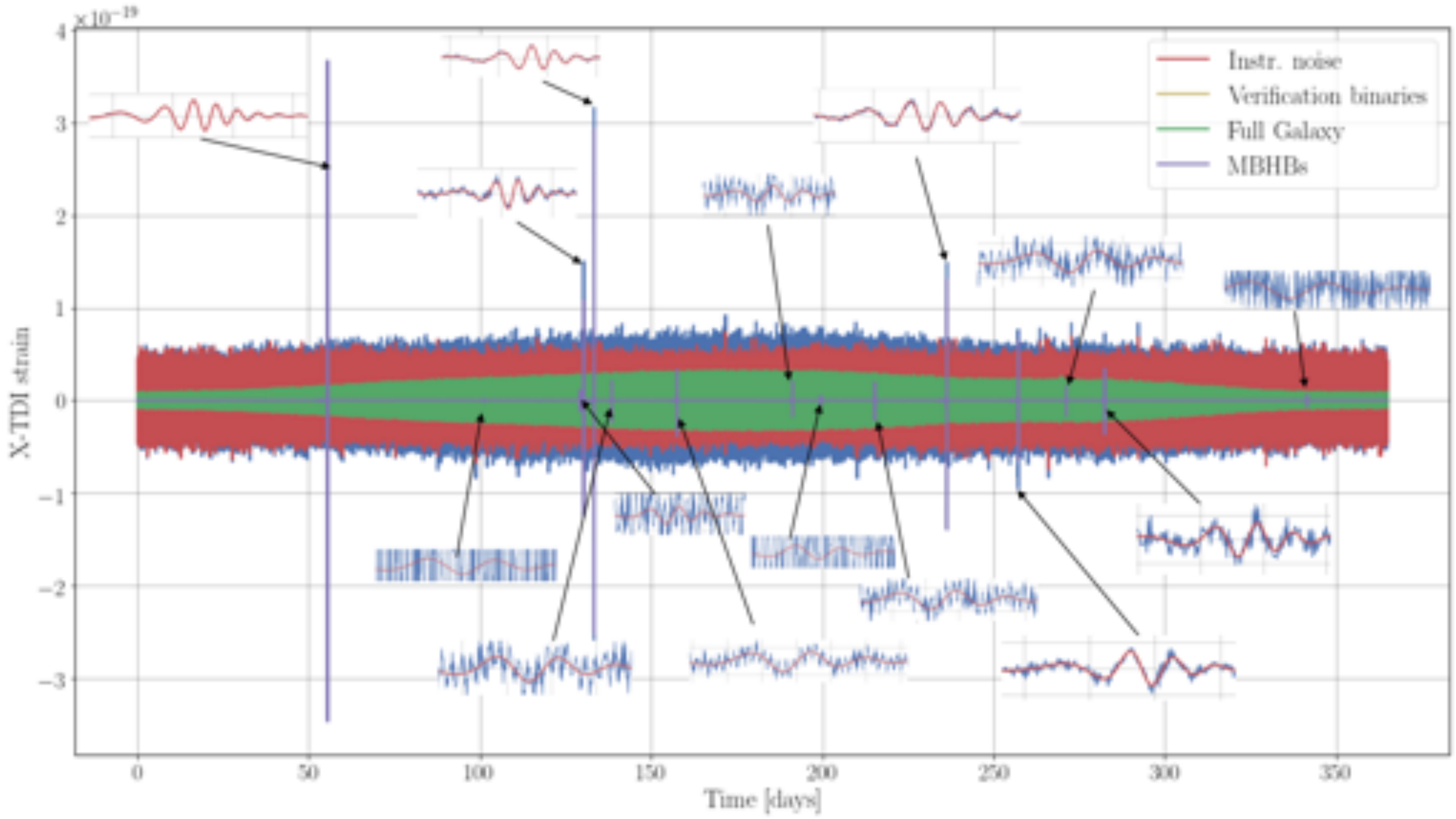
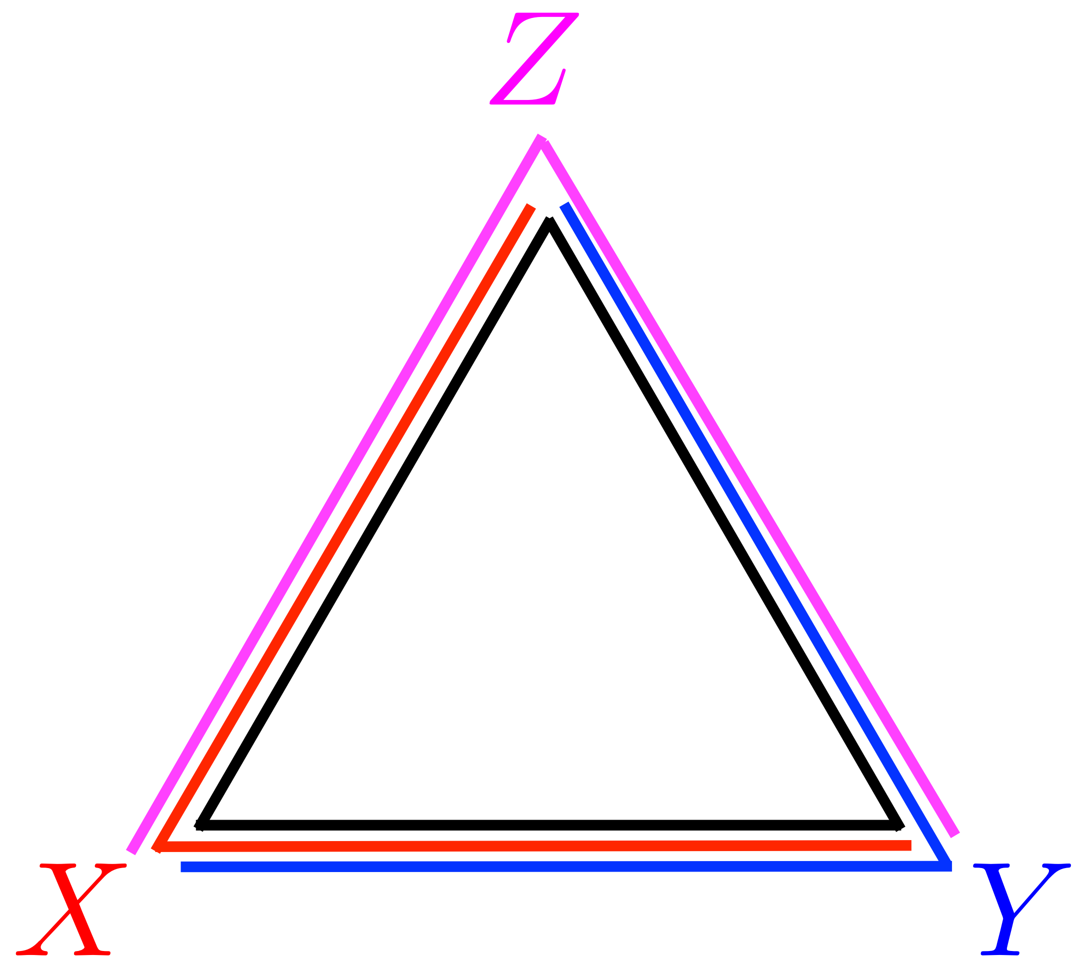
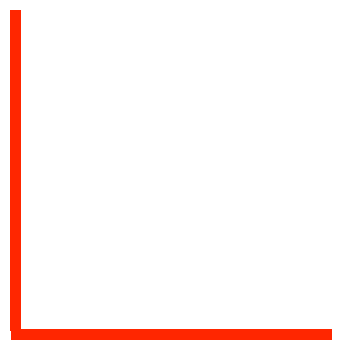
A global perspective on LISA analysis and cosmological backgrounds

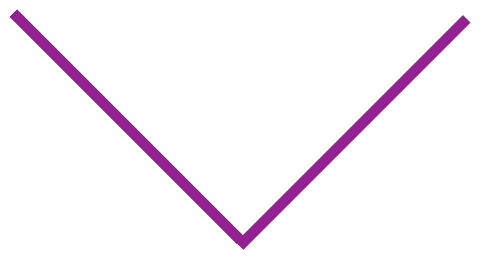


Neil Cornish

LISA - Three interferometers in one



$$A = \frac{\sqrt{3}}{2} X \quad \Rightarrow$$ 

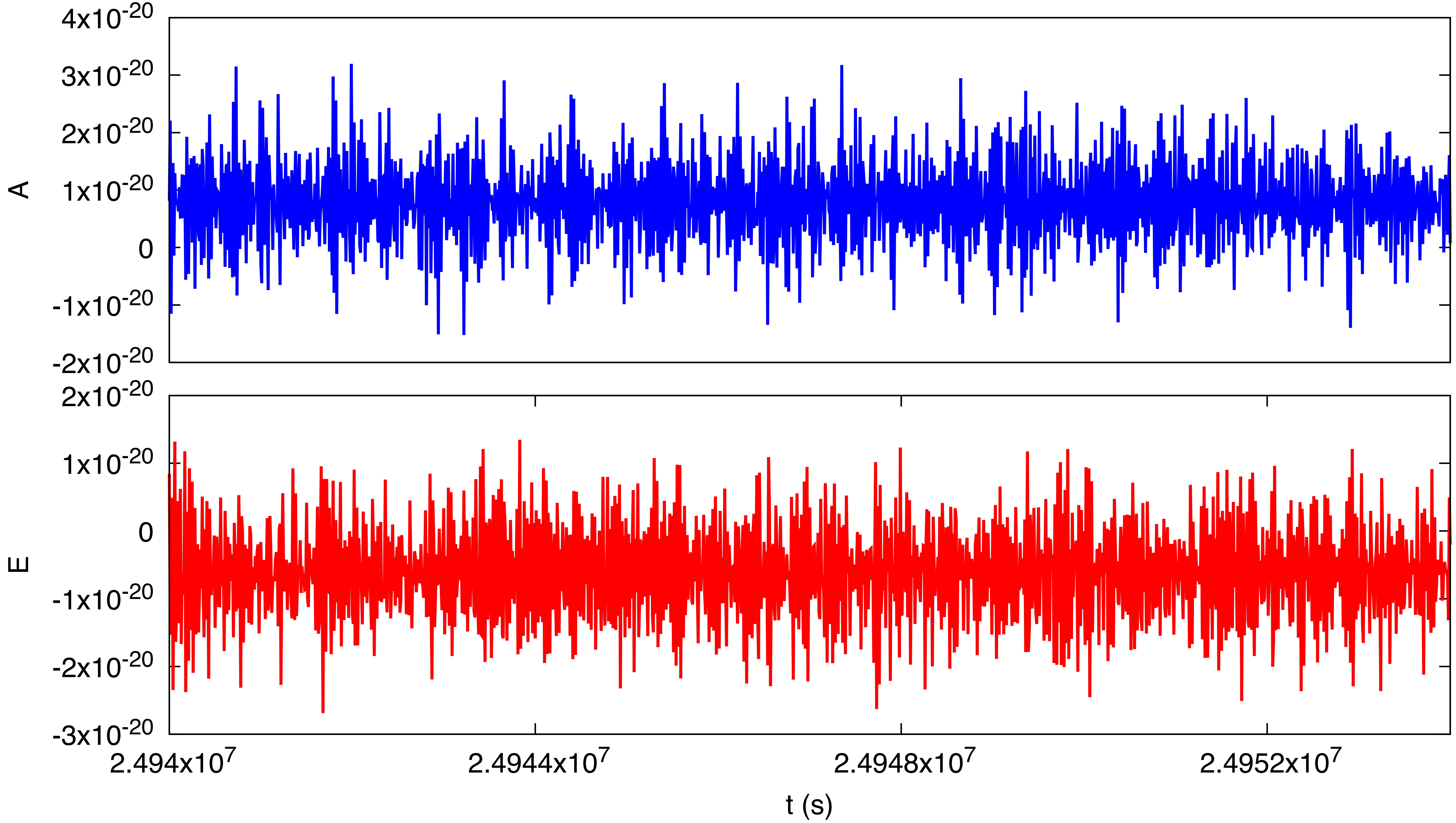
$$E = \frac{1}{2}(X + 2Y) \quad \Rightarrow$$ 

Instantaneous measurement of both polarization states and increased signal-to-noise

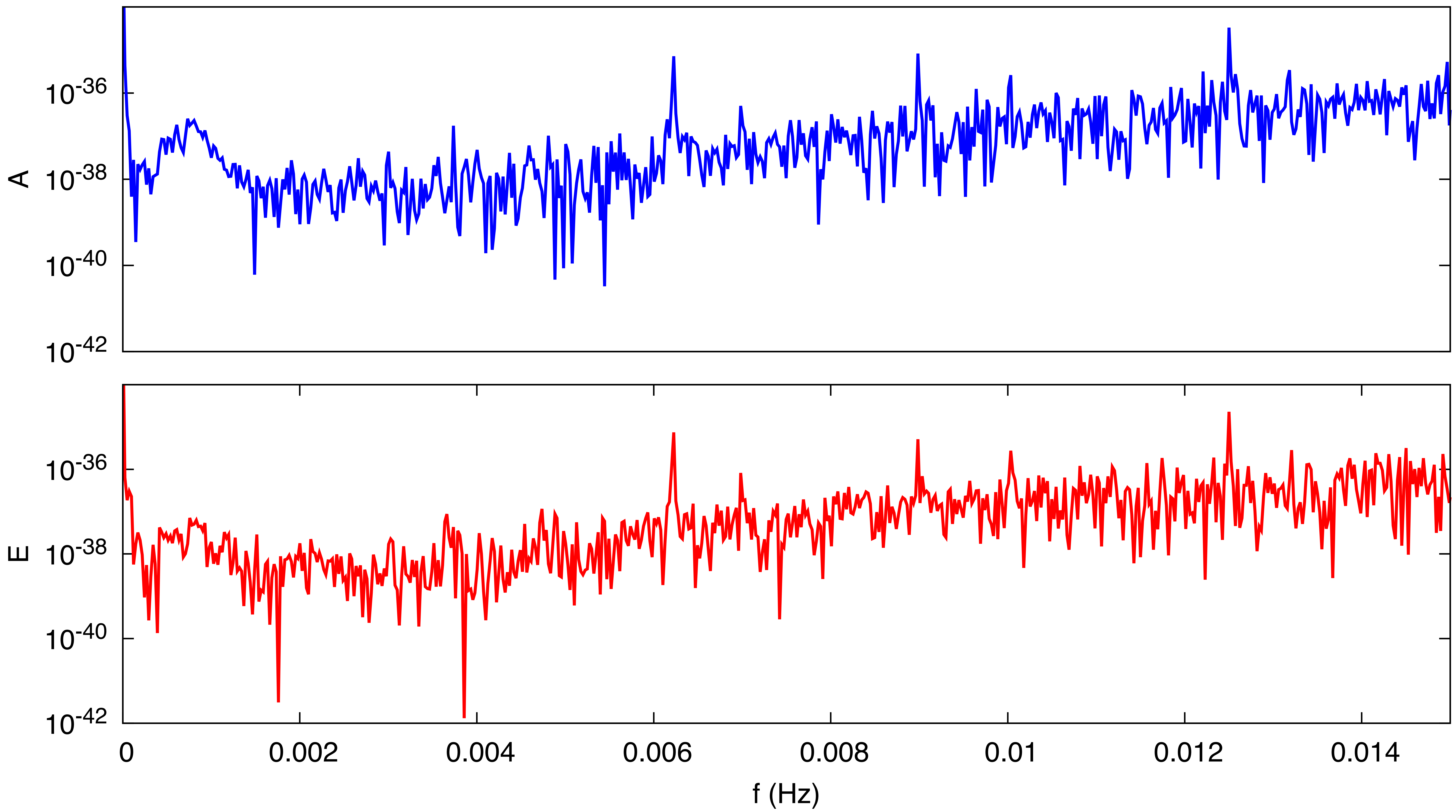
$$T = \frac{1}{3}(X + Y + Z) \quad \Rightarrow$$

Null channel to monitor average low frequency instrument noise

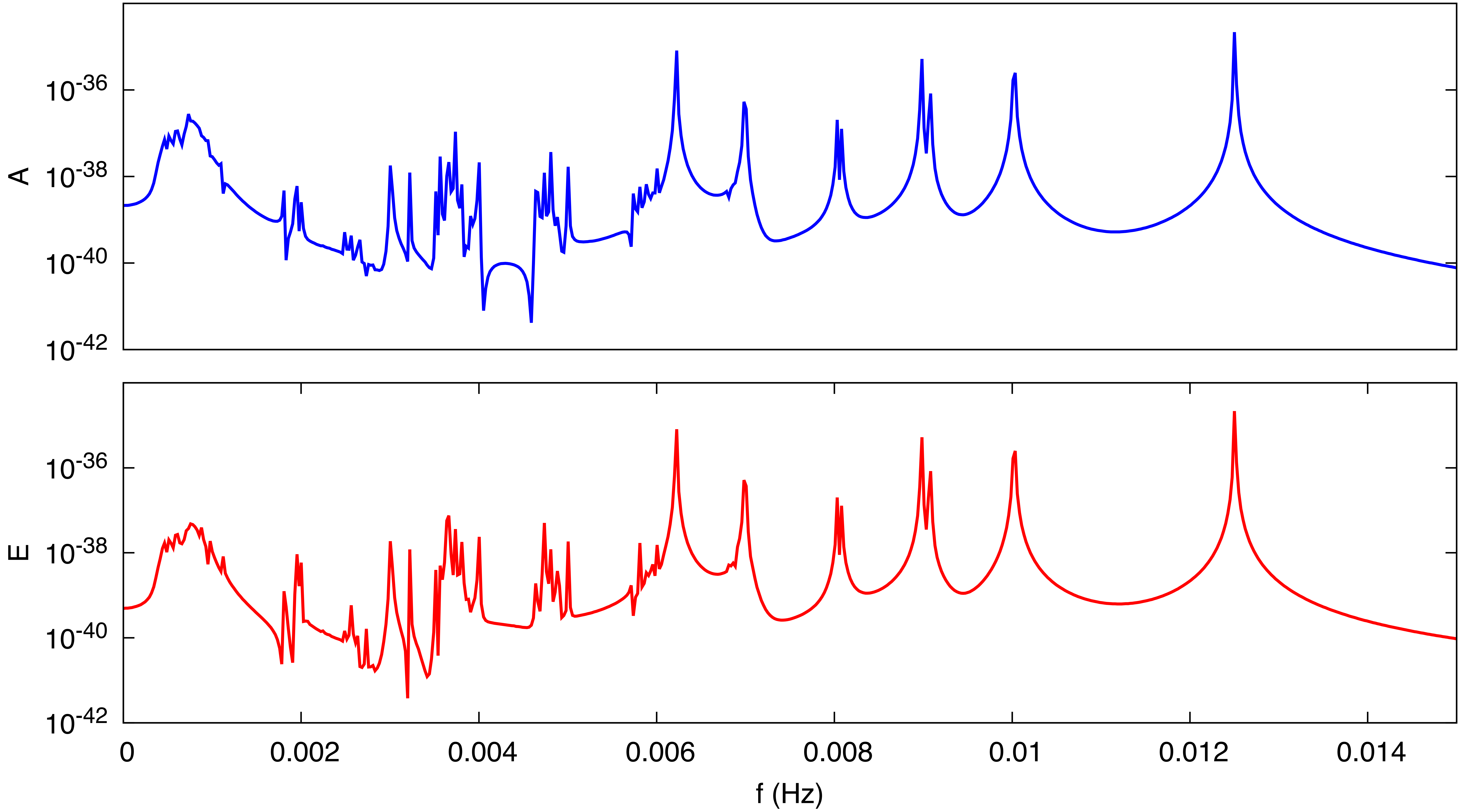
Four hours of simulated LISA data



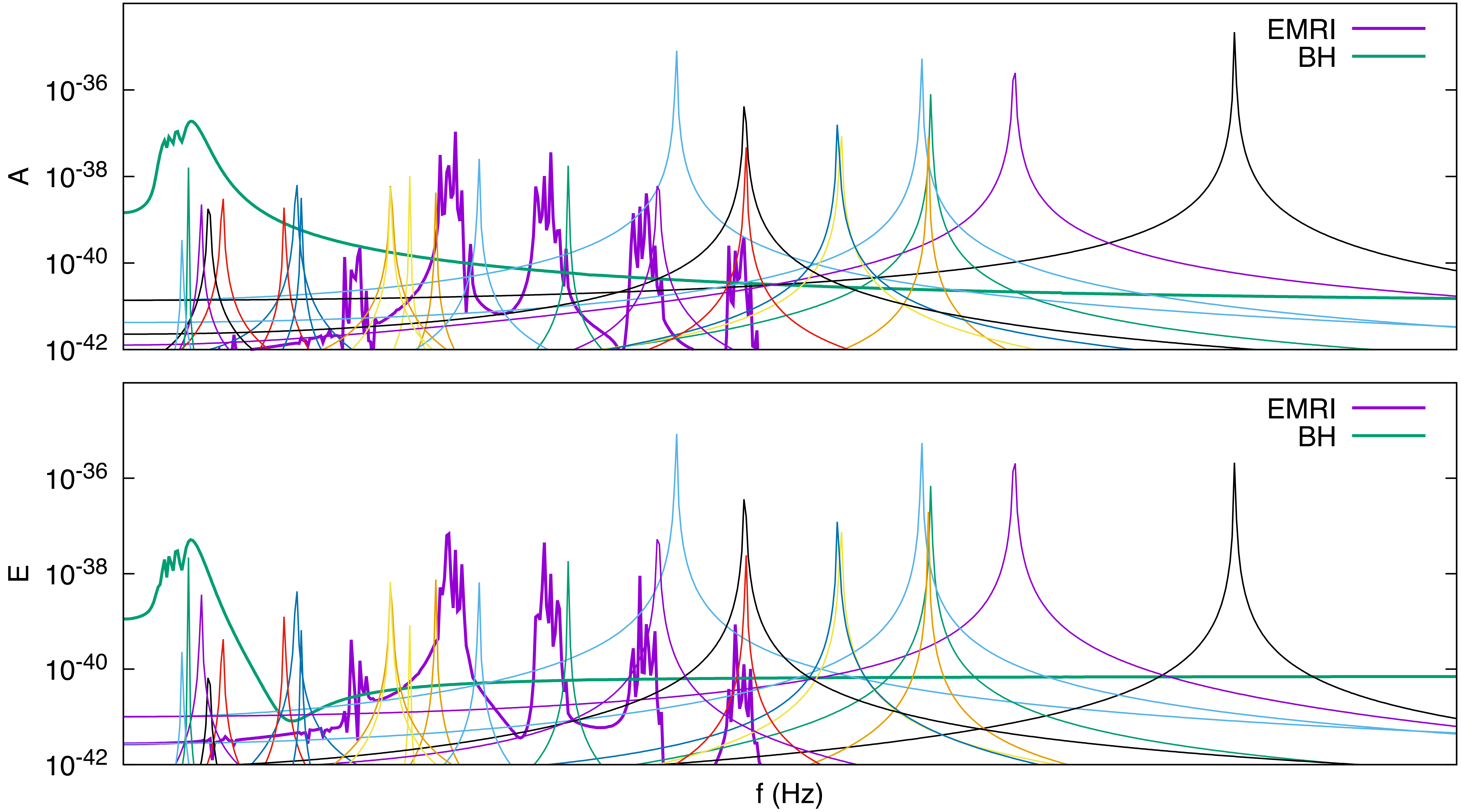
Four hours of simulated LISA data (Fourier domain)



Four hours of simulated LISA data - noise removed



1 SMBHB, 1 EMRI, 25 Galactic Binaries



The LISA Global Solution

Likelihood function for Gaussian noise

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det \mathbf{C}}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

$$\mathbf{d} = \text{data}$$

$$\mathbf{h} = \sum_{i=1}^{N} \mathbf{h}_i = \text{GW signal model}$$

$$\mathbf{C} = \text{noise correlation matrix}$$

$$\vec{\lambda} = \text{model parameters (signals, noise)}$$

$$M = \text{size of data}$$

The LISA Global Solution

Likelihood function for Gaussian noise

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det \mathbf{C}}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

$\vec{\lambda}$ = model parameters (Signals, Noise)

$\dim[\vec{\lambda}] \sim 200{,}000 \rightarrow 500{,}000$

Galactic Binaries
Massive Black Holes
Extreme Mass Ratio Inspirals
Stellar Origin Black Holes
Astrophysics Backgrounds
Primordial Backgrounds
Un-modeled Bursts

Acceleration Noise
Position Noise
Glitches
Gaps
Orbital Dynamics

Note: Number of resolvable sources *a priori* unknown - requires trans-dimensional model selection

The Global Solution

$$\chi^2 = (\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h}) = (\mathbf{d}-\mathbf{h}|\mathbf{d}-\mathbf{h})$$

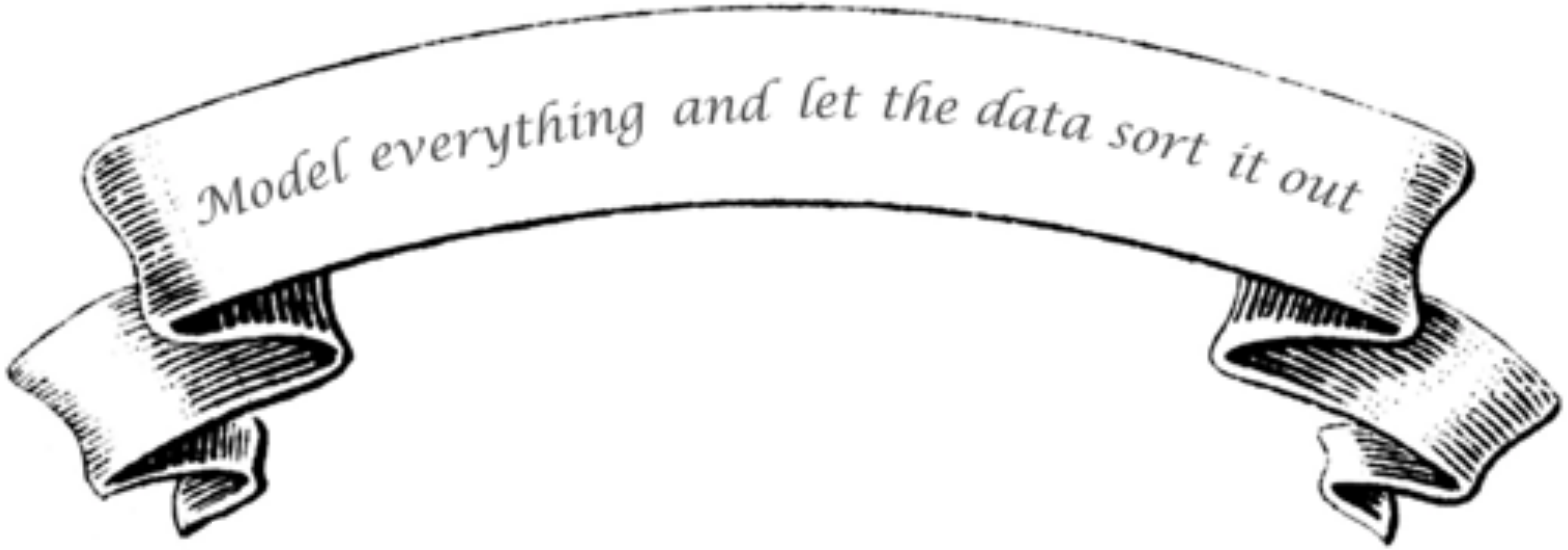
$$\chi^2 = \sum_i(\mathbf{d}-\mathbf{h}_i|\mathbf{d}-\mathbf{h}_i) + (N-1)(\mathbf{d}|\mathbf{d}) - \sum_{i\neq j}(\mathbf{h}_i|\mathbf{h}_j)$$

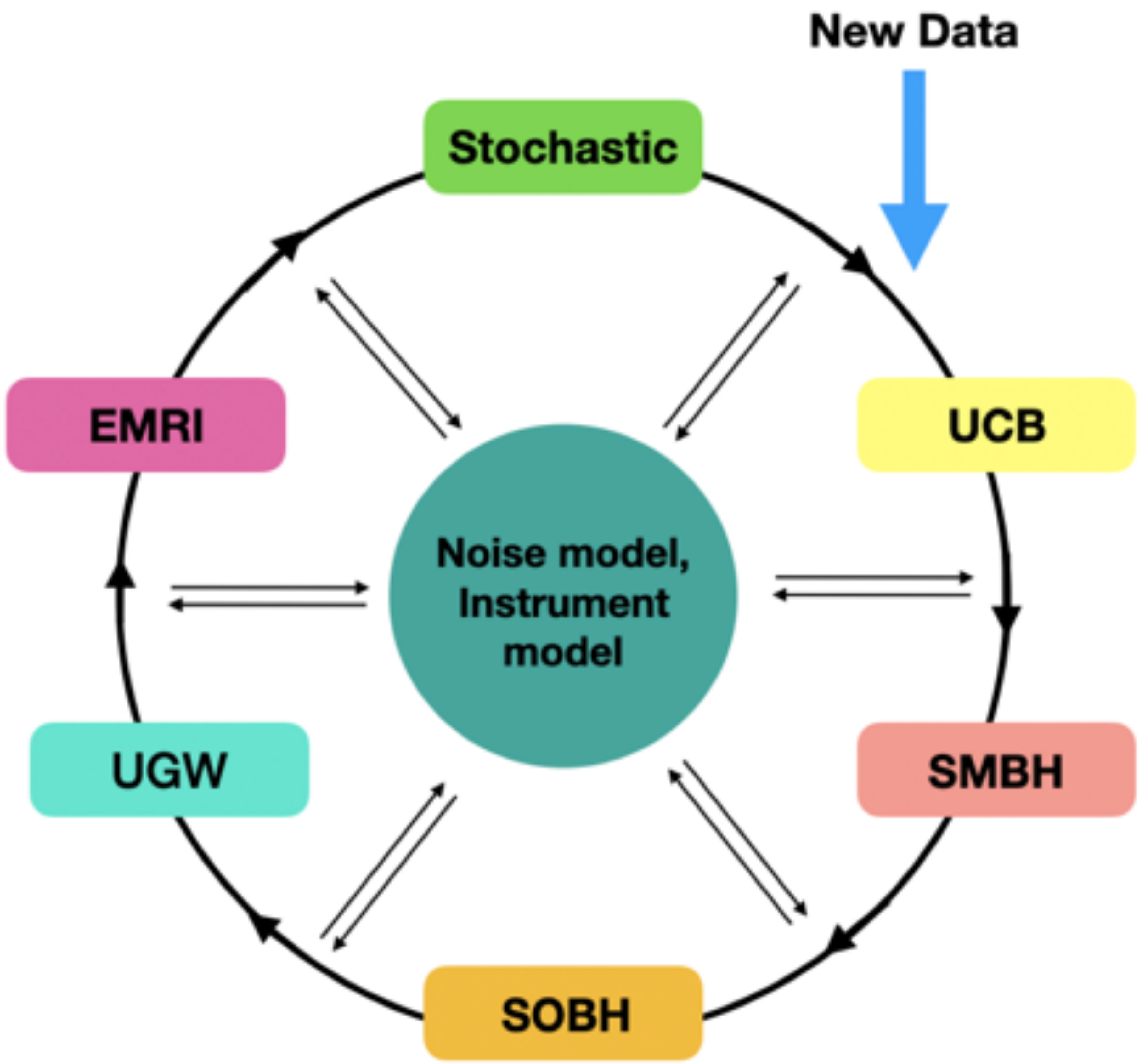
Individual source chi-squared

Overlap between sources

If the overlap terms were zero we wouldn’t need a global solution

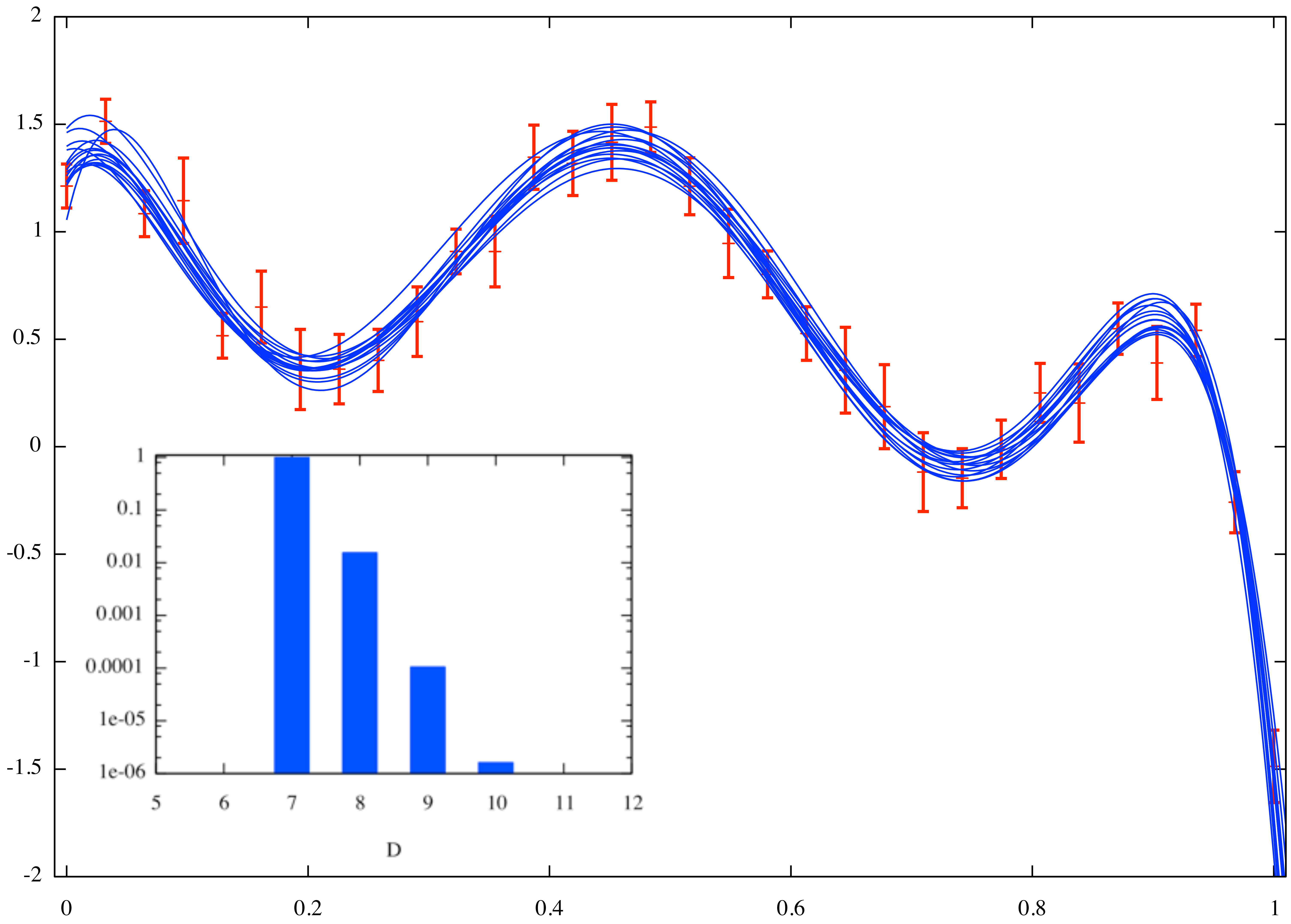
The overlaps between any two sources are typically small, but there are lots of sources





- Trans-dimensional Bayesian Inference
- Time-Frequency (Wavelet) Domain Analysis
- Blocked Gibbs - update each component of the signal/noise model in circular sweeps
- Time-evolving solution as new data arrives

Trans-dimensional Markov Chain Monte Carlo

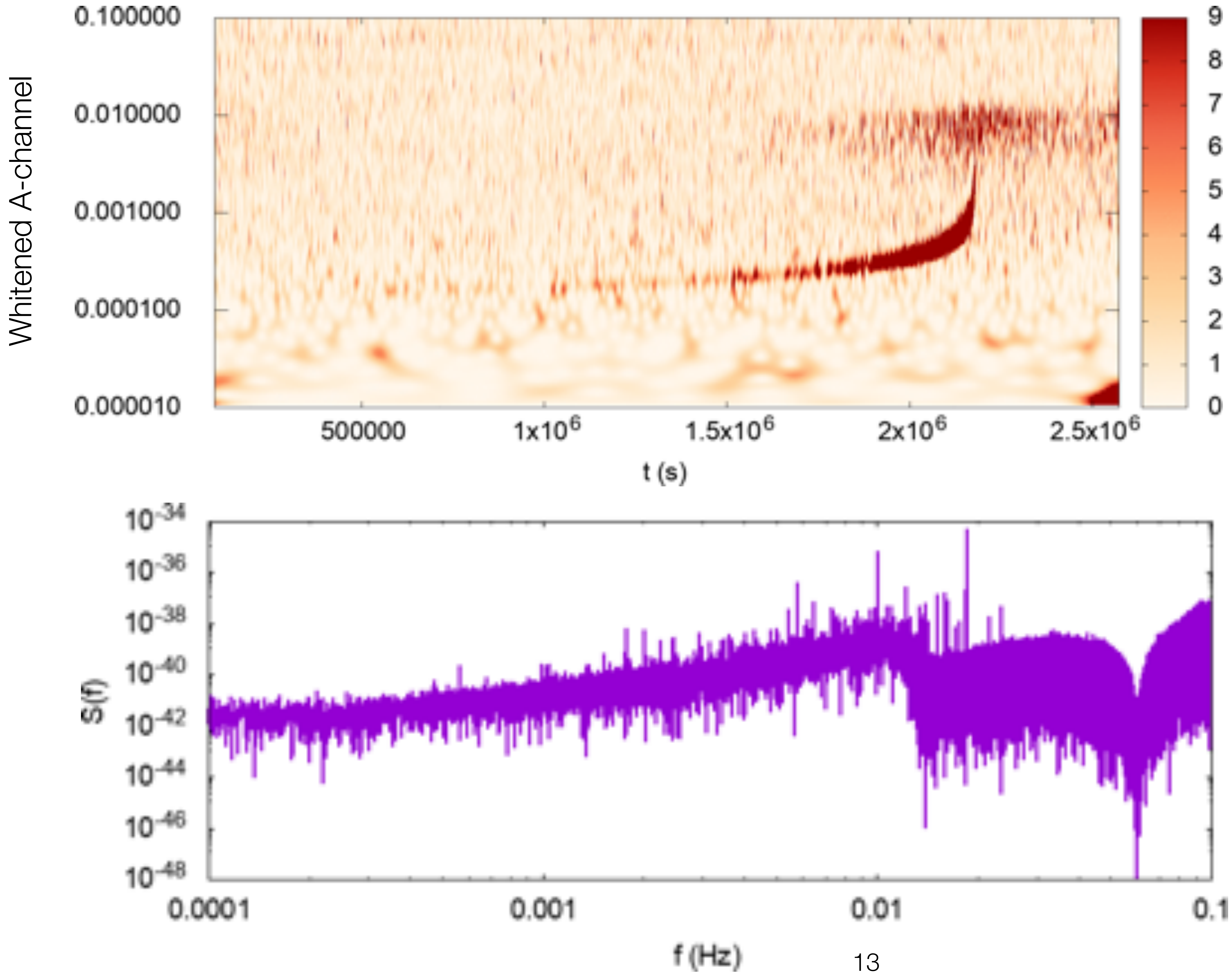


Ultra Compact Binaries

https://github.com/tlittenberg/ldasoft

[Littenberg, Cornish, Lackeos & Robson, PRD 101 123021 (2020)]

Joint Inference of Signals and Power Spectra



Example from month 2 of the LDC "Sangria" data set

Loud SMBHB merger distorts the power spectrum

Non-stationary Noise

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^{\dagger}\mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

Cost of computing the likelihood is far less in a representation where the noise correlation matrix **C** is diagonal

For a large class of discrete wavelet transformations and locally stationary noise [1]

$$C_{(i,j)(k,l)} = \delta_{ij}\delta_{kl}\, C_{ik}$$ [2]

Time Frequency

This is the likelihood used by the LIGO coherent WaveBurst algorithm

1. [“Fitting time series models to nonstationary processes”. Dahlhaus, Ann. Statist., 25, 1 (1997)]

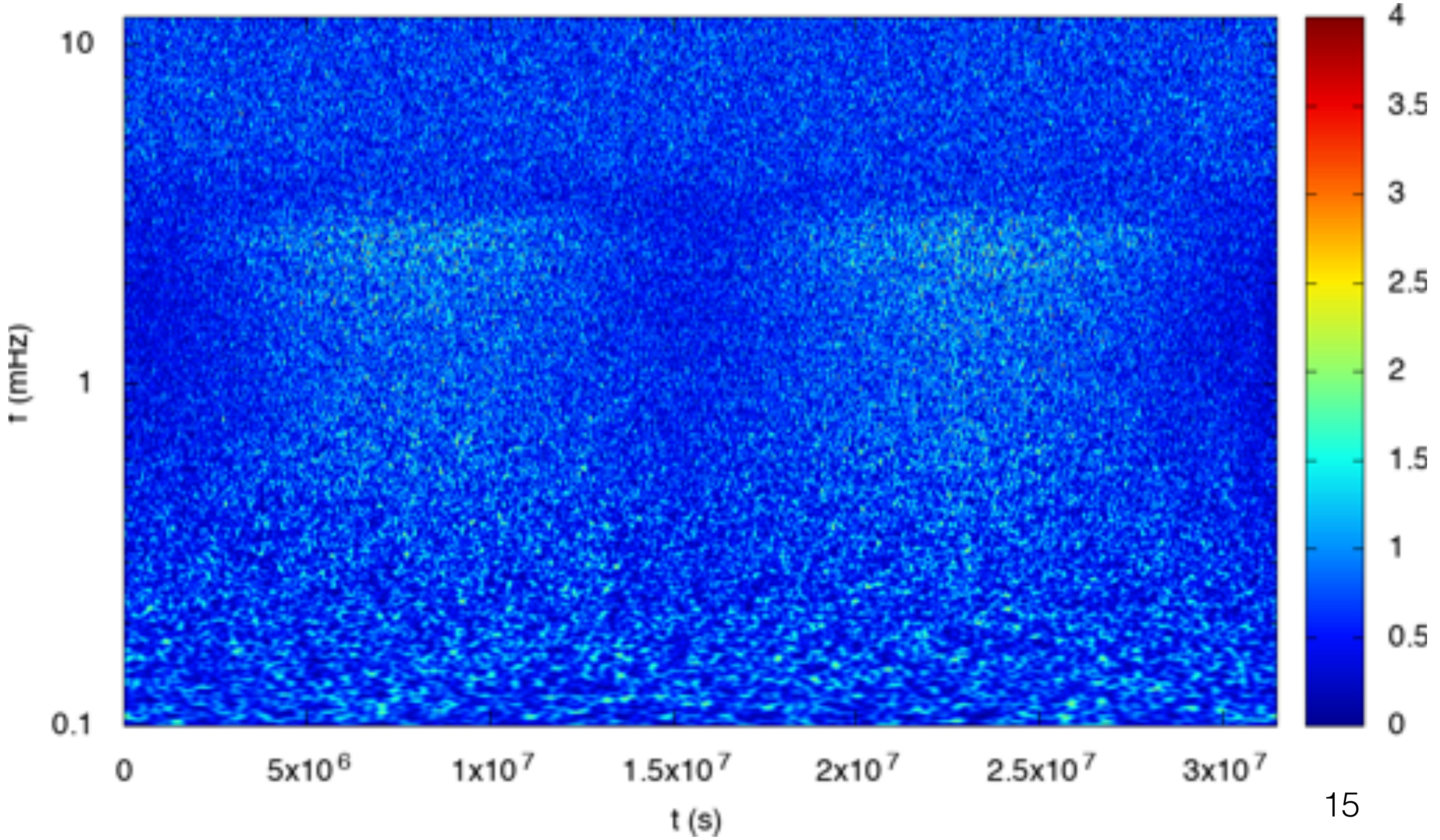
2. [“Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum”, Nason, von Sachs, & Kroisandt, J. R. Statist. Soc. Series B**62**, 271 (2000)]

Non-stationary Noise - Moving LISA analysis to the wavelet domain

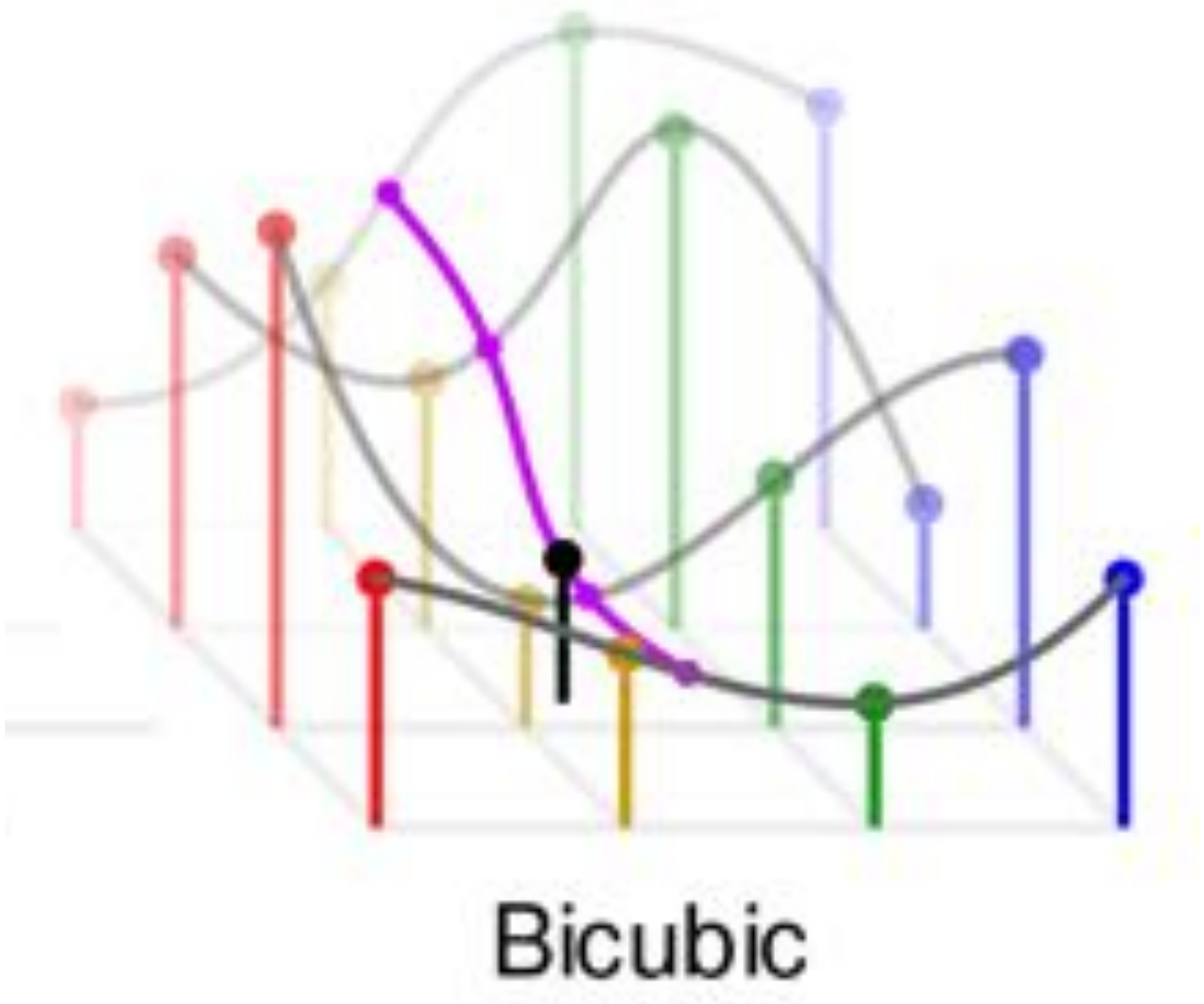
$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

$$C_{(i,j)(k,l)} = \delta_{ij}\delta_{kl}\, C_{ik}$$

LISA data whitened assuming the noise is stationary

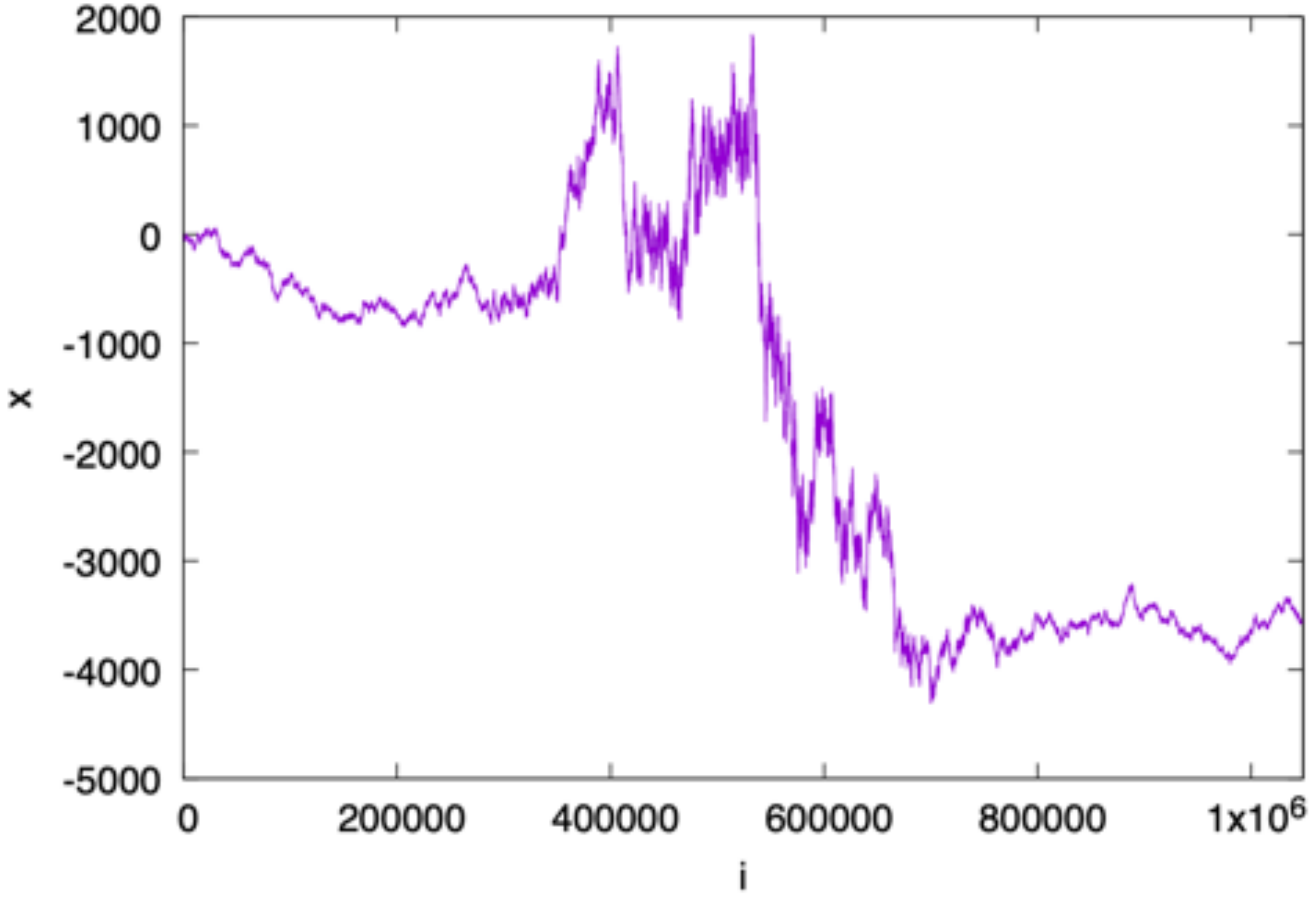


Model the wavelet spectrum C_{ik} as a smooth function in frequency and time. E.g. Trans-dimensional Bicubic spline

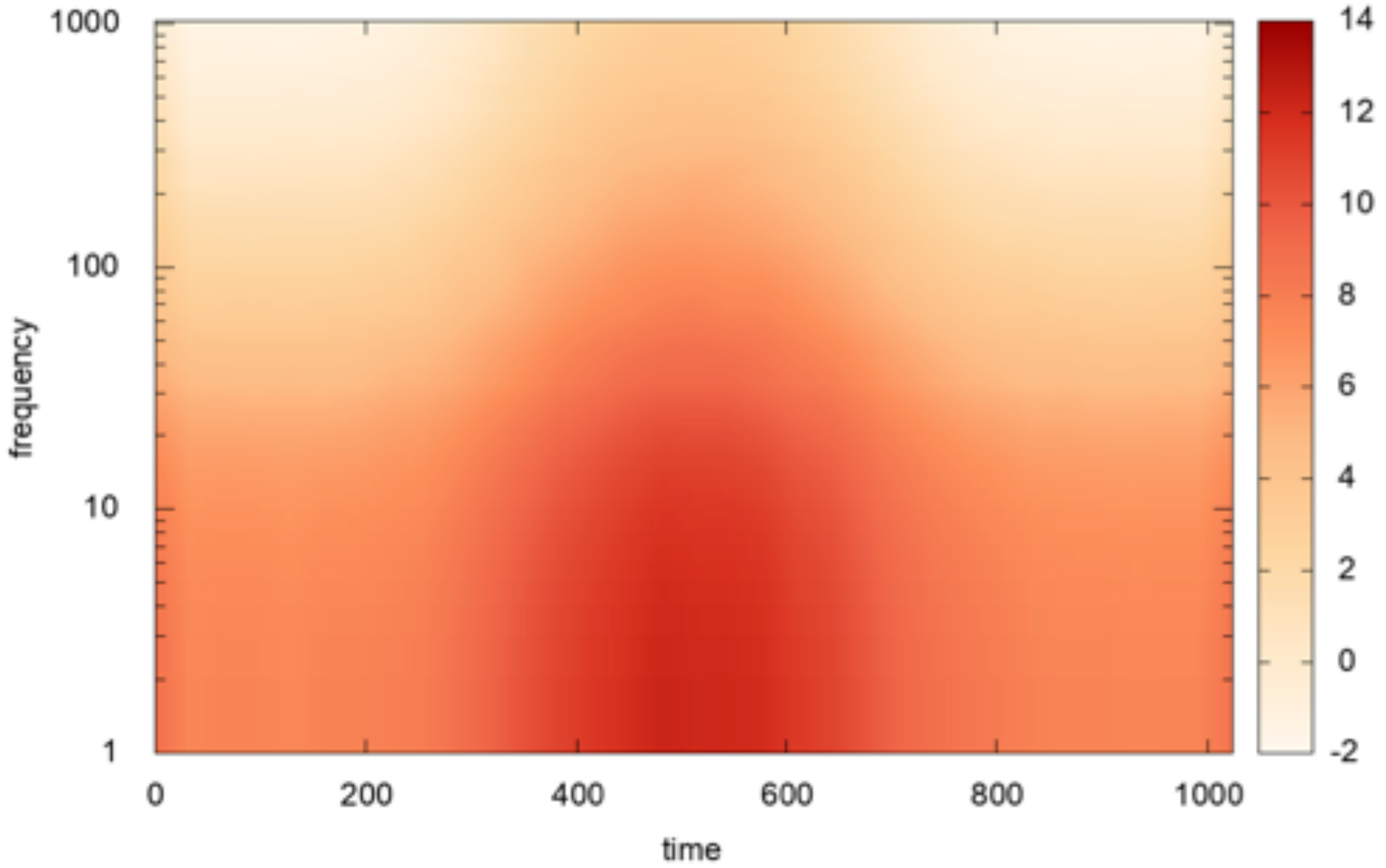


[Cornish, Phys Rev D **102,** 124038 (2020)]

Non-stationary Noise



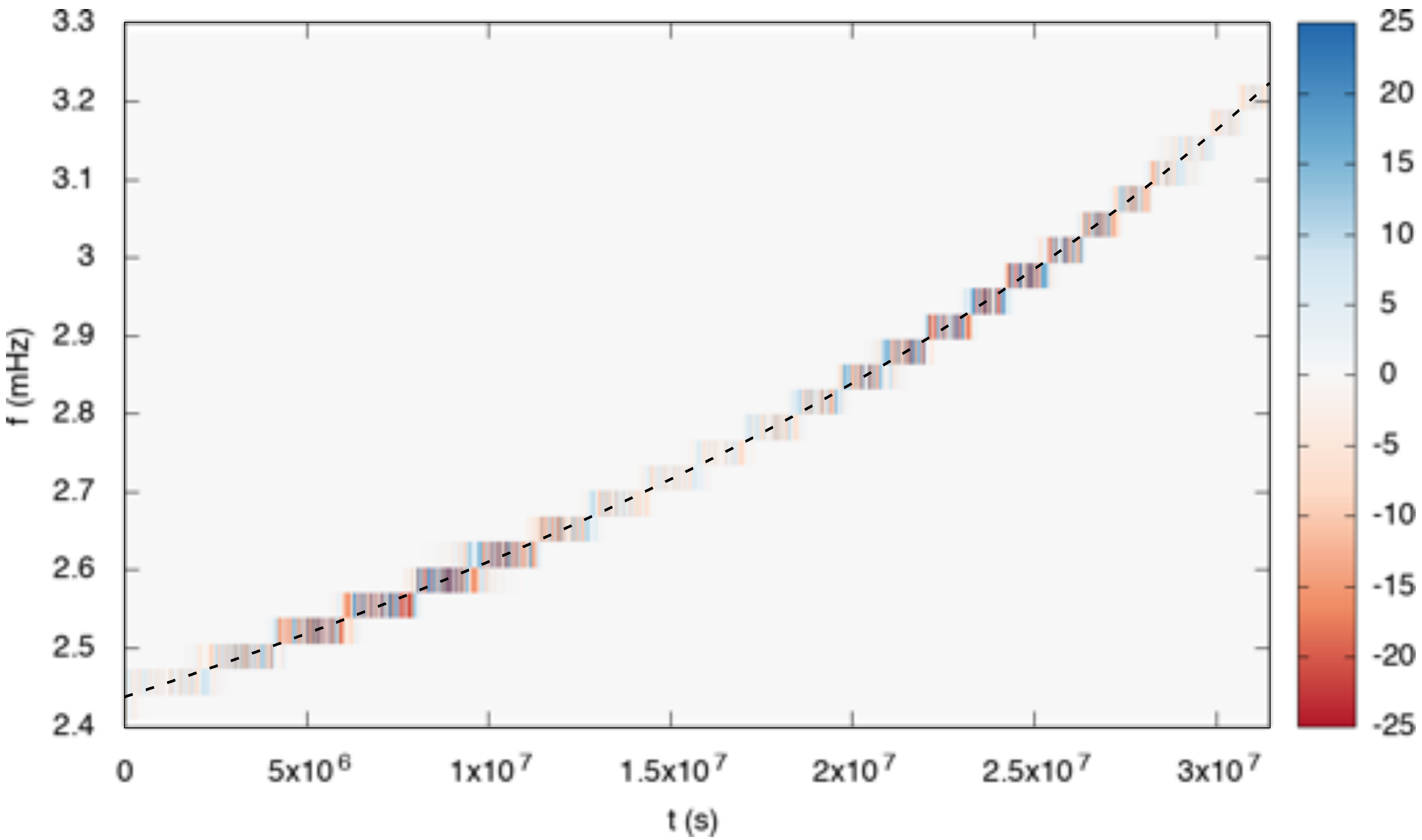
Non-stationary time series



Bi-cubic spline fit to the dynamic spectrum

[Cornish, Phys Rev D **102,** 124038 (2020)]

Wavelet domain waveforms



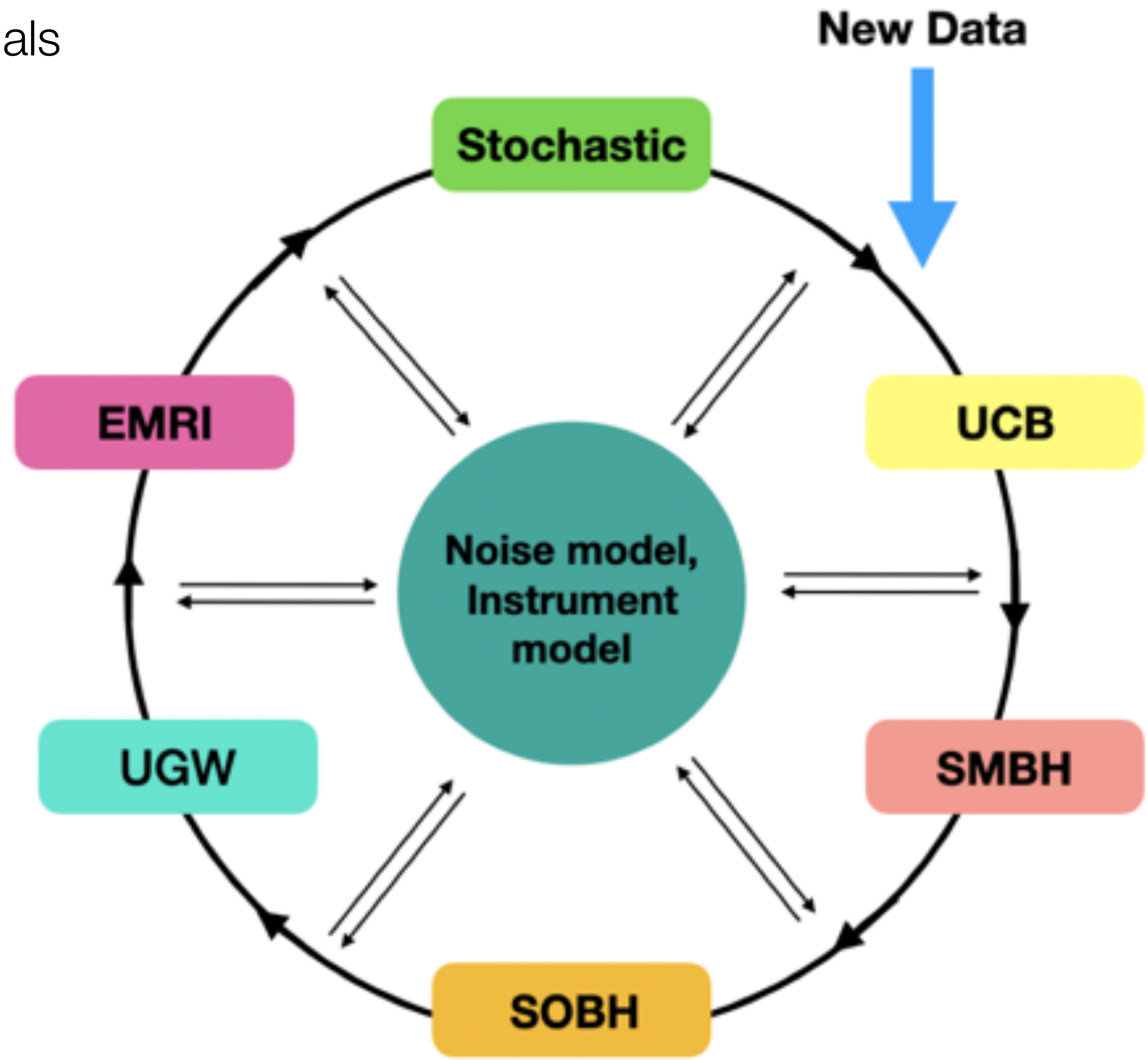
Fast wavelet transforms of the signals for computational efficiency

Faster than frequency domain, $\sqrt{N}$ scaling

[Cornish, Phys Rev D **102,** 124038 (2020)]

Wavelet domain - Faster, Better, Cheaper!

How do cosmological stochastic signals fit into the LISA global solution?



Stochastic Signals

Likelihood
$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det\mathbf{C}}}\, e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

Prior
$$p(\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{S_h})}}\, e^{-\frac{1}{2}(\mathbf{h}^\dagger\mathbf{S_h}^{-1}\mathbf{h})}\; p(\mathbf{S_h})$$

We are not interested in the value of each GW signal sample $\tilde{h}(f)$. Want to infer the power spectrum $S_h(f)$

Marginalize over h:
$$p(S_h|\mathbf{d}) = \int \frac{p(\mathbf{d}|\mathbf{h})p(\mathbf{h})}{p(\mathbf{d})}\, d\mathbf{h} = \frac{p(\mathbf{d}|S_h)p(S_h)}{p(\mathbf{d})}$$

Stochastic Signals

The integration over h is easy as it just involves Gaussians [Cornish & Romano, PRD 2013]

$$p(\mathbf{d}|S_h) \propto e^{-\frac{1}{2}(\mathbf{d}|\mathbf{d})_S}$$

Where
$$(\mathbf{a}|\mathbf{b})_S = 2\sum_{I,J}\int_0^\infty \left(\tilde{a}_I(f)\tilde{b}_J^*(f) + \tilde{a}_I^*(f)\tilde{b}_J(f)\right) S_{IJ}^{-1}(f)\, df$$

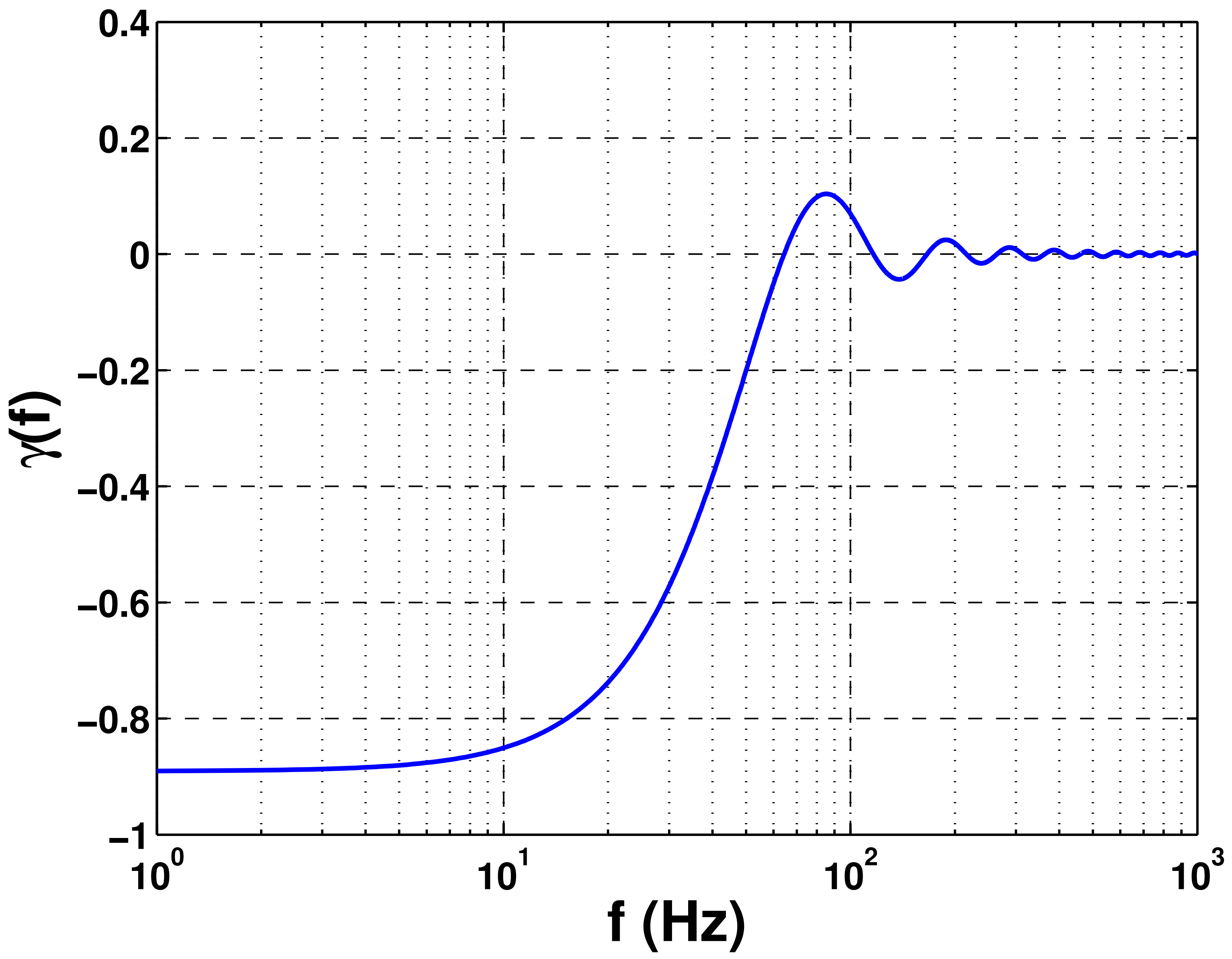
and
$$S_{IJ}(f) = S_{n,I}(f)\,\delta_{IJ} + S_h(f)\,\gamma_{IJ}(f)$$

Stochastic Signals: Putting the signal on the denominator

$$S_{IJ}(f) = S_{n,I}(f)\,\delta_{IJ} + S_h(f)\,\gamma_{IJ}(f)$$

The quantity $\gamma_{IJ}(f)$ is a geometrical factor that encodes the response of the detectors. In the long wavelength limit it is called the overlap reduction function

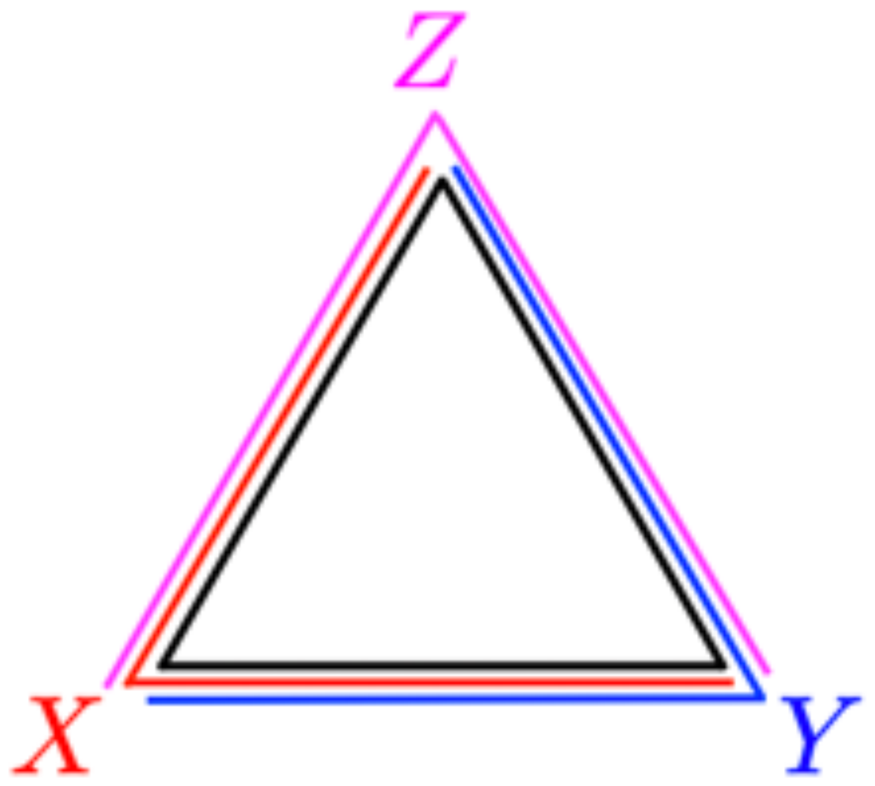
$$\gamma_{IJ}(f) = \frac{1}{4\pi}\int (F_I^+(\hat{n})F_J^+(\hat{n}) + F_I^+(\hat{n})F_J^+(\hat{n}))e^{2\pi i f(\vec{x}_I - \vec{x}_J)\cdot\hat{n}} d\Omega_{\hat{n}}$$



Normalized overlap for LIGO Hanford/Livngston (normalization factor is x 5)

Stochastic Signals: LISA

$$S_{IJ}(f) = S_{n,I}(f)\,\delta_{IJ} + S_h(f)\,\gamma_{IJ}(f)$$



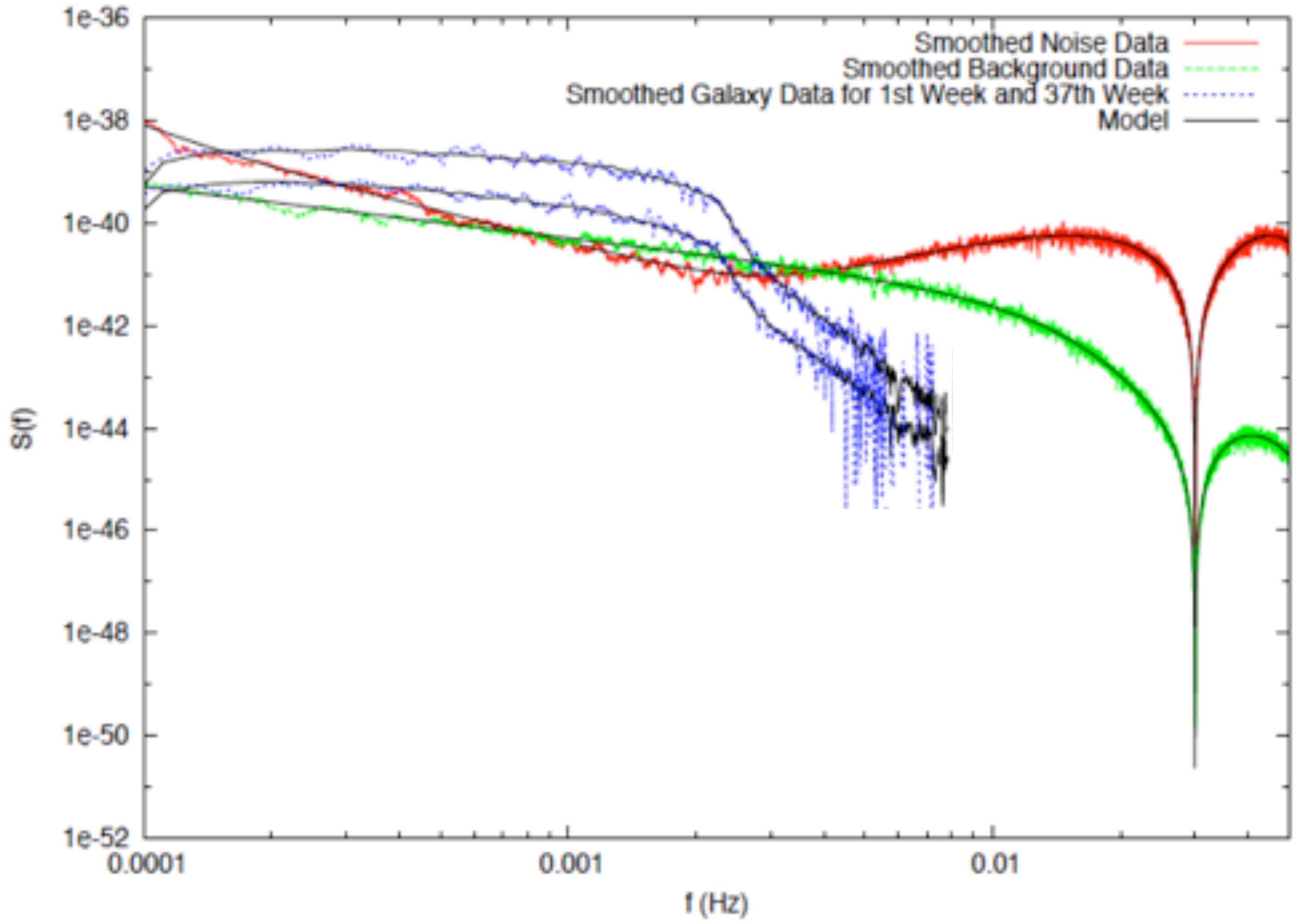
$$A = \frac{\sqrt{3}}{2}X$$

$$E = \frac{1}{2}(X + 2Y)$$

$$T = \frac{1}{3}(X + Y + Z)$$

$$\gamma_{IJ}(f) \simeq 0$$

Channels have zero overlap at low frequencies

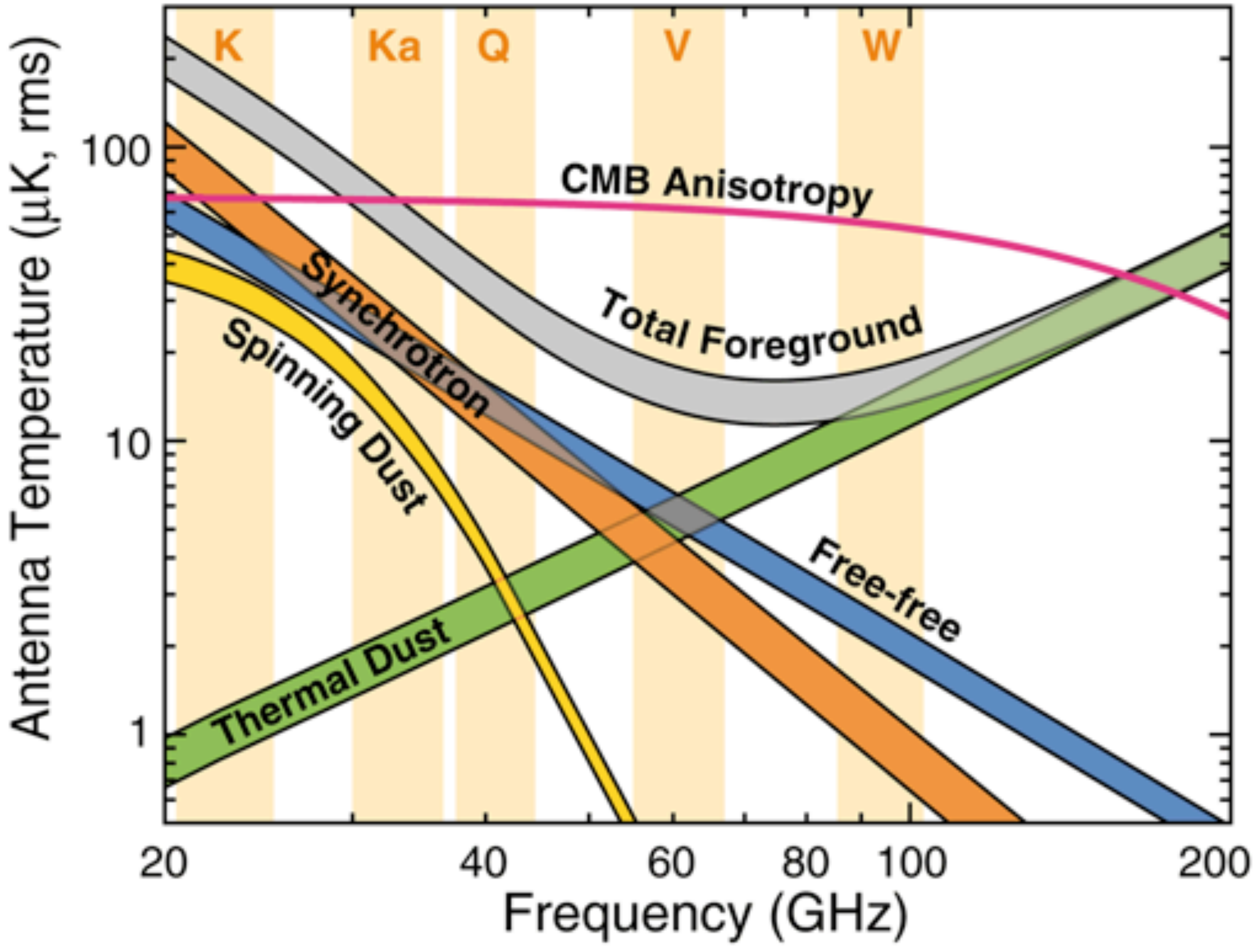


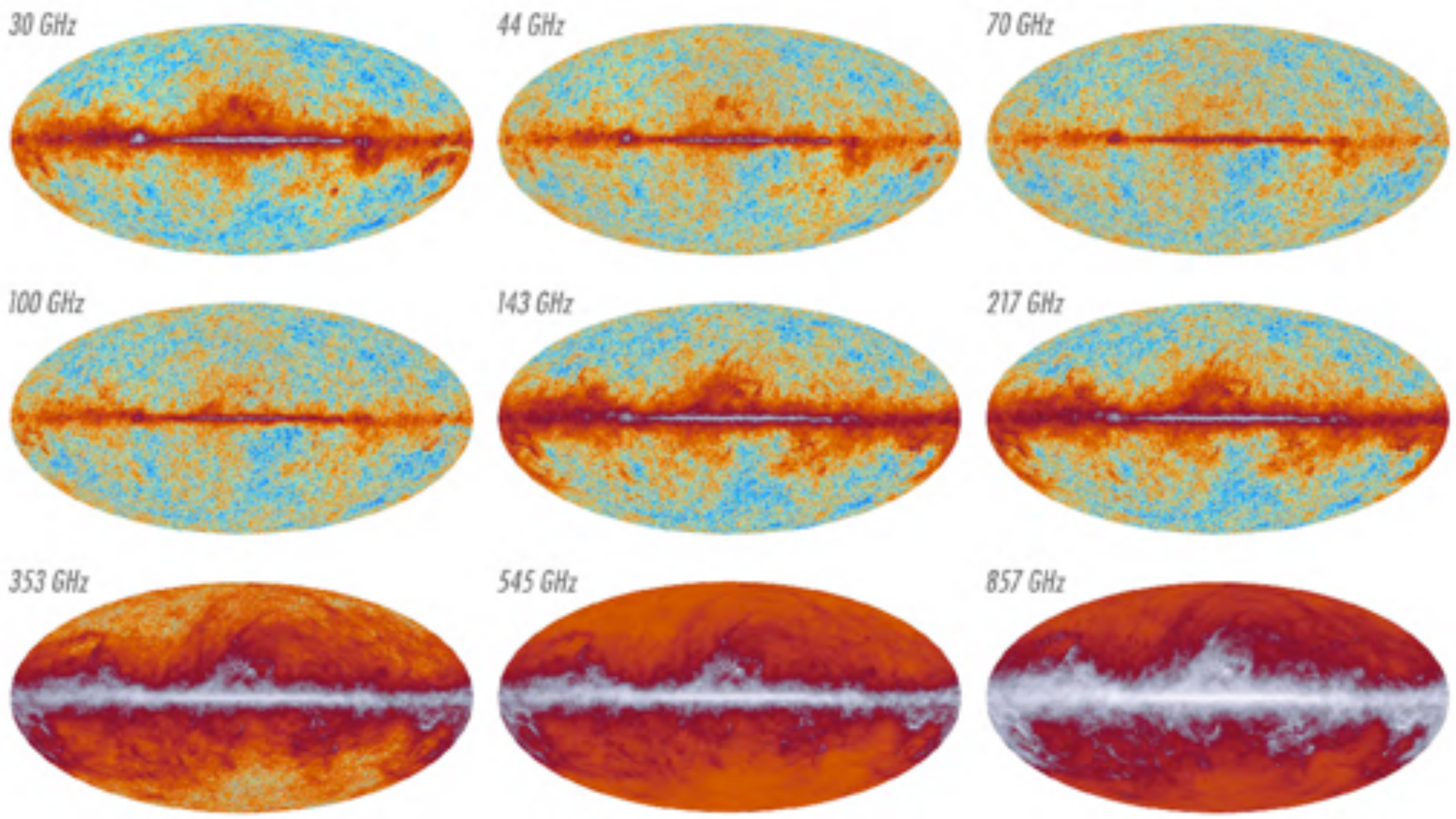
Have to use spectral separation - works when there are different spectra for noise and signals

[Adams & Cornish 1307.4116 (2013)]

[Boileau, Christensen, Meyer & Cornish 2011.05055 (2020)]

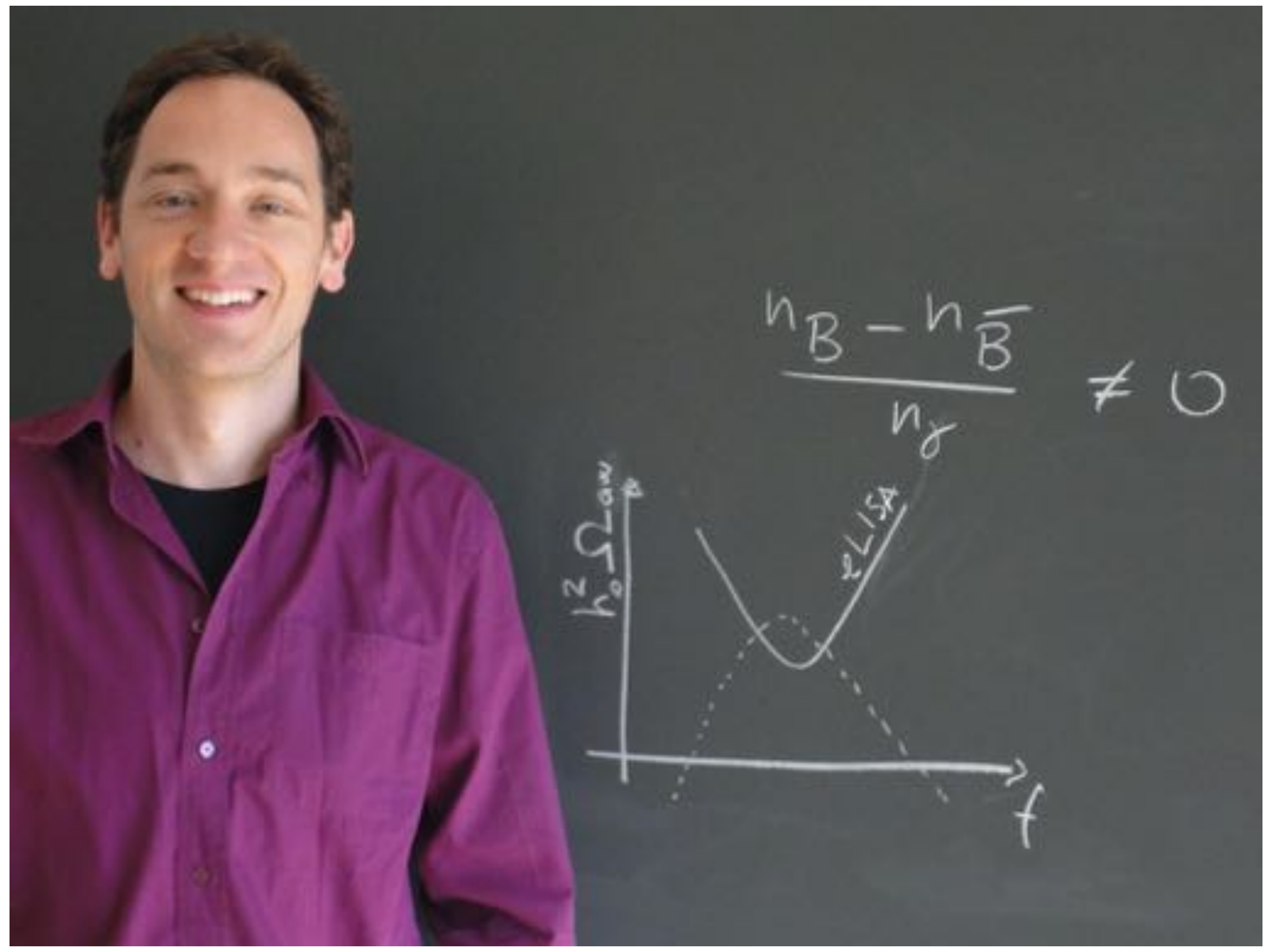
CMB Spectral Component Separation





LISA Cosmology Working Group Workshop
DESY Hamburg (October 2016)

Chiara Caprini

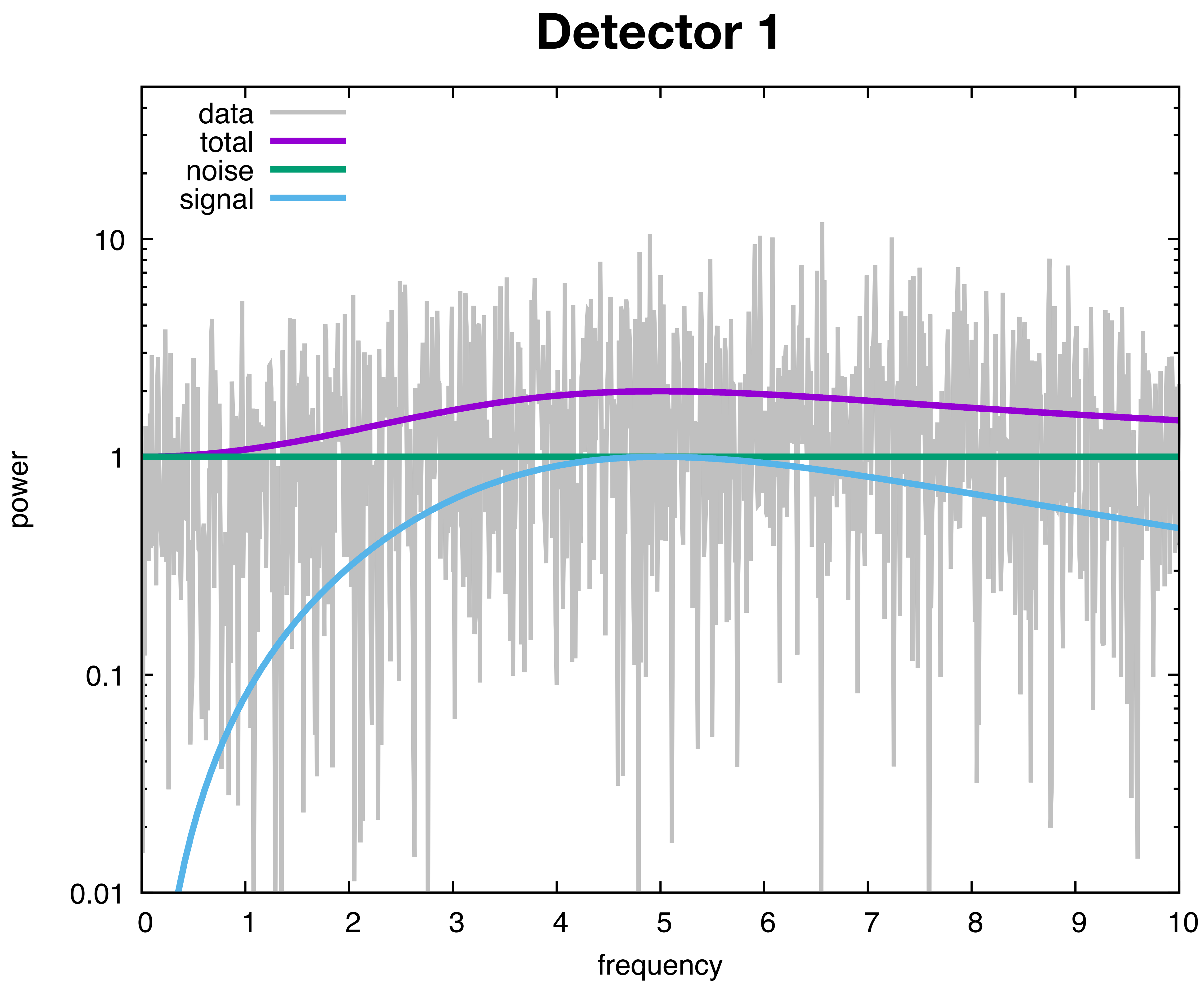


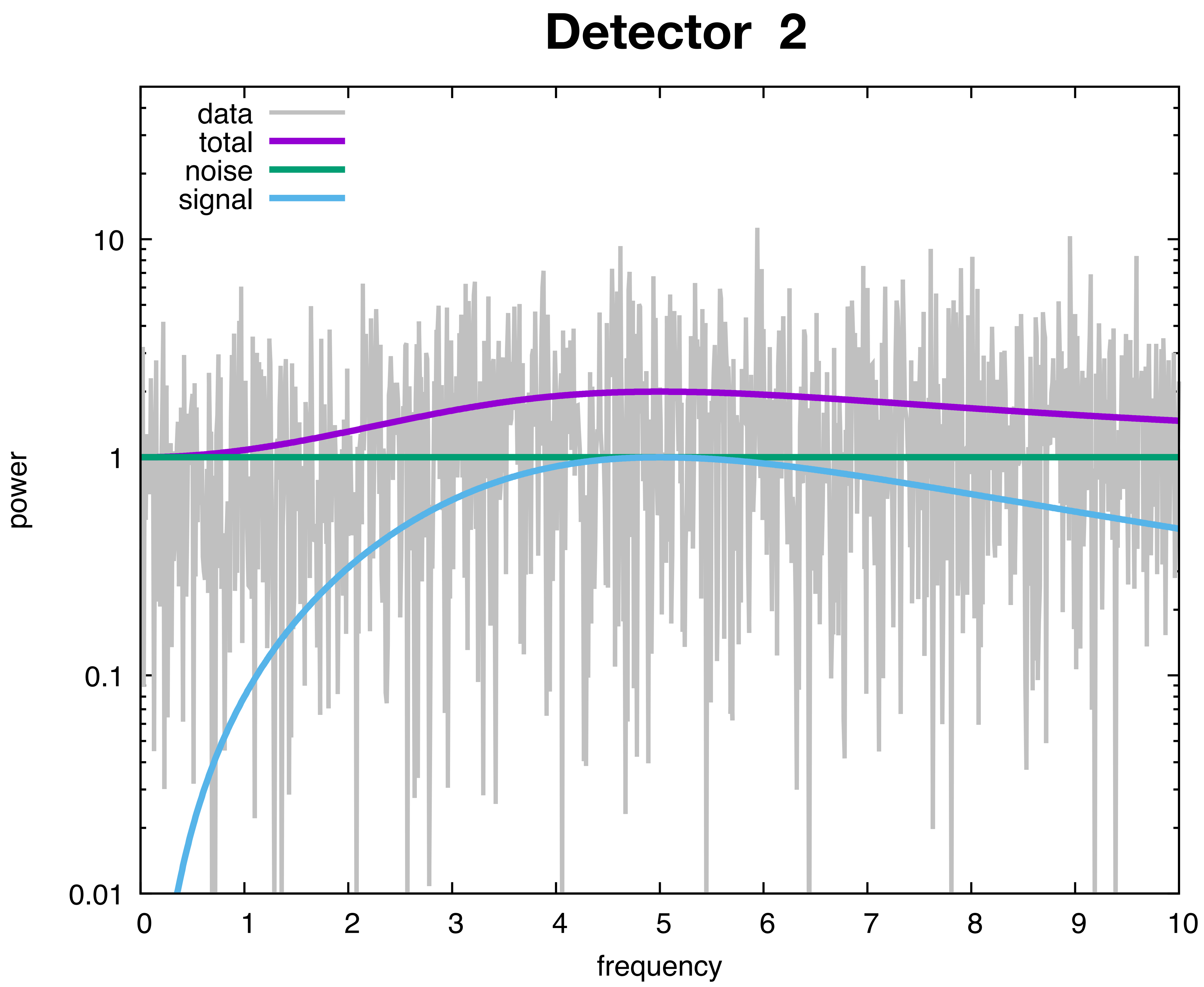
Germano Nadini

“How well can you subtract a stochastic signal?”

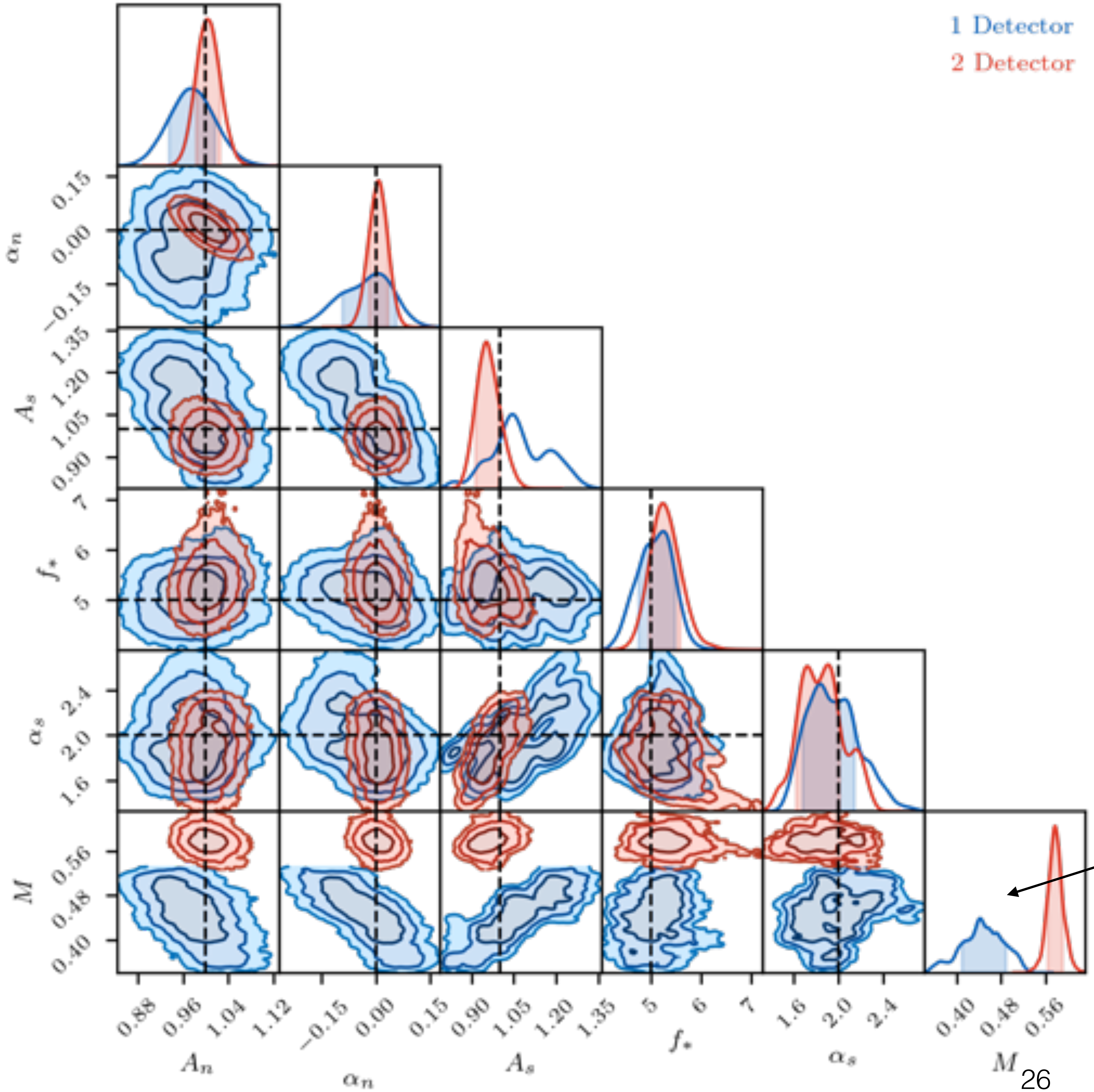
Stochastic Signals - Toy model for subtraction

$$S_n(f) = \frac{A_n^2}{f^{\alpha_n}} \qquad S_h(f) = \frac{2A_s^2}{\left(\frac{f}{f_*}\right)^{\alpha_s} + \left(\frac{f_*}{f}\right)^{\alpha_s}}$$





Stochastic Signals - Toy model for subtraction



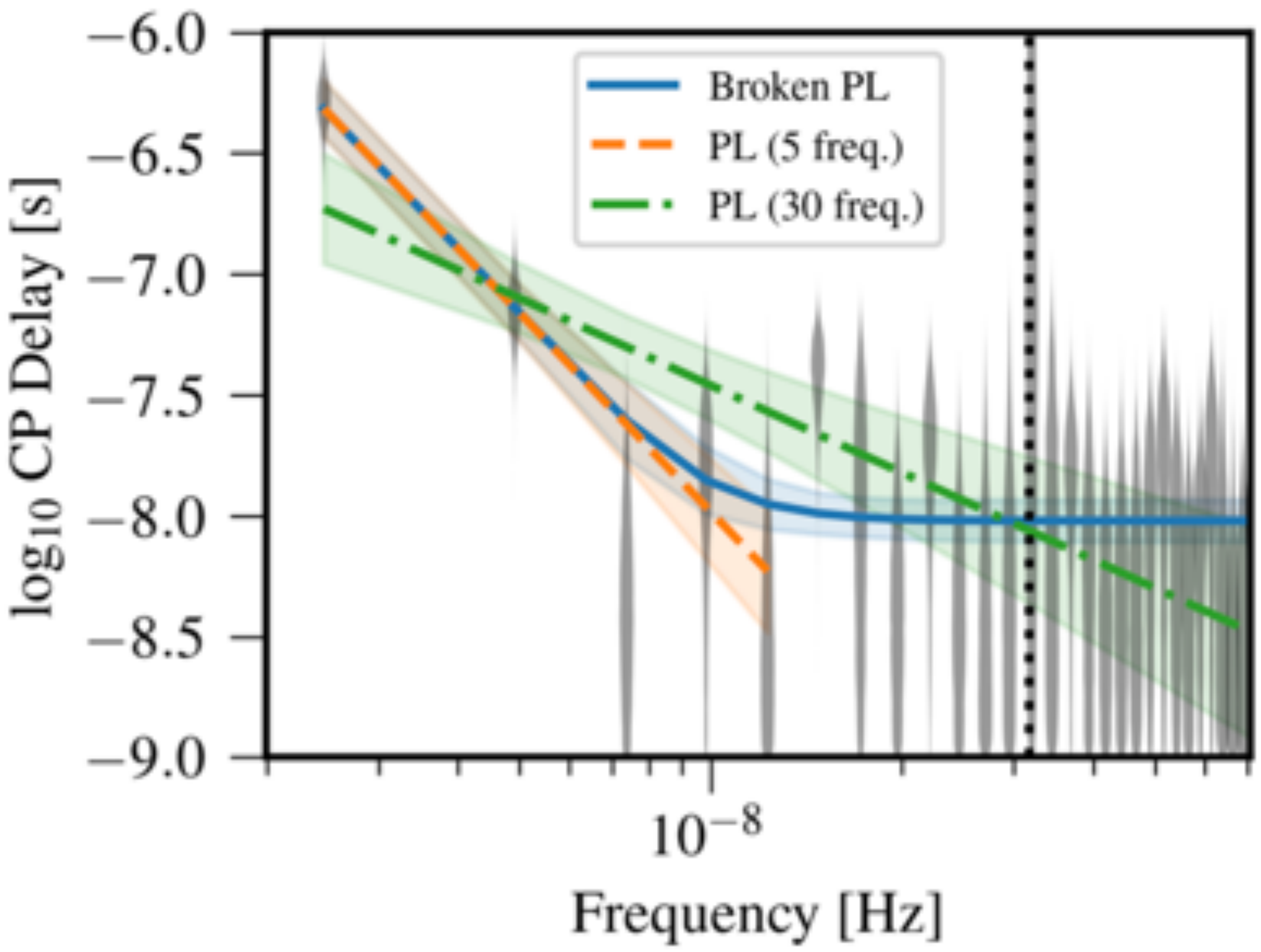
$$S_h(f) = \frac{2A_s^2}{\left(\frac{f}{f_*}\right)^{\alpha_s} + \left(\frac{f_*}{f}\right)^{\alpha_s}}$$

$$S_n(f) = \frac{A_n^2}{f^{\alpha_n}}$$

Not only do we recover the signal and noise model (hyper) parameters, we also reconstruct the stochastic signal pretty well - match of 0.45 for one detector, 0.58 for two detectors. Decent signal subtraction!

Keeping stochastic waveforms in the numerator

[Lentati, Alexander, Hobson, Taylor, Gair, Balan & van Haasteren, 1210.3578 (2012)]



Modern pulsar timing analysis solves directly for the stochastic signals using a pseudo Fourier basis

[Arzoumanian et al, 2009.04496 (2012)]

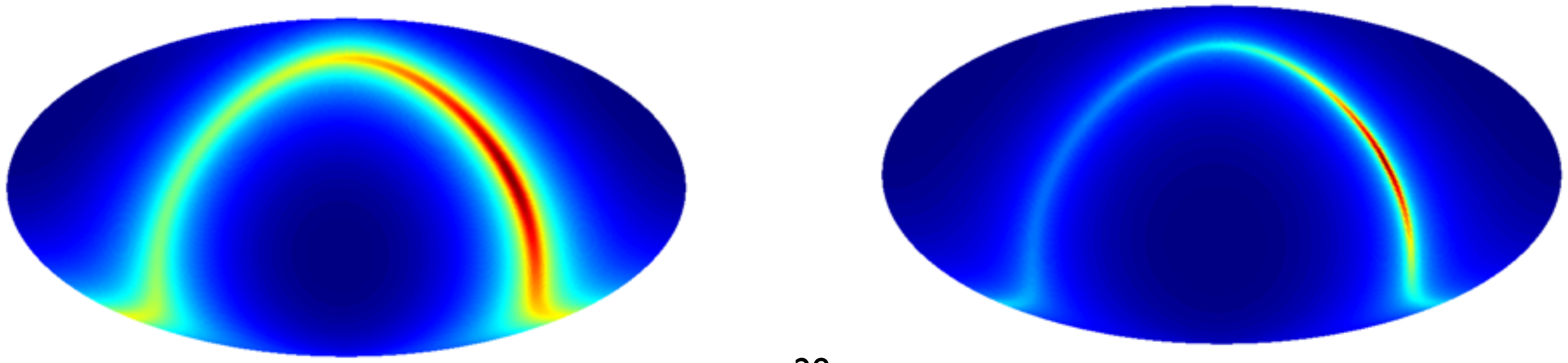
Back to LISA - Modeling Instrument Noise, Galactic Binaries and an Isotropic Stochastic Background

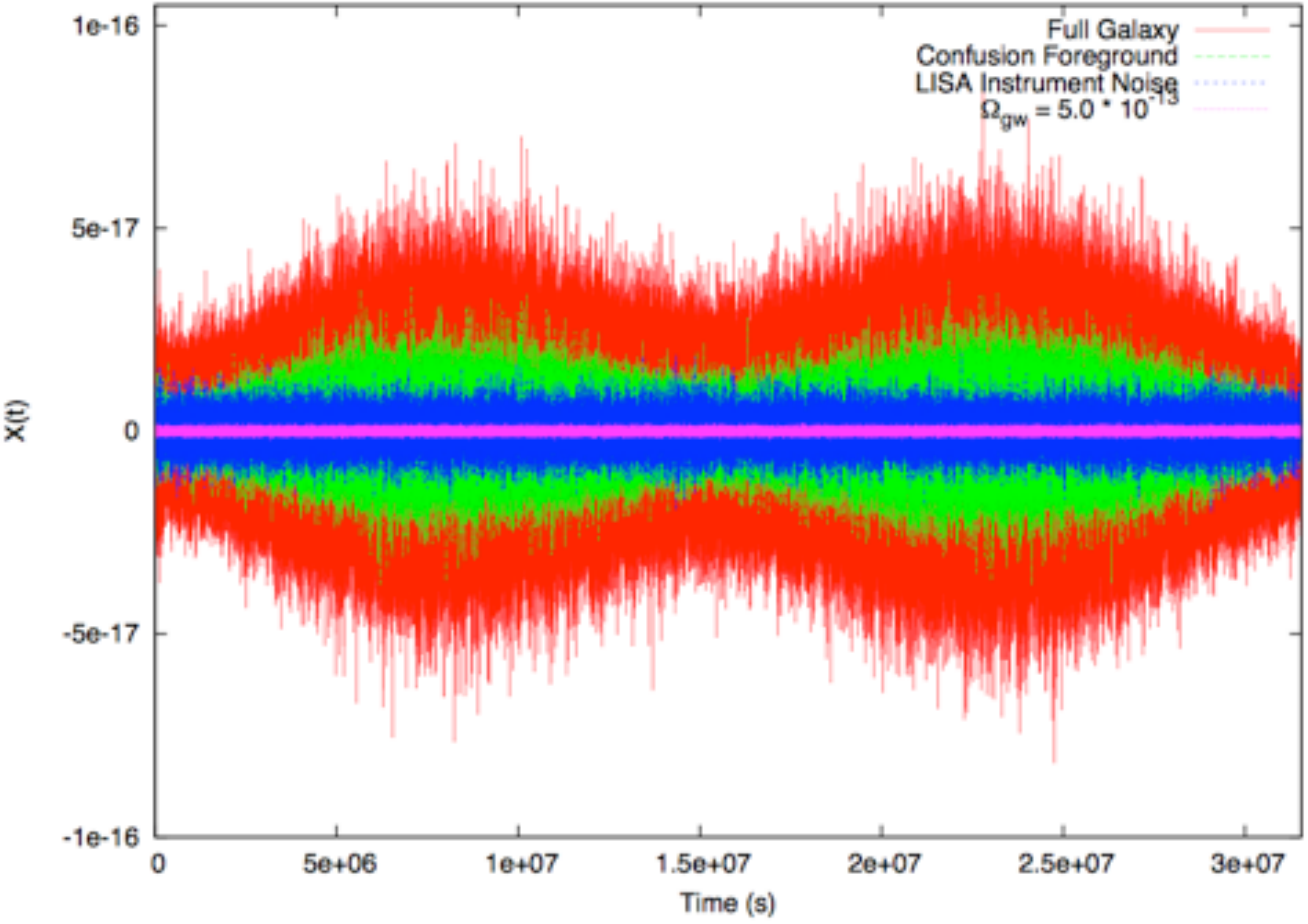
Likelihood $$p(X|S_a, S_p, S_h) = \prod_f \frac{1}{(2\pi)^{3/2}|C|} e^{-(X_i C_{ij}^{-1} X_j)/2}$$

$$\text{where} \quad C(f) = \begin{pmatrix} \langle AA \rangle & \langle AE \rangle & \langle AT \rangle \\ \langle EA \rangle & \langle EE \rangle & \langle ET \rangle \\ \langle TA \rangle & \langle TE \rangle & \langle TT \rangle \end{pmatrix}$$

Spectral Priors $$p(S_a),\ p(S_p)\ p(S_h)$$

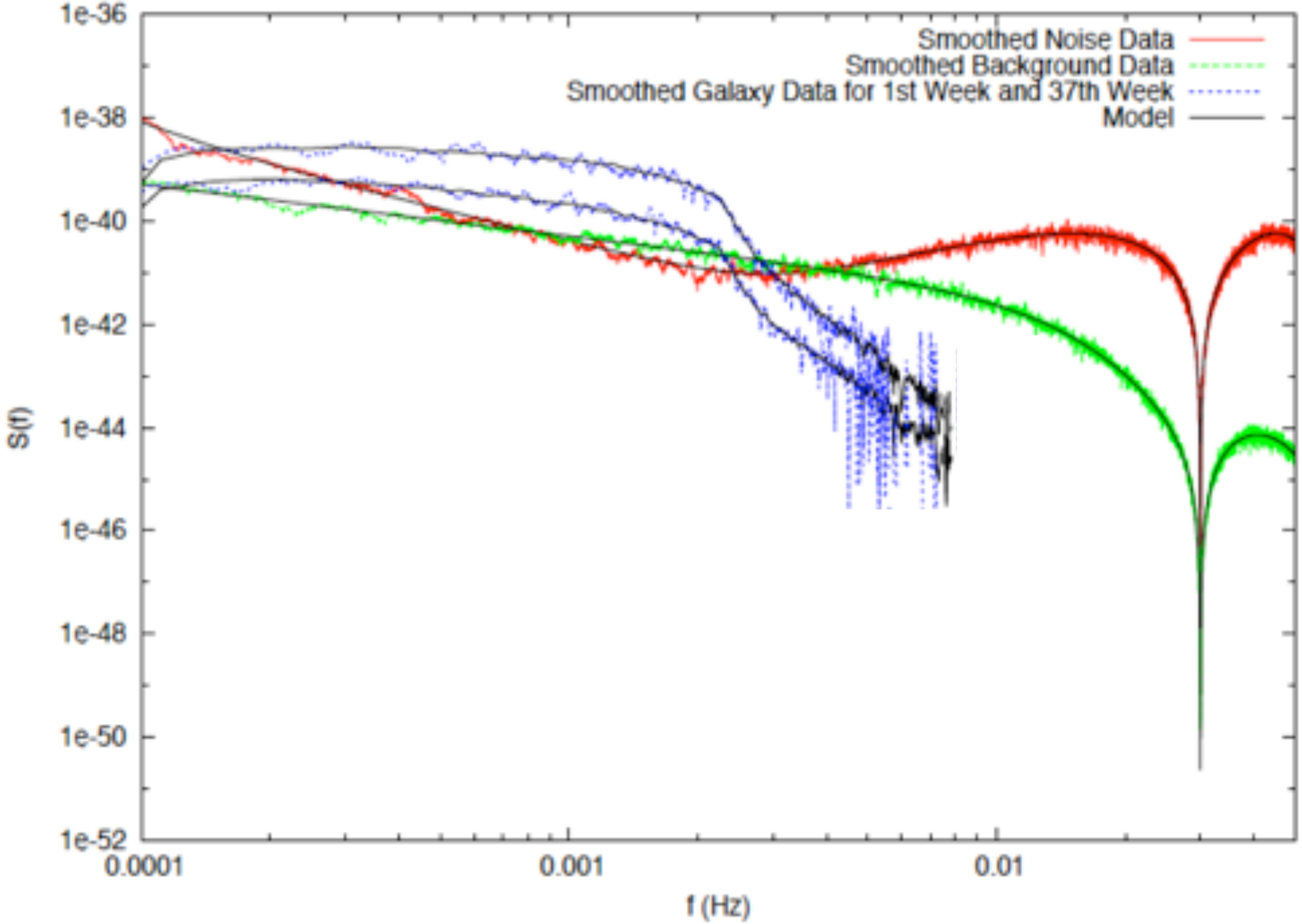
Galaxy Shape Priors $$\rho(x, y, z) = \rho_0\, e^{-\sqrt{x^2+y^2}/R_d}\, \mathrm{sech}^2(z/Z_d)$$





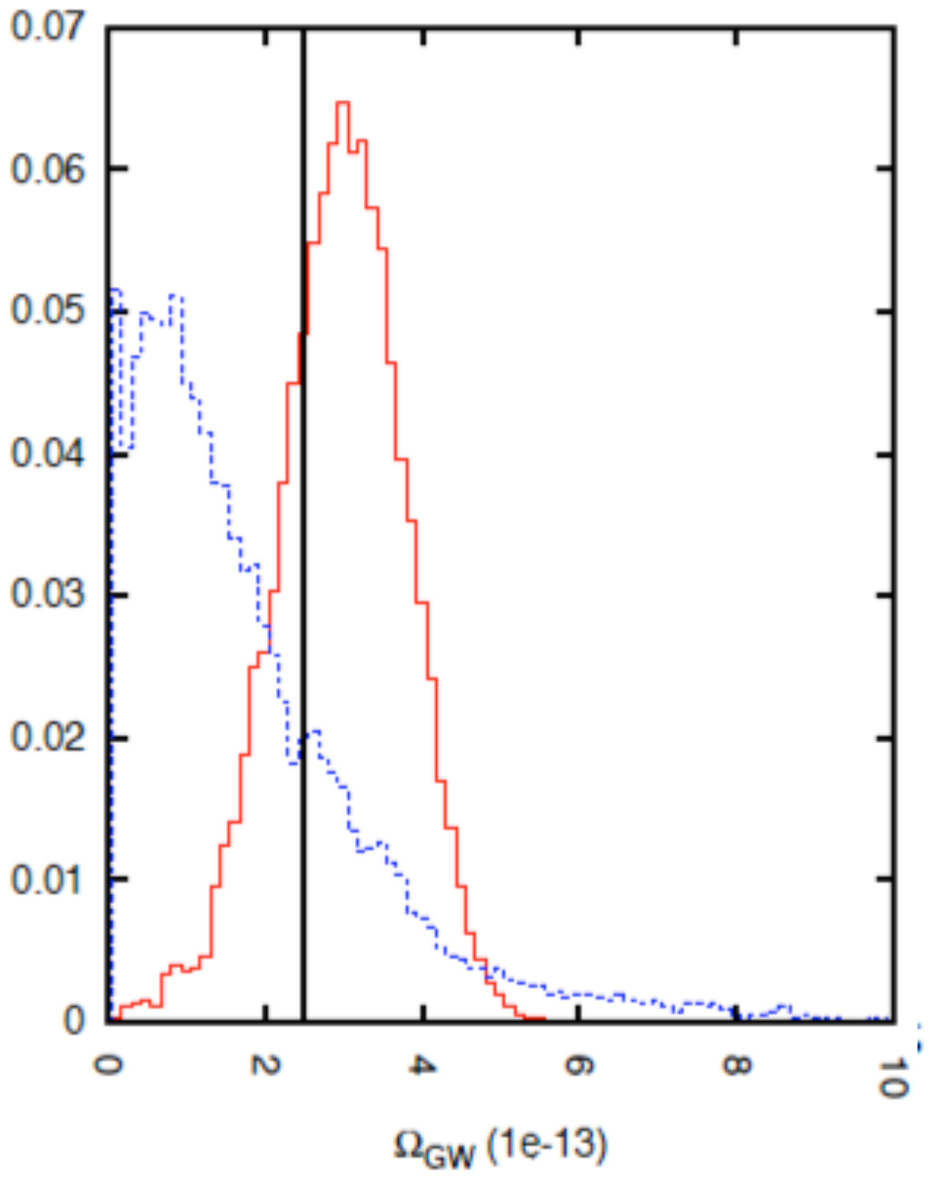
Key Point: The resolved galactic binaries fix the time variation and spectra of the unresolved “confusion noise”

[Adams & Cornish 1307.4116 (2013)]

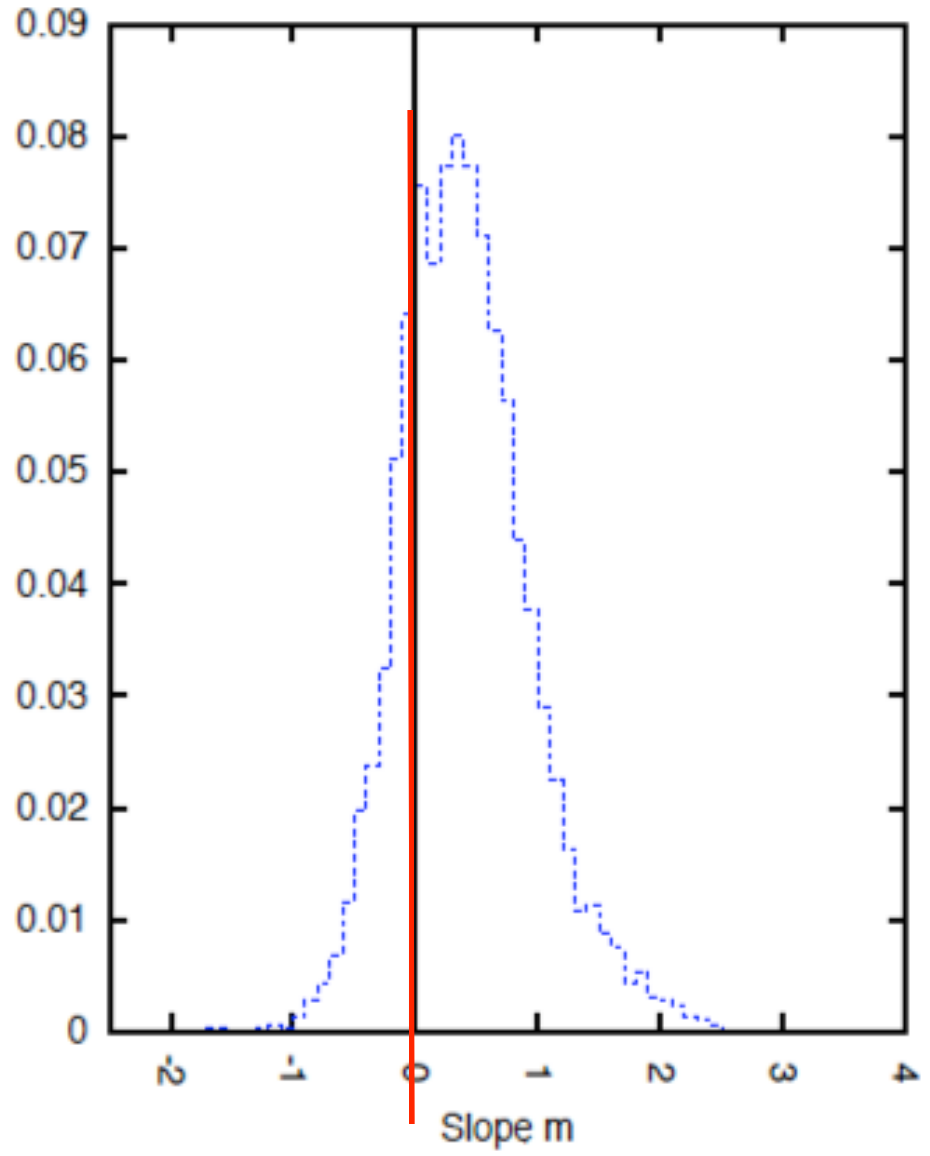


[Adams & Cornish 1307.4116 (2013)]

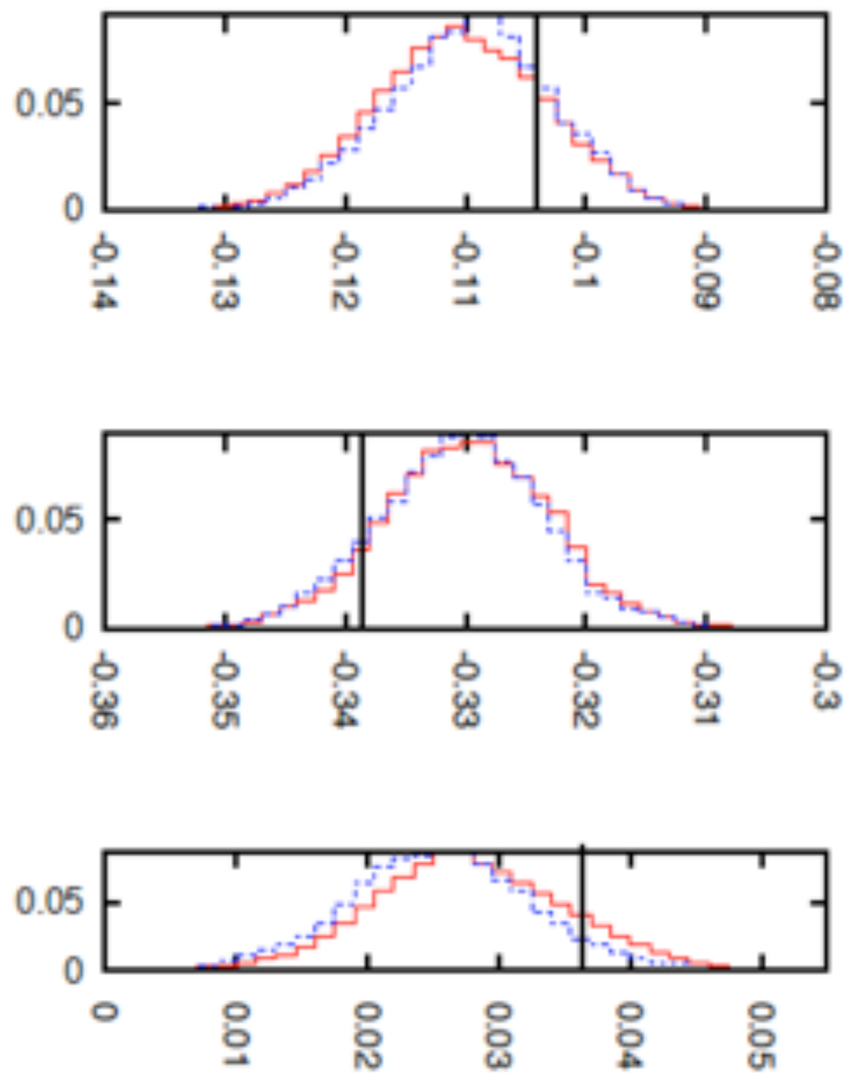
Simultaneously solve for amplitude of instrument noise, stochastic background and galactic white dwarf density and distribution



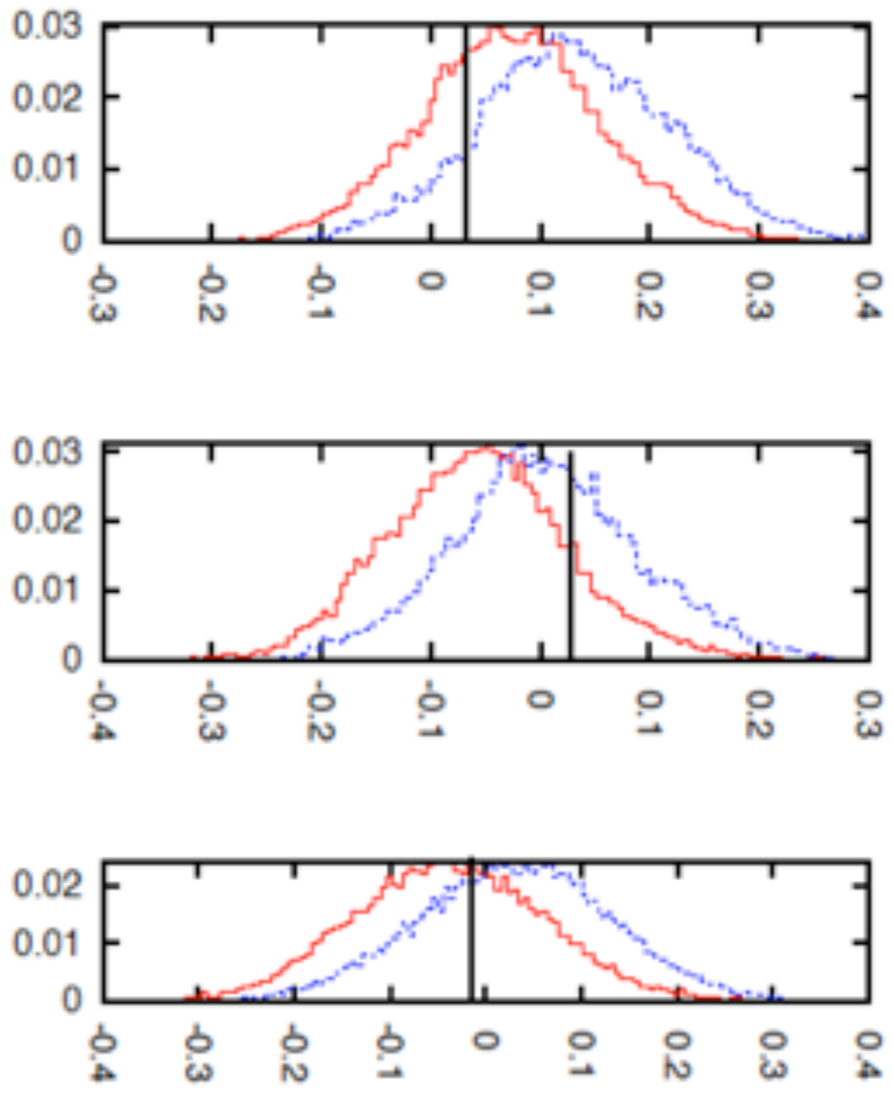
GW amplitude



GW slope



Position noise



Acceleration noise

[Adams & Cornish 1307.4116 (2013)]

Conclusions

- Stochastic signals fit naturally into the LISA global solution
- You can indeed "subtract" stochastic signals
- Stochastic components due to unresolved astrophysical signals are highly constrained by the resolved component