Imperial College London
Claudia de Rham

The Speed of Gravity

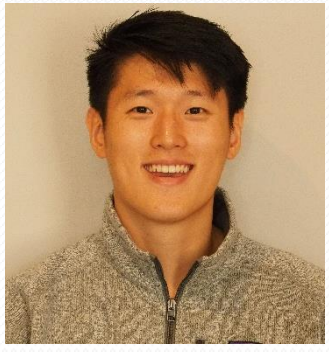
Meeting of the National Research Group on Gravitational Waves

Institut Poincaré

erc
THE ROYAL SOCIETY
SF

Thanks to some of amazing collaborators

Lasma Alberte
(@ Imperial)



Calvin Chen
(@ Imperial)

Jeremie Francfort
(@ Geneva)

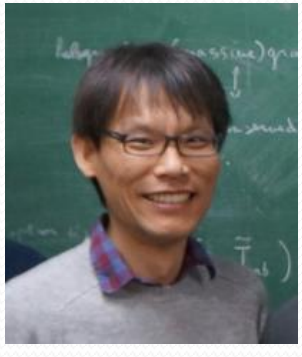
Sumer Jaitly
(@ Imperial)

Aoibheann Margalit
(@ Imperial)

Scott Melville
(@ Cambridge)

Johannes Noller
(@ Portsmouth)

Andrew Tolley
(@ Imperial)

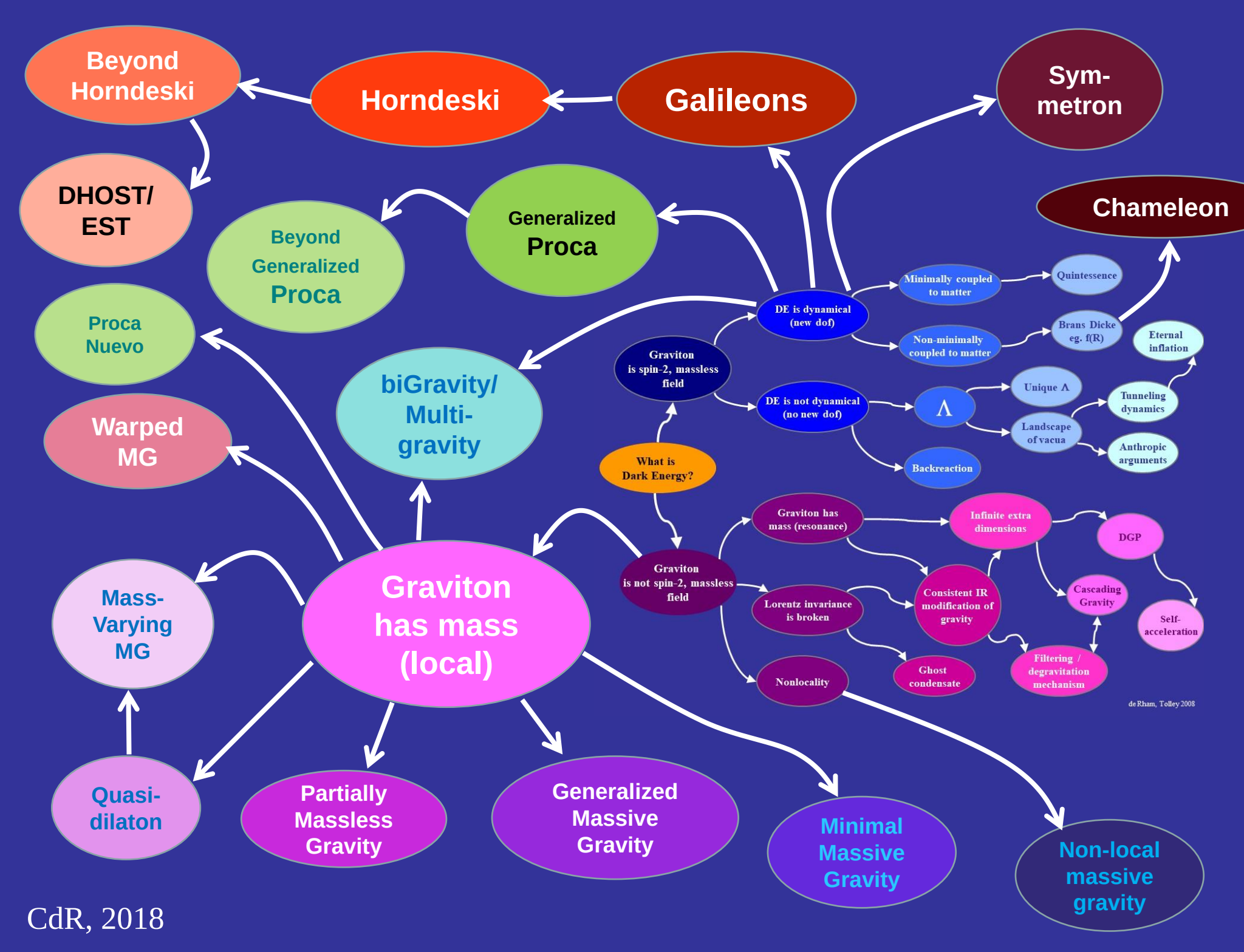


Jun Zhang
(@ Imperial)

Shuang-Yong Zhou
(@ USTC)

1806.09417, 1909.00881, 2005.13923, 2007.01847, 2007.12667, 2012.05798, 2103.06855


What is Dark Energy?
Graviton is spin-2, massless field
DE is dynamical (new dof)
Minimally coupled to matter
Quintessence
Non-minimally coupled to matter
Brans Dicke eg. f(R)
DE is not dynamical (no new dof)
Λ
Unique Λ
Landscape of vacua
Tunneling dynamics
Eternal inflation
Anthropic arguments
Backreaction
Graviton is not spin-2, massless field
Graviton has mass (resonance)
Infinite extra dimensions
DGP
Lorentz invariance is broken
Consistent IR modification of gravity
Cascading Gravity
Self-acceleration
Nonlocality
Ghost condensate
Filtering / degravitation mechanism
de Rham, Tolley 2008


Beyond Horndeski
Horndeski
Galileons
Sym-metron
DHOST/ EST
Generalized Proca
Beyond Generalized Proca
Chameleon
Proca Nuevo
biGravity/ Multi-gravity
Warped MG
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de Rham, Tolley 2008
Mass-Varying MG
Graviton has mass (local)
Quasi-dilaton
Partially Massless Gravity
Generalized Massive Gravity
Minimal Massive Gravity
Non-local massive gravity
CdR, 2018

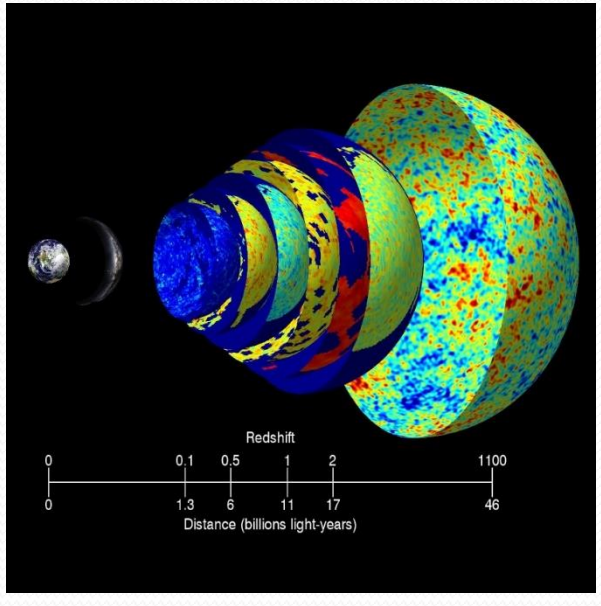
Diagnostic

- Does the model even make sense? (is it stable? Classically? QM??)

Diagnostic

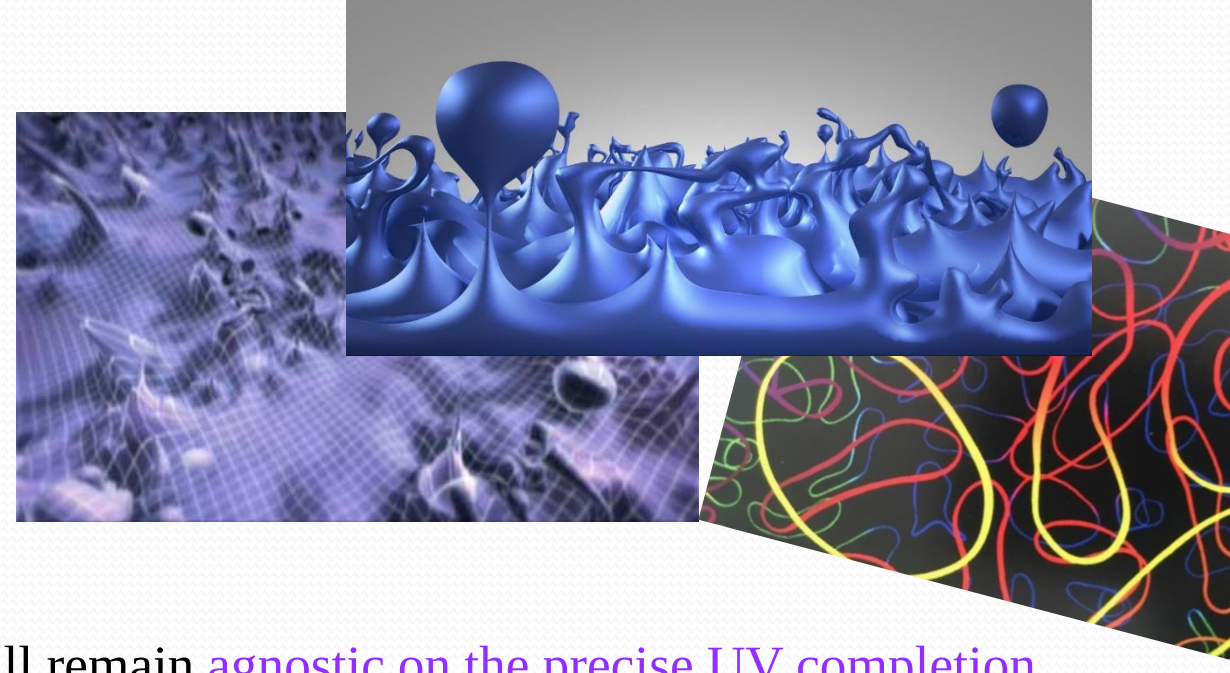
Low-energy criteria

- Does the model even make sense? (is it stable? Classically? QM??)
- Does it fit the multitude of existing observational constraints? (signature for future ones?)

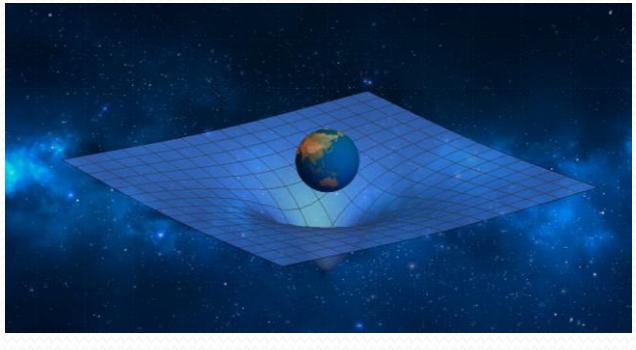


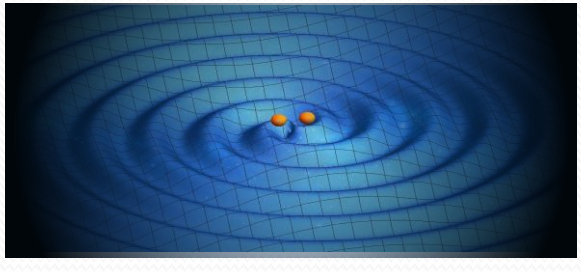
- High energy completion?

Energy



Will remain agnostic on the precise UV completion (field content, realization,…) so long as it is local, unitary, Lorentz invariant and CAUSAL





Within low-energy gravitational EFTs

(relevant for EFT of Inflation, dark energy, dark matter, BSM,…)

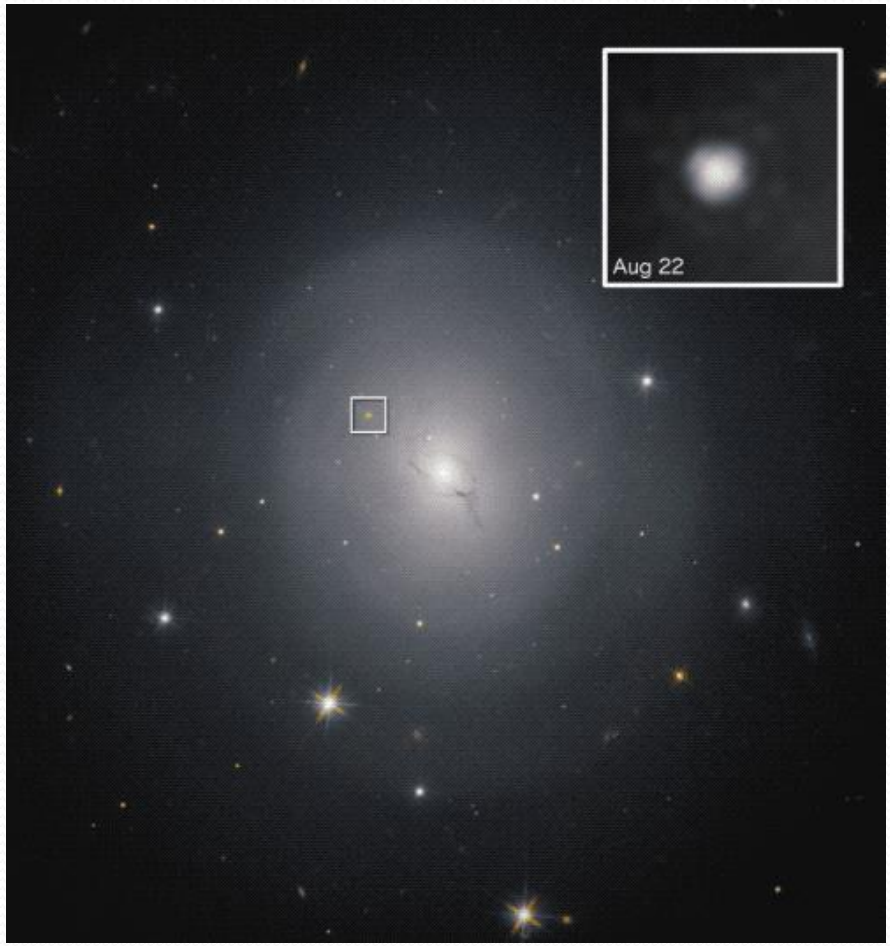
- Can Gravitational Waves be Superluminal?
- Can Other Species (eg light) be Superluminal?
- Is this consistent with Causality?
- Is this consistent with a Standard UV completion?

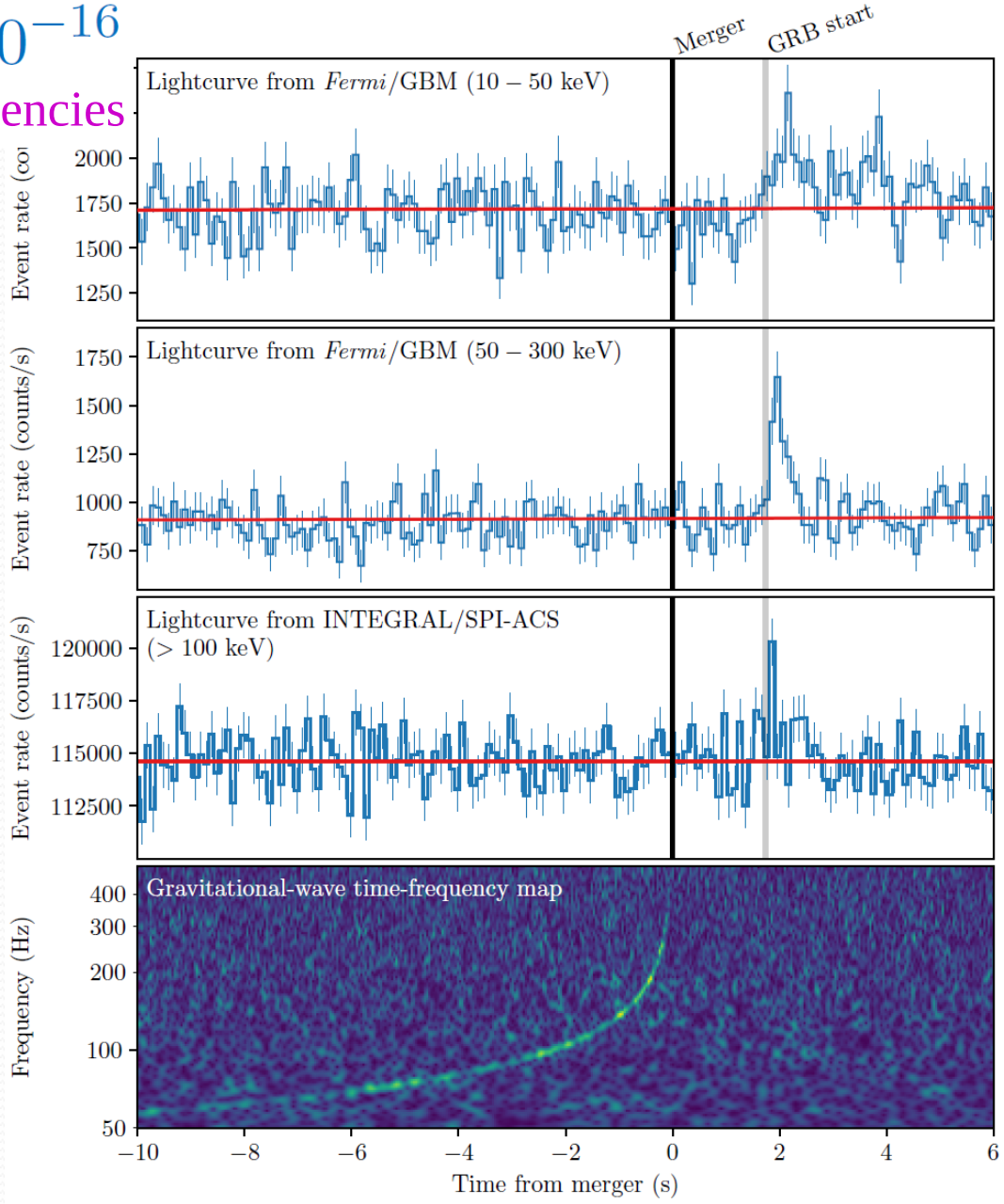
Abbott *et. al.* 1710.05832, 1710.05833, 1710.05834

GW&GBR 170817

$$-3 \times 10^{-15} \leq \frac{c_T}{c_\gamma} - 1 \leq 7 \times 10^{-16}$$

c_γ =1 at those frequencies





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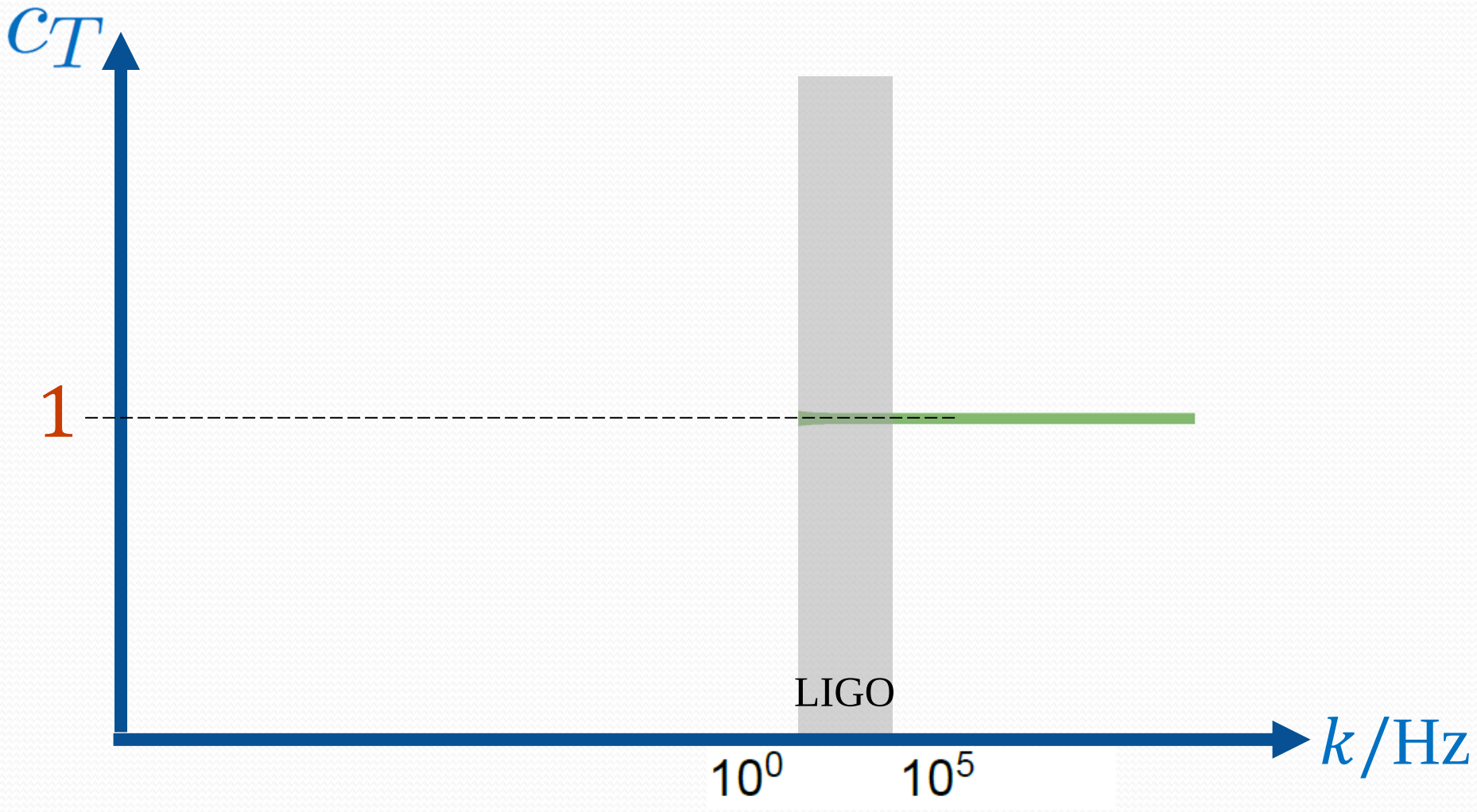
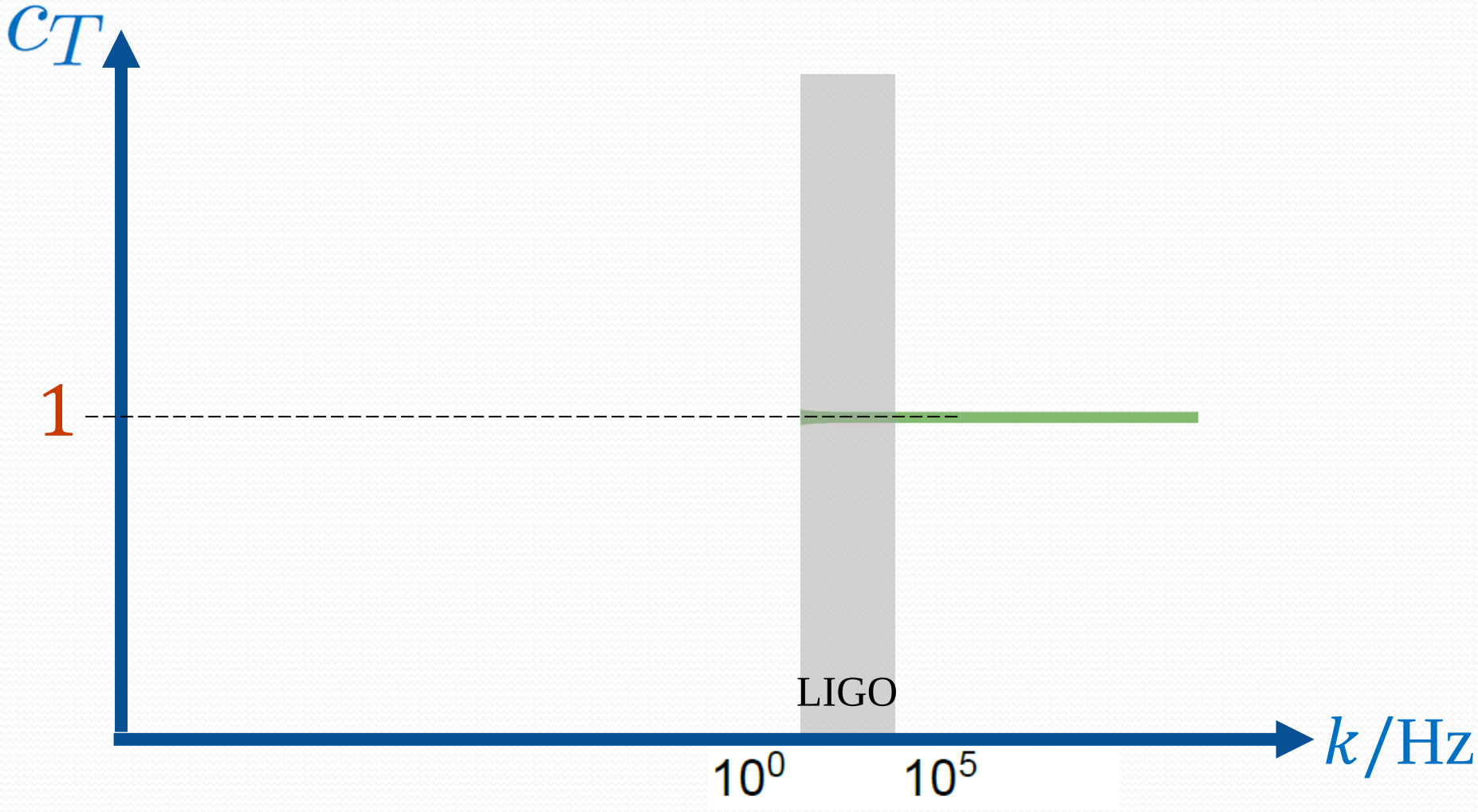


Many EFT for dark energy "predict" a
non-luminal sound speed for GW on FLRW
by an amount larger than 10^{-15}



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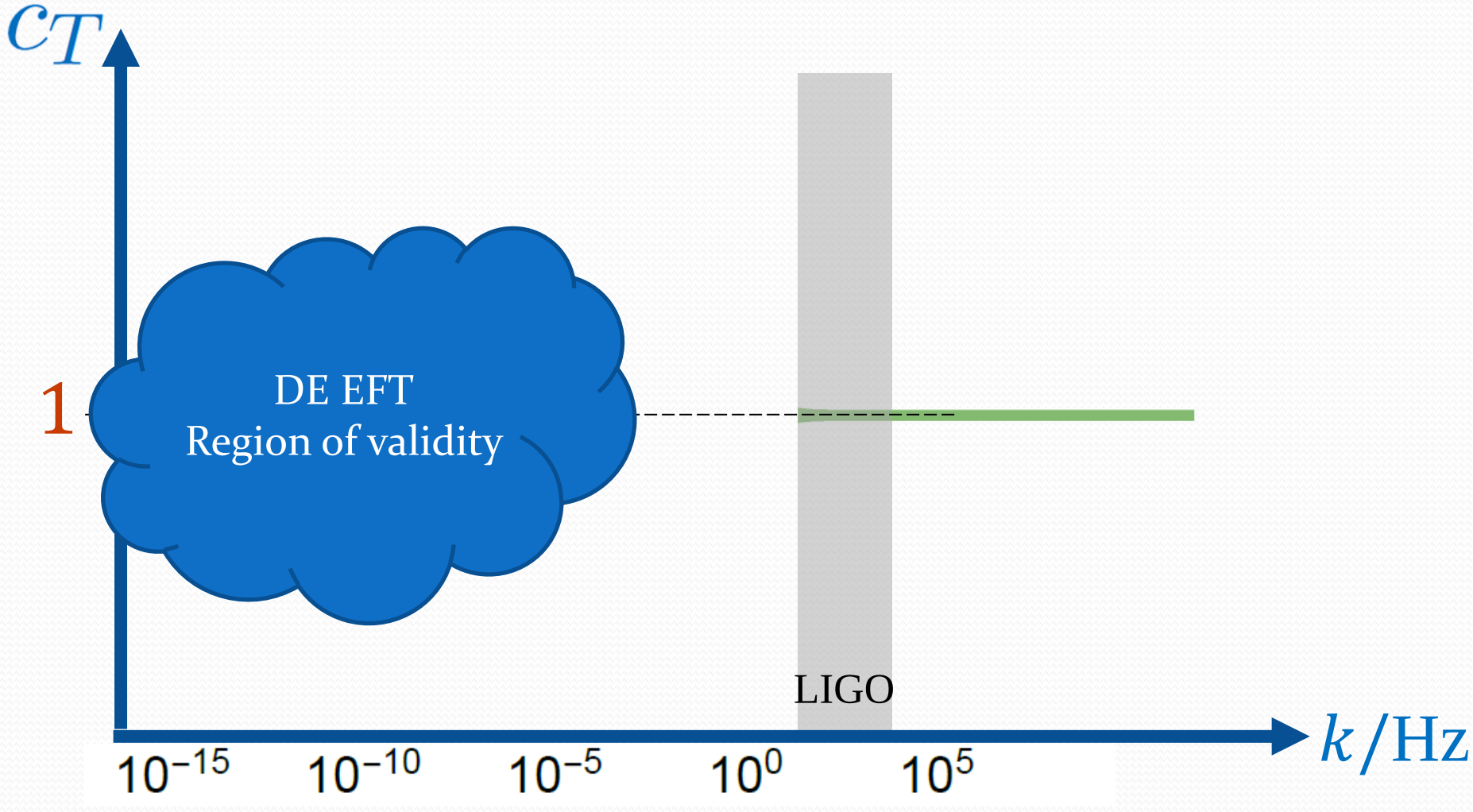

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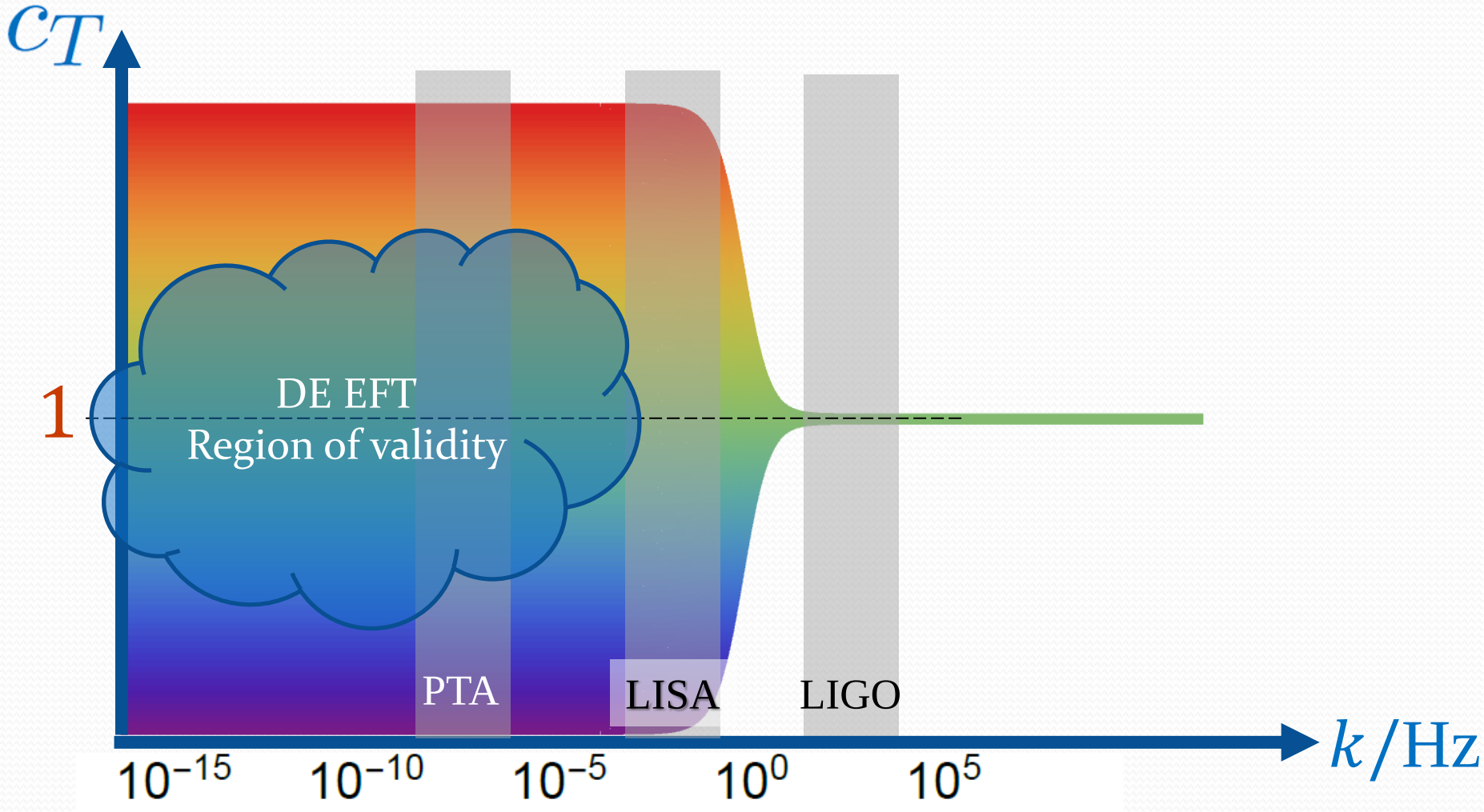


with Scott Melville, 1806.09417

Abbott *et. al.* 1710.05832, 1710.05833, 1710.05834

From GW&GBR 170817

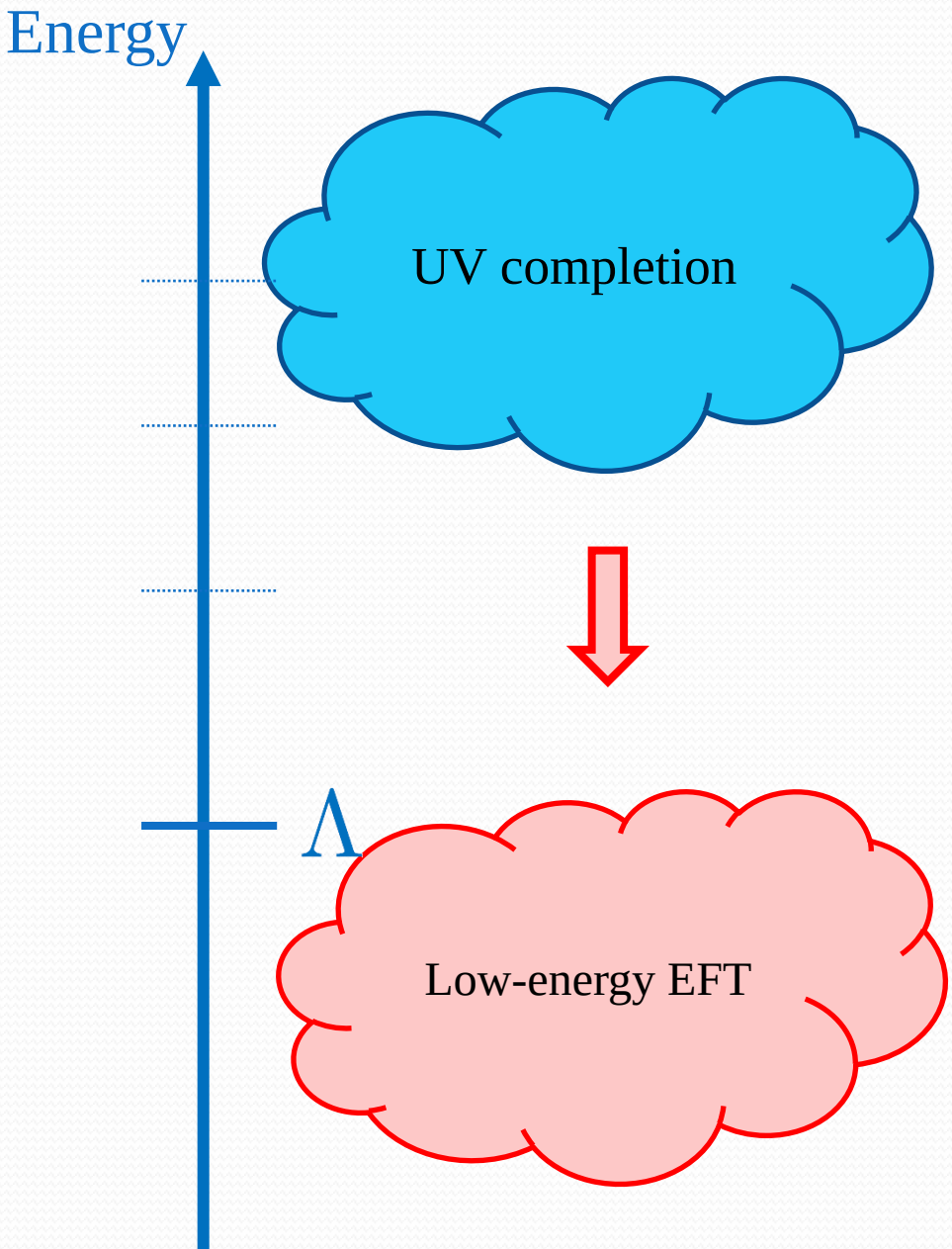
$$-\mathcal{O}\left(10^{-15}\right) < c_T - 1 < \mathcal{O}\left(10^{-16}\right)$$

Is this even a possibility
Or should we disregard this option???

$$-\mathcal{O}\left(10^{-15}\right) < c_T - \underbrace{1 \leq 0}$$

type of prior often imposed

Non-Gravitational EFT



- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound)

positivity bounds
(applied to low-energy scattering amplitude or refractive index)

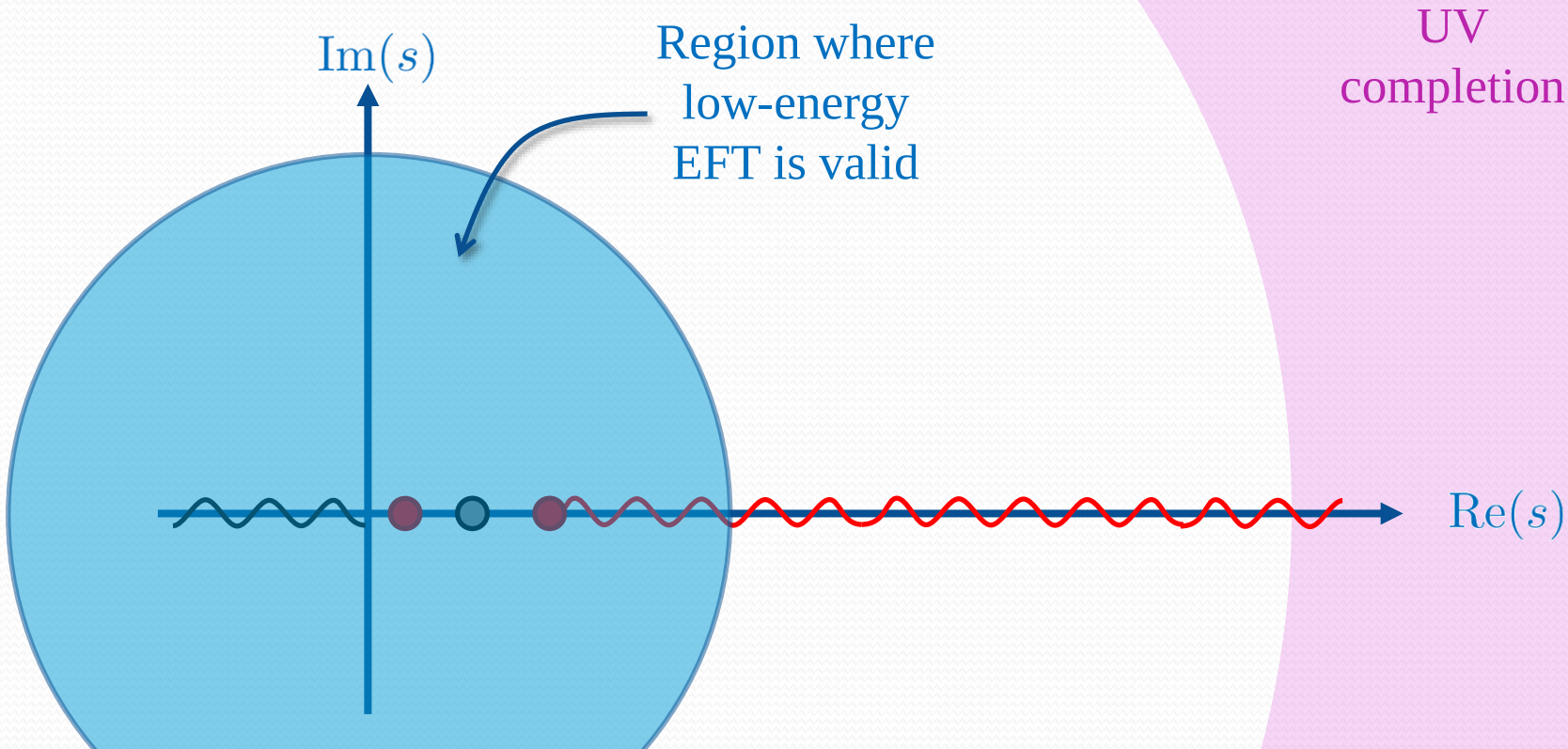
(sub)luminal sound speed

Low-energy EFT

$\mathcal{A}(s)$: $2-2$ elastic scattering amplitude

s: center of mass energy^2

t: momentum transfer



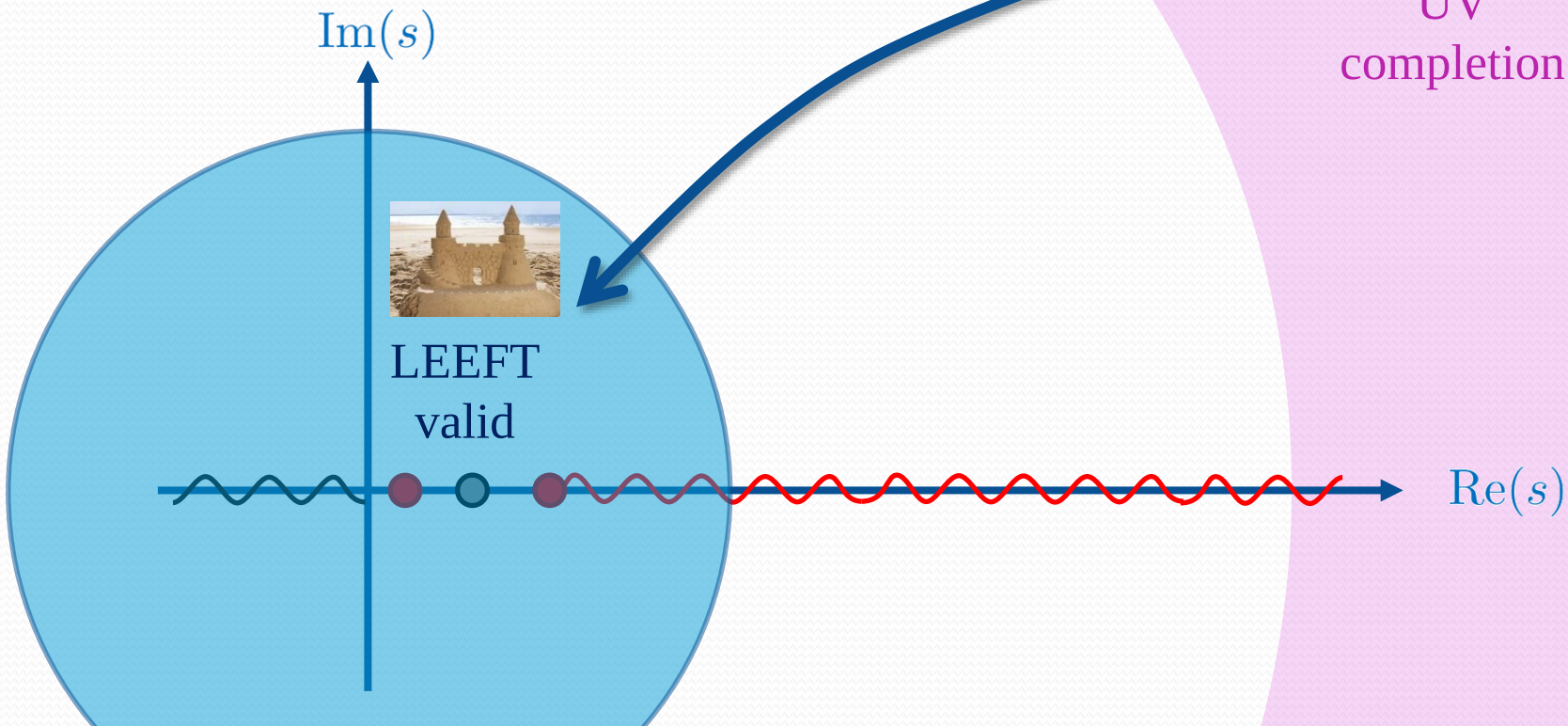
Analyticity - Causality

Causality

– encoded by requirement of analyticity –

is what connects UV to IR

UV completion



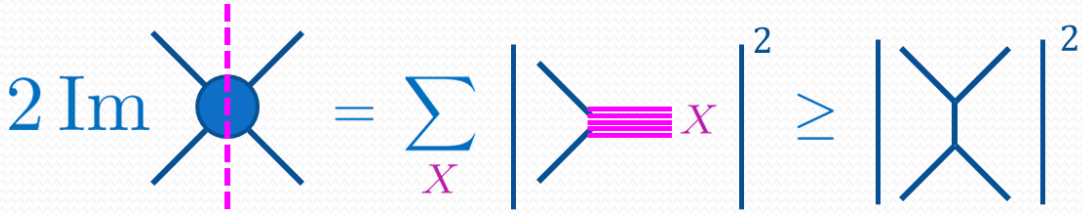
Non-Gravitational EFT

✓ Unitary (optical theorem)

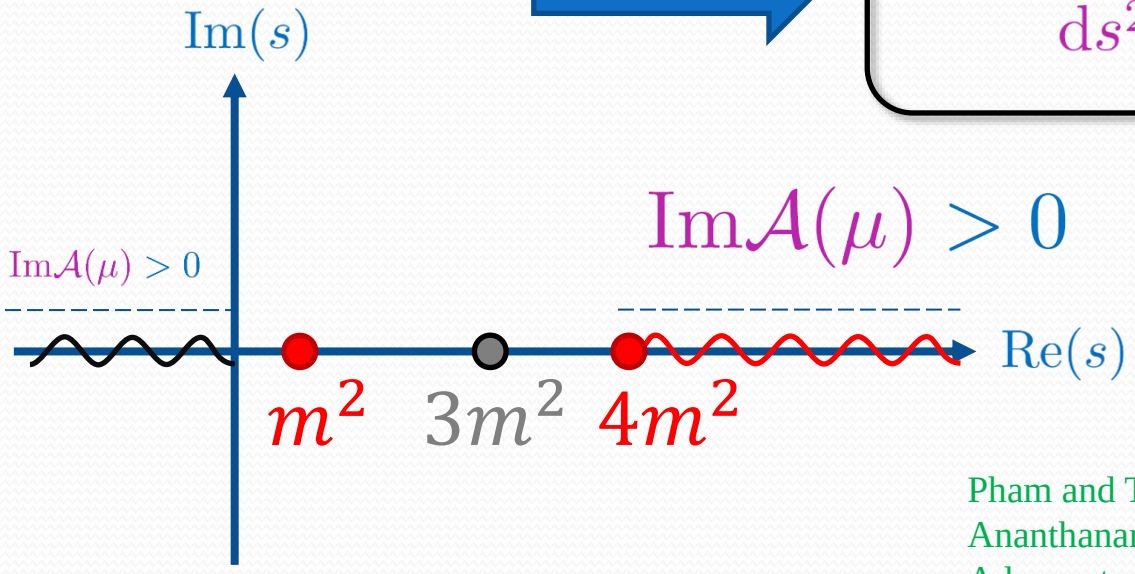
✓ Lorentz invariant (crossing symmetry)

UV completion

$\mathcal{A}$: $2-2$ elastic scattering amplitude

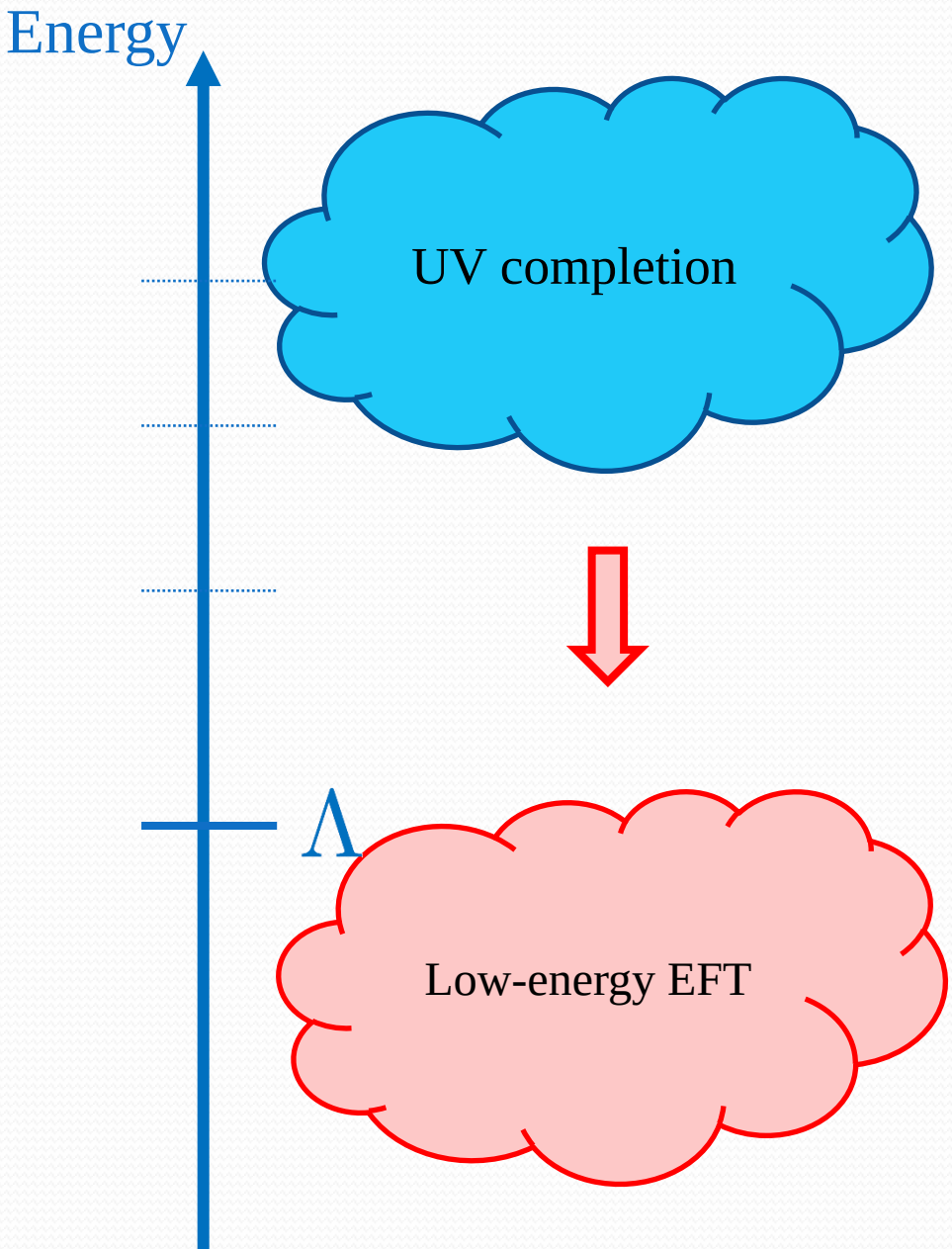


$$\left.\frac{\mathrm{d}^2\mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0$$



Pham and Truong 1985
Ananthanarayan, Toublan and Wanders, 1994
Adams et. al. 2006

Non-Gravitational EFT



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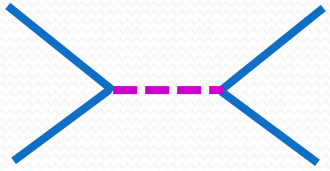
positivity bounds
(applied to low-energy scattering amplitude or refractive index)

(sub)luminal sound speed

EFT for a Scalar Field ϕ

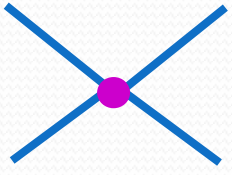
Energy



$$\mathcal{L}[\phi, H] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial H^2) - \frac{1}{2}M^2H^2 + \frac{\alpha}{M}H(\partial\phi)^2$$

M

$$\int \mathcal{D}H$$



$$\mathcal{L}[\phi] = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha^2}{2M^4}(\partial\phi)^4 + \cdots$$

EFT for a Scalar Field ϕ

$$\mathcal{L}[\phi] = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha^2}{2M^4}(\partial\phi)^4 + \cdots$$

On any Lorentz symmetric background, the sound speed is exactly luminal

$$\phi = f(\eta_{\mu\nu}x^\mu x^\nu) + \chi \qquad \Rightarrow \qquad c_\chi = 1$$

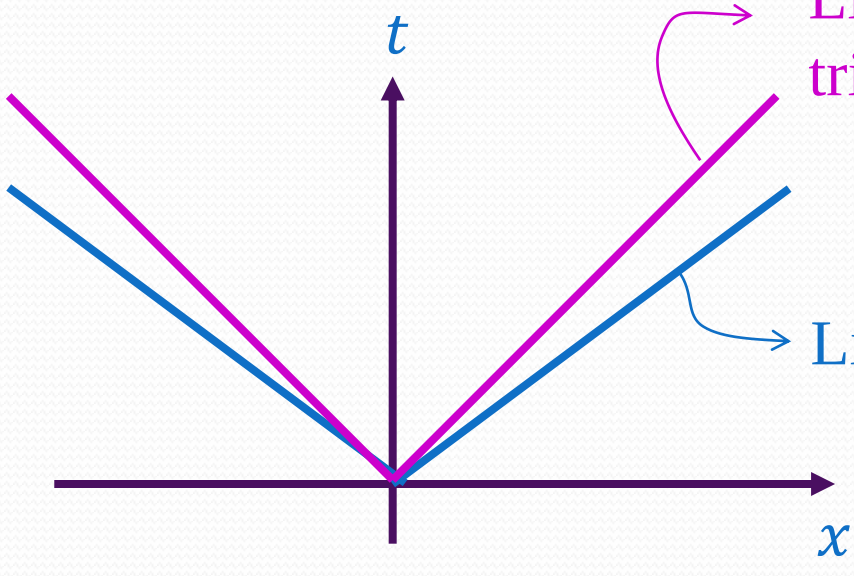
On backgrounds that spontaneously break Lorentz Invariance, the sound speed can be subluminal

$$\phi = \phi(t) + \chi \qquad \Rightarrow \qquad c_\chi = 1 - \frac{\alpha^2}{M^4}\dot{\phi}^2$$

Scalar Field minimally coupled to gravity

No gravity: $$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \cdots$$

In the absence of gravity, positivity bounds
assuming Unitarity, Analyticity, Causality require $c > 0$



Light-cone of $\delta\phi$ fluctuations on a non-trivial background if $c > 0$

Light light-cone

Non-Gravitational EFT

UV completion

- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound)

Eg of LEEFT

$$\mathcal{L}[\phi] = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha^2}{2M^4}(\partial\phi)^4 + \cdots$$

$$\mathcal{A}''(s)\big|_{t=0} \sim \alpha^2 > 0$$

(improved)
positivity bounds

$$\phi = \phi(t) + \chi \qquad \Rightarrow \qquad c_\chi = 1 - \frac{\alpha^2}{M^4}\dot{\phi}^2$$

(sub)luminal
sound speed

Adding Gravity

UV completion

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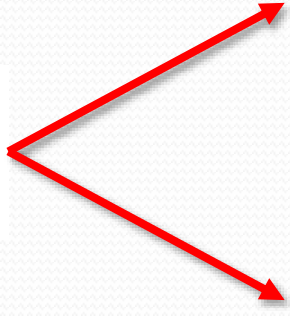
More subtle for
gravitational EFTs



(sub)luminal
sound speed

Adding Gravity

Causality ⟷ analyticity

Locality, Froissart Bound with Gravity???

✓ Unitary (optical theorem)

✓ Lorentz invariant (crossing symmetry)

✓ CAUSAL (analyticity)

✓ Local (Froissart Bound)

(improved)
positivity bounds

More subtle for
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(sub)luminal
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Adding Gravity

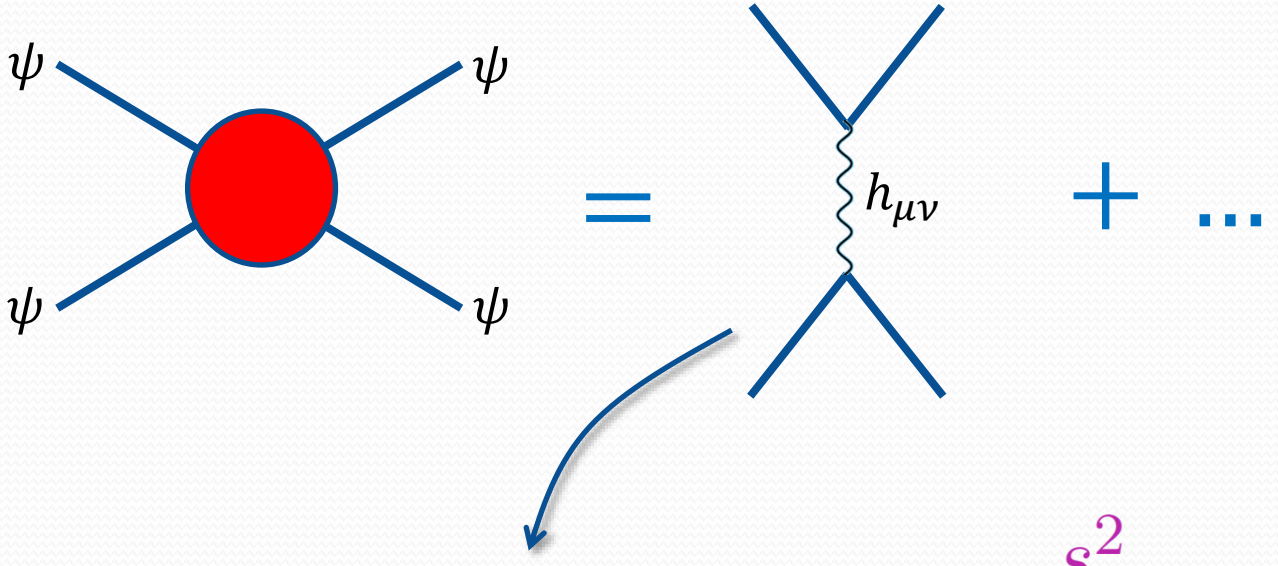
CAUSALITY  (sub)luminal sound speed

Connection more subtle with gravity for 2 reasons:

1. **Gravitational exchange**
   cannot prevent gravity from coupling to everyone and having an effect

2. **Frame artefacts**
   (speed not invariant under frame transformations, notion of causality is)
   trivial artefact yet important implications

Positivity Bounds in Gravitational LEEFT



$$\mathcal{A}_{\mathrm{t-channel}} = \# \frac{s^2}{M_{\mathrm{Pl}}^2 t} + \cdots$$

t-channel pole from gravity exchange compromises positivity bound

$$\left.\frac{\mathrm{d}^2 \mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0$$

Causality in a Negative World

In non-gravitational EFTs

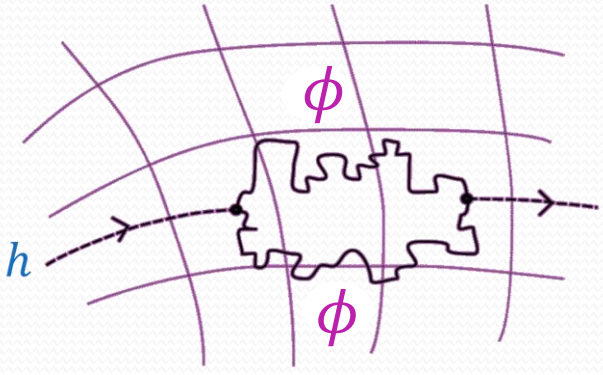
causality $\Longrightarrow$ positivity bounds $\mathcal{A} \sim \mathcal{C}\, s^2 + \cdots \Rightarrow \mathcal{C} > 0$ $\Longrightarrow$ (sub)luminal sound speed $0 < c_s \leq 1$

In gravitational EFTs at scale *M*

causality $\Longrightarrow$ Approximate positivity bounds $\mathcal{C} > -\dfrac{\mathcal{O}(1)}{M^2 M_{\rm Pl}^2}$ $\Longrightarrow$ sound speed $0 < c_s \leq 1 + \mathcal{O}\left(\dfrac{M^2}{M_{\rm Pl}^2}\right)$

Adapted from Hollowood & Shore

EFT for Gravity



Energy

M

High-energy theory with gravity and light & heavy modes

$$\int \mathcal{D}H$$

Integrate out heavy modes

Low-energy EFT of gravity

$$\mathcal{L}_{\rm IR} = \sqrt{-g}\left[-\Lambda^{\rm IR} + \frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_{\psi}^{(\rm light)}(g,\psi) + \mathcal{L}_{R^2} + \frac{1}{M^2}\mathcal{L}_{R^3} + \cdots\right]$$

Speed of Gravity

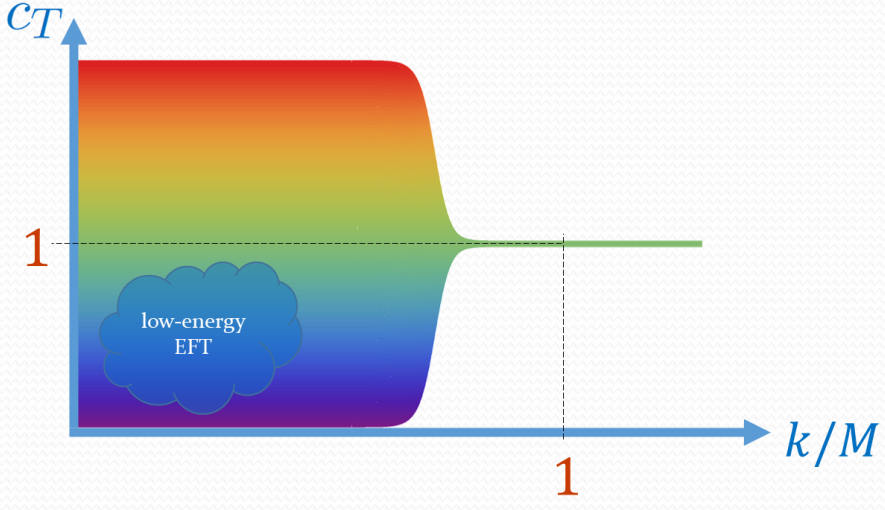
$$\mathcal{L}_{\rm IR} = \sqrt{-g}\left[-\Lambda^{\rm IR} + \frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_{\psi}^{(\rm light)}(g,\psi) + \mathcal{L}_{R^2} + \frac{1}{M^2}\mathcal{L}_{R^3} + \cdots\right]$$

For GWs on curved background (e.g. FLRW, Schwarzschild,…)
even in a frame where high-frequency travel luminally,
at low-frequency speed may be qualified as *super*luminal

$$c_s^2 = 1 + \mathcal{O}(\pm 1)\frac{(-\dot{H})}{M_{\rm Pl}^2} + \cdots$$

$$c_s^2 = 1 + \mathcal{O}(\mp 1)\frac{1}{M^2 M_{\rm Pl}^2 r_g^4} + \cdots$$

+ corrections at high energy



Speed of Gravity

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+ corrections at high energy

Typical approach (eg. for EFT of DE)
is to constrain low-energy EFT
so as to ensure **SUB luminality!**

Speed of Gravity

$$\mathcal{L}_{\rm IR} = \sqrt{-g}\left[-\Lambda^{\rm IR} + \frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_{\psi}^{(\rm light)}(g,\psi) + \mathcal{L}_{R^2} + \frac{1}{M^2}\mathcal{L}_{R^3} + \cdots\right]$$

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+ corrections at high energy

Such levels of Superluminality are not in tension with causality
and should not be used as a way to discard some operators
in the low-energy EFT of gravity

Adding Gravity

CAUSALITY  (sub)luminal sound speed

Connection more subtle with gravity for 2 reasons:

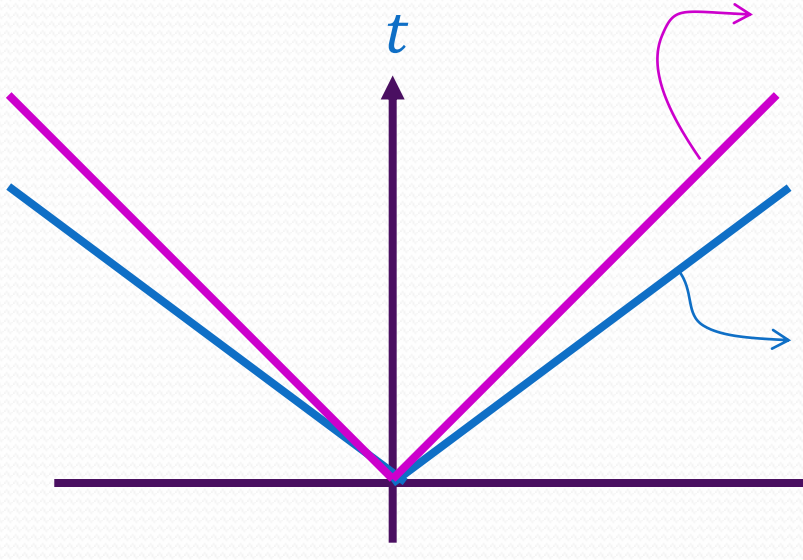
1. **Gravitational exchange** ✓
   cannot prevent gravity from coupling to everyone and having an effect

2. **Frame artefacts**
   (speed not invariant under frame transformations, notion of causality is)
   trivial artefact yet important implications

Scalar Field minimally coupled to gravity

Minimally-coupled to gravity:

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \cdots\right]$$



Einstein frame

Light-cone of $\delta\phi$ fluctuations on a non-trivial background if $c > 0$

Light light-cone
& light-cone of GWs

Scalar Field minimally coupled to gravity

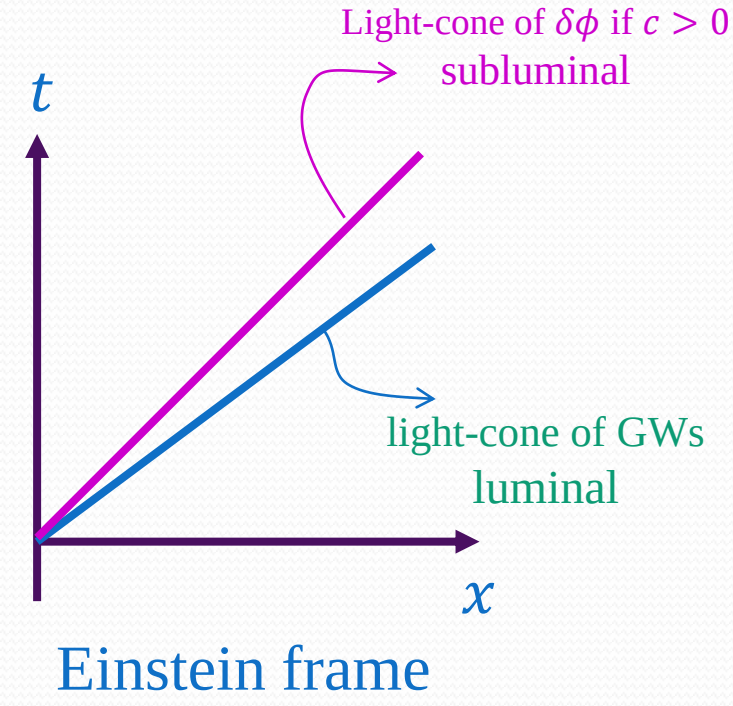
Einstein Frame:

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \cdots\right]$$

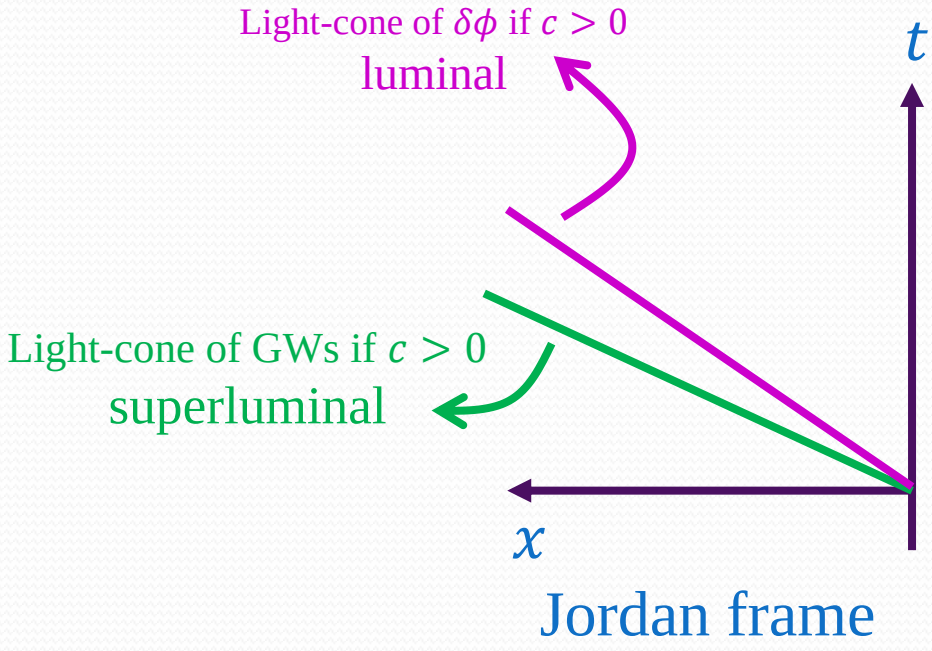
$$g_{\mu\nu} = \tilde{g}_{\mu\nu} - \frac{c}{\Lambda^4}\partial_\mu\phi\partial_\nu\phi + \frac{cM_{\rm Pl}^2}{\Lambda^4}G_{\mu\nu}$$

Jordan Frame:

$$\mathcal{L} = \sqrt{-\tilde{g}}\left[\frac{M_{\rm Pl}^2}{2}\tilde{R} - \frac{1}{2}(\tilde{\partial}\phi)^2 + \tilde{c}\,\tilde{G}^{\mu\nu}\tilde{R}_{\mu\nu} + \cdots\right]$$



Einstein frame

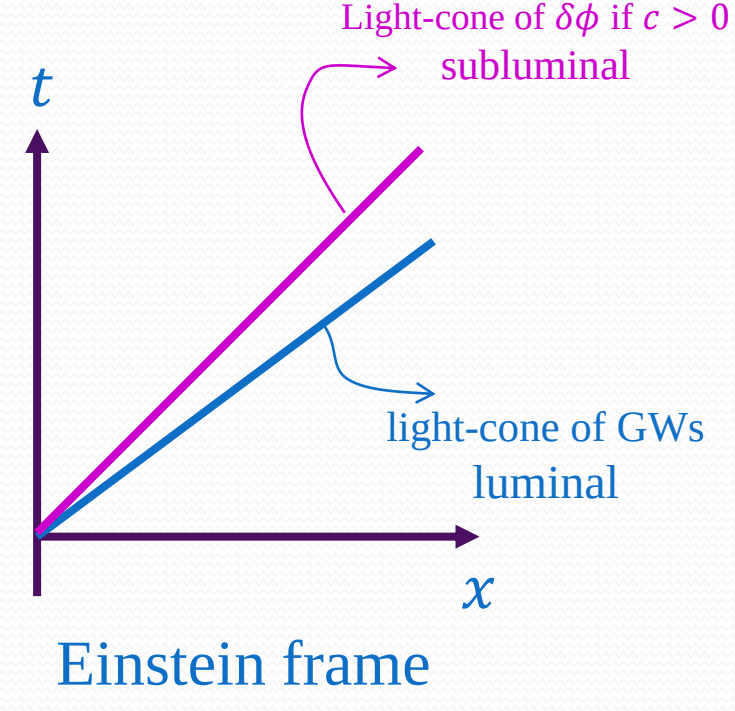


Jordan frame

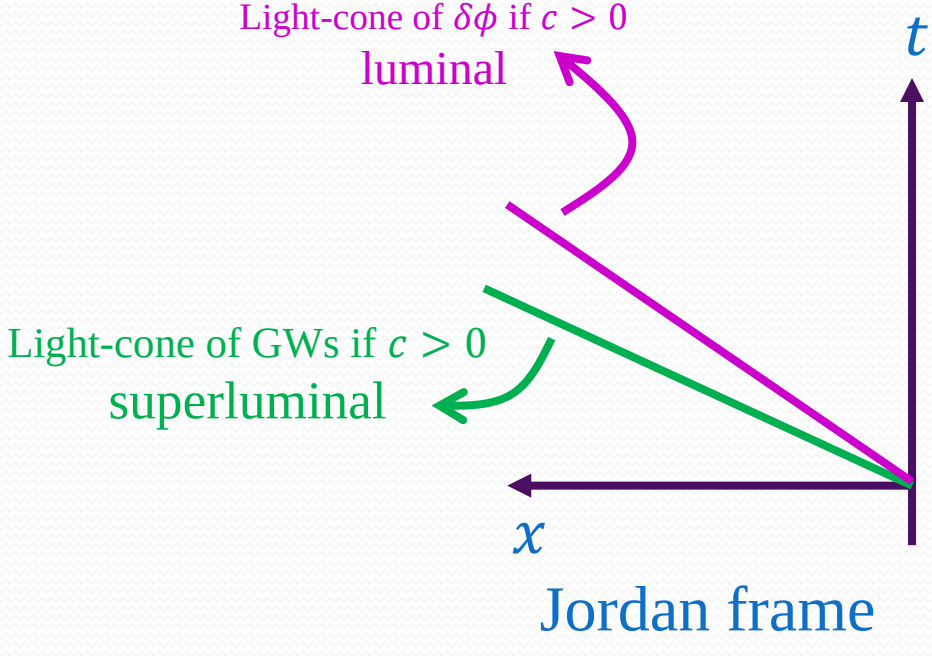
Frame artefacts

In some low-energy EFTs, causality imposes superluminal GWs…
just a trivial (yet Important!) frame artefact

particularly important when dealing with potential
modifications of Gravity (e.g. à la Horndeski,…) but not only



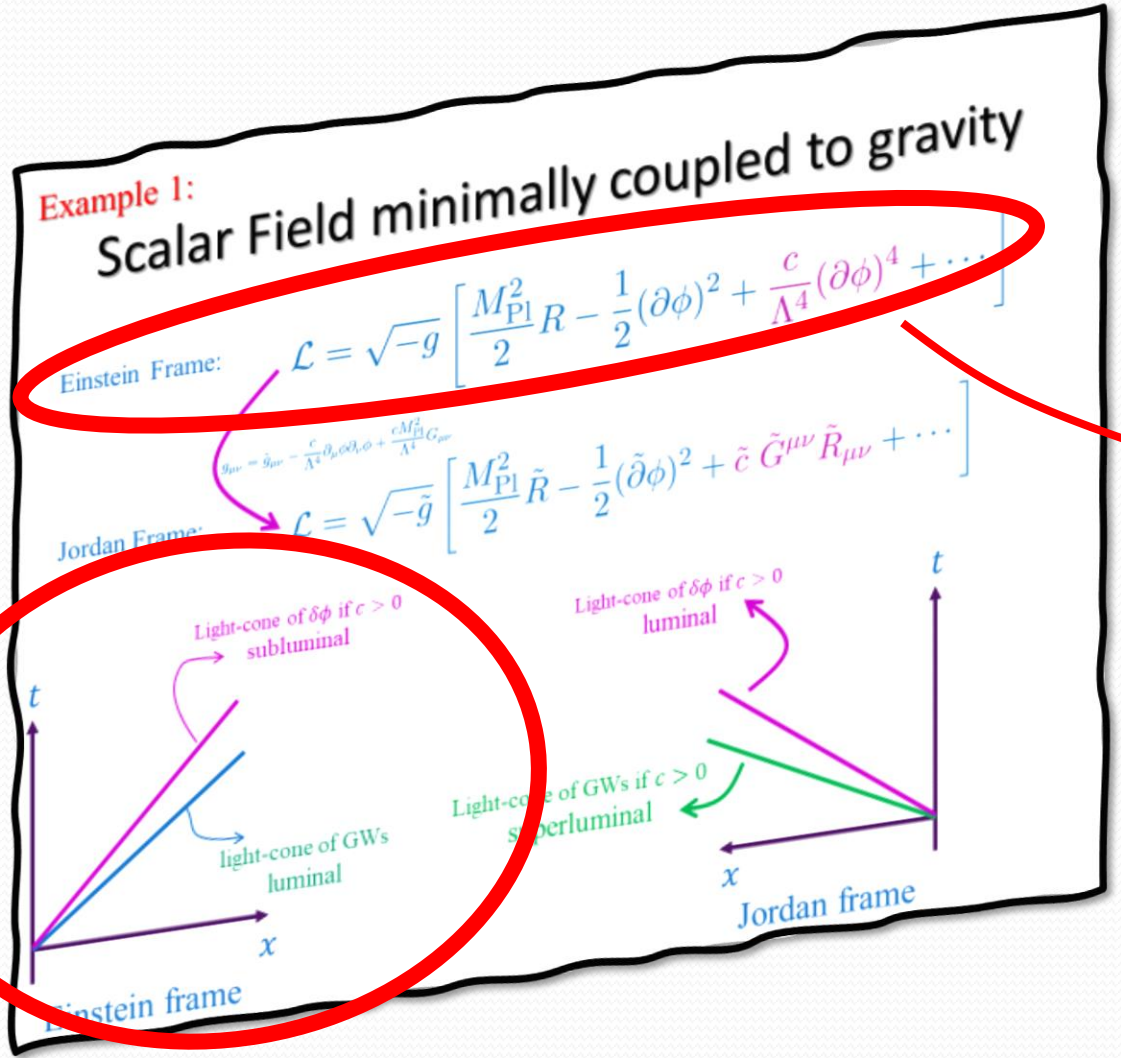
Einstein frame



Jordan frame

Causality

In some low-energy EFTs, causality imposes superluminal GWs…
just a trivial (yet Important!) frame artefact



To 0th order, it is safe to impose subluminality for all the fields once we are in the frame where gravity can be decoupled

Frame where we can take a smooth limit $M_{Pl} \to \infty$

The change of frame is singular in that limit

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In some low-energy EFTs, causality imposes superluminal GWs…
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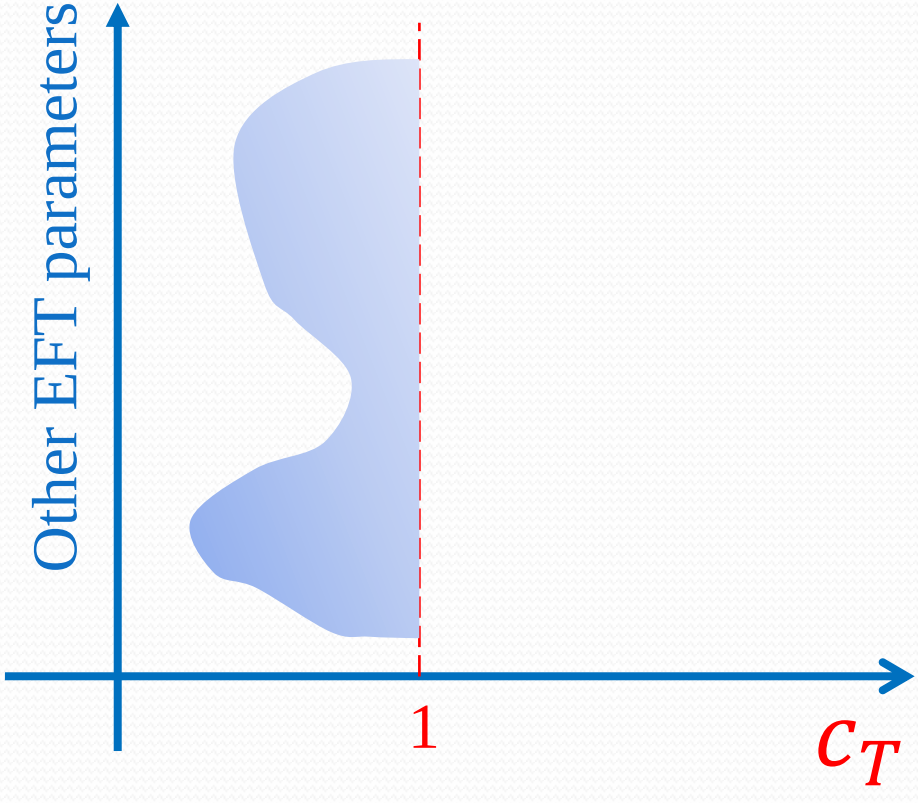

To 0^{th} order, it is safe to impose subluminality for all the fields once we are in the frame where gravity can and is decoupled

Frame where we can take a smooth limit $M_{Pl} \to \infty$

This may imply a large amount of superluminality in original frame

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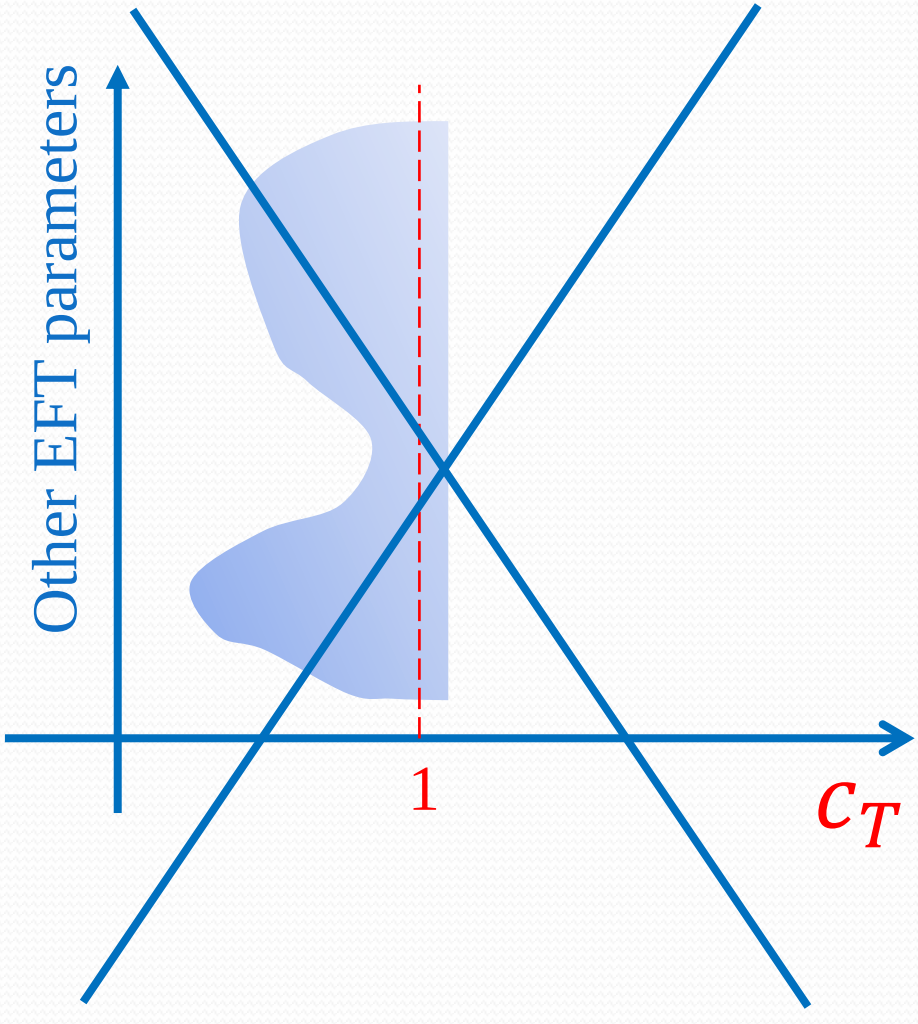

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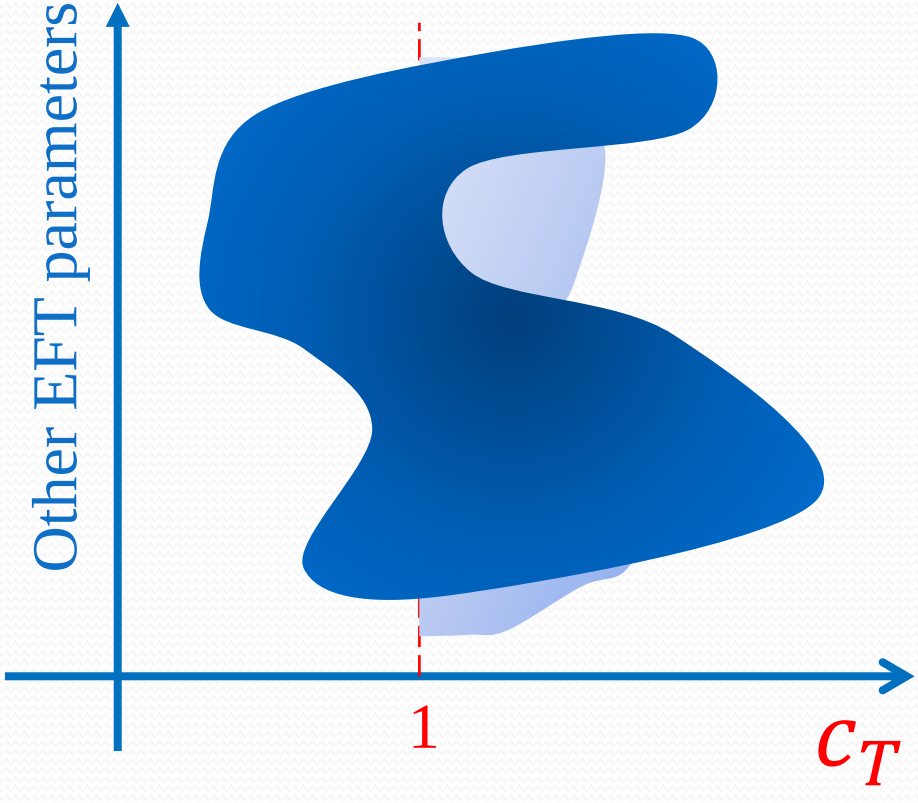

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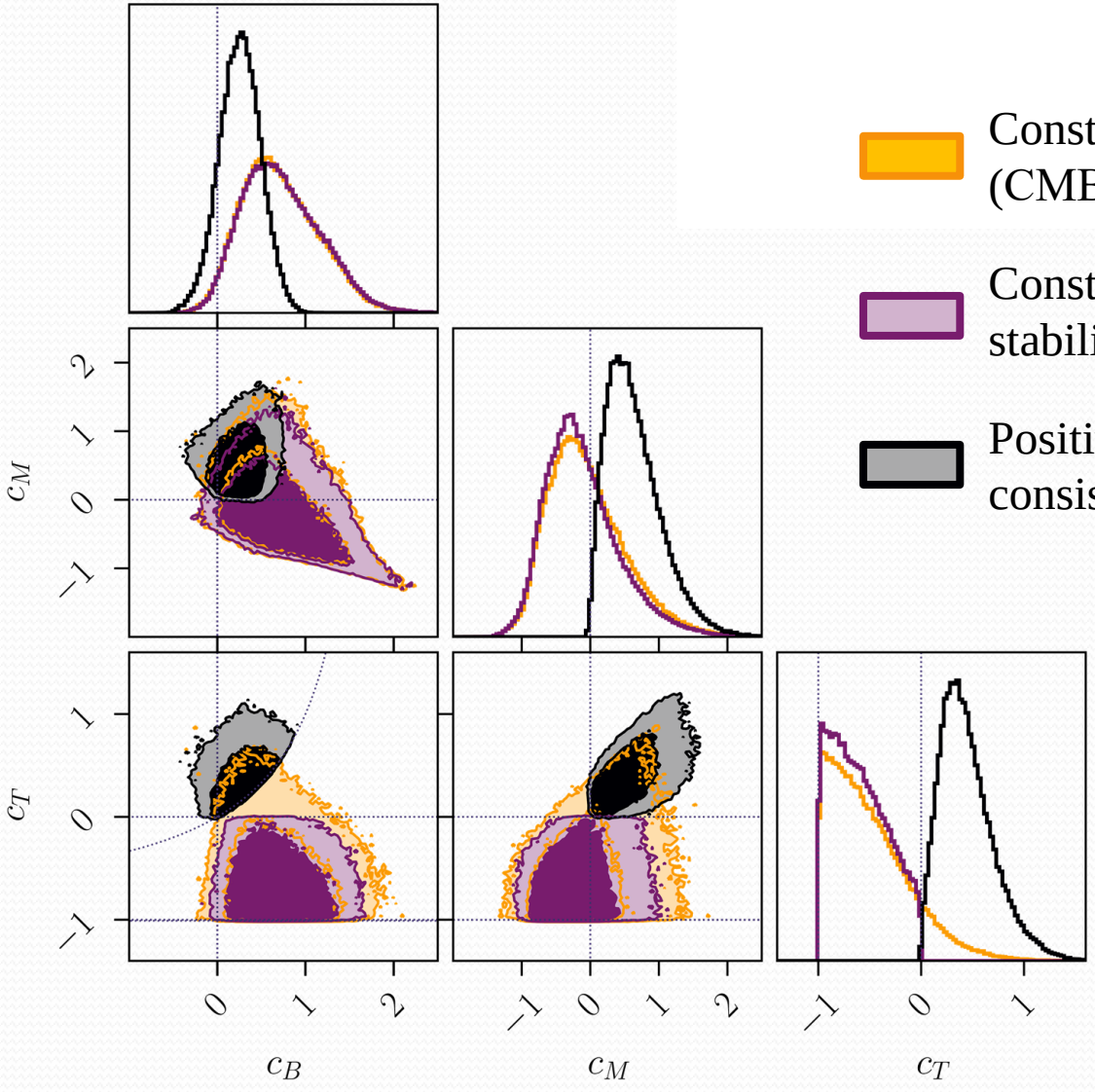
Frame where we can take a smooth limit $M_{Pl} \to \infty$

This may imply a large amount of superluminality in original frame

Positivity bounds are frame independent – agnostic to such considerations
In doubts, apply (approximate) positivity bounds (with allowed negativity from t-channel pole)

Constraints on Low-energy Models

Example of Dark Energy model (quartic Horndeski)
with parameters $c_{B,M,T}$



Constraints from observations
(CMB, BAO, redshift space distortion,…)

Constraints we would have imposed from stability and subluminality at low-energy

Positivity Priors (constraints from causality and consistent high energy completion)

with Melville and Noller, 2103.06855

Living with Superluminality

- Gravitational Waves are luminal to a (VERY) good accuracy at LIGO frequencies $-\mathcal{O}\left(10^{-15}\right) < c_T - 1 < \mathcal{O}\left(10^{-16}\right)$

- Within the standard EFT of gravity, GWs are no longer perfectly luminal on backgrounds that spontaneously break Lorentz invariance (eg Schwarzschild, FLRW, the real world,…)

Lesson 1:

- In an arbitrary frame, GWs may be superluminal
- Imposing subluminality priors only makes sense in a frame where gravity can be decoupled
- In doubts, to be derived from positivity bounds
- In the original frame this may correspond to GWs being superluminal by a 'considerable' amount

Living with Superluminality

Lesson 2:

- Even in the frame where matter and gravity can decouple, a tiny amount of SL *or a negative phase shift* – be it for GWs or other fields – is not in conflict with causality.
  It may even follow from consistent causal and Lorentz invariant UV completions.

- In the frame where matter and gravity can decouple, superluminality is consistent with causality so long as

$$\lim_{M_{\rm Pl}\to\infty} |c_s^2 - 1| \sim M_{\rm Pl}^{-\alpha} \quad \text{with} \quad \alpha \geq 2$$

Living with Superluminality

Lesson 2:

- Even in the frame where matter and gravity can decouple, a tiny amount of SL *or a negative phase shift* – be it for GWs or other fields – is not in conflict with causality.
  It may even follow from consistent causal and Lorentz invariant UV completions.

- superluminality not in conflict with causality so long as amplitude respects some (approximate) positivity bounds

$$\mathcal{A} \sim -\frac{s^2}{M_{\rm Pl}^2 t} + \frac{c\, s^2}{M^4} \qquad \text{with} \qquad c > -\frac{M^2}{M_{\rm Pl}^2} \times \mathcal{O}(1)$$