Minimalism in modified gravity

1. Introduction
2. Minimally modified gravity (MMG)
3. Examples of type-I & type-II MMG theories
4. $D\rightarrow 4$ EGB gravity with 2 dof
5. Summary

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Based on collaborations with
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INTRODUCTION
Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
  
...
# of d.o.f. in general relativity

- 10 metric components $\rightarrow$ 20-dim phase space @ each point
ADM decomposition

• Lapse $N$, shift $N^i$, 3d metric $h_{ij}$

\[
\begin{align*}
    ds^2 &= -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \\
\end{align*}
\]

• Einstein-Hilbert action

\[
I = \frac{M_{Pl}^2}{2} \int d^4 x \sqrt{-g} \left(^{(4)}R \right) \\
= \frac{M_{Pl}^2}{2} \int dt d^3 \vec{x} N \sqrt{h} \left[ K_{ij} K_{ij} - K^2 + ^{(3)}R \right]
\]

• Extrinsic curvature

\[
K_{ij} = \frac{1}{2N} \left( \partial_t h_{ij} - D_i N_j - D_j N_i \right)
\]
# of d.o.f. in general relativity

• 10 metric components $\rightarrow$ 20-dim phase space @ each point

• Einstein-Hilbert action does not contain time derivatives of $N$ & $N^i$ $\rightarrow$ $\pi_N = 0$ & $\pi_i = 0$
# of d.o.f. in general relativity

- 10 metric components $\rightarrow$ 20-dim phase space @ each point

- Einstein-Hilbert action does not contain time derivatives of $N$ & $N^i$ $\rightarrow$ $\pi_N = 0$ & $\pi_i = 0$
  All constraints are independent of $N$ & $N^i$ $\rightarrow$ $\pi_N$ & $\pi_i$
  “commute with” all constraints $\rightarrow$ 1st-class
1\textsuperscript{st}-class vs 2\textsuperscript{nd}-class

• 2\textsuperscript{nd}-class constraint $S$
  \{ $S$, $C_i$ \} $\not\approx$ 0 for $\exists i$
  Reduces 1 phase space dimension

• 1\textsuperscript{st}-class constraint $F$
  \{ $F$, $C_i$ \} $\approx$ 0 for $\forall i$
  Reduces 2 phase space dimensions
  Generates a symmetry
  Equivalent to a pair of 2\textsuperscript{nd}-class constraints

\{ $C_i$ | $i = 1,2,...$\} : complete set of independent constraints

$A \approx B$ $\iff$ $A = B$ when all constraints are imposed
(weak equality)
# of d.o.f. in general relativity

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- Einstein-Hilbert action does not contain time derivatives of $N$ & $N^i$ $\rightarrow$ $\pi_N = 0$ & $\pi_i = 0$
  All constraints are independent of $N$ & $N^i$ $\rightarrow$ $\pi_N$ & $\pi_i$
  “commute with” all constraints $\rightarrow$ 1$^{\text{st}}$-class
- 4 generators of 4d-diffeo: 1$^{\text{st}}$-class constraints
- $20 - (4+4) \times 2 = 4$ $\rightarrow$ 4-dim physical phase space @ each point $\rightarrow$ 2 local physical d.o.f.

Minimal # of d.o.f. in modified gravity = 2
# of d.o.f. in general relativity

- 10 metric components $\rightarrow$ 20-dim phase space @ each point

- **Einstein-Hilbert action does not contain time derivatives of** $N$ & $N^i \rightarrow \pi_N = 0$ & $\pi_i = 0$

  All constraints are independent of $N$ & $N^i \rightarrow \pi_N$ & $\pi_i$

  “commute with” all constraints $\rightarrow$ 1\textsuperscript{st}-class

- 4 generators of 4d-diffeo: 1\textsuperscript{st}-class constraints

- $20 - (4+4) \times 2 = 4 \rightarrow$ 4-dim physical phase space @ each point $\rightarrow$ 2 local physical d.o.f.

**Minimal # of d.o.f. in modified gravity = 2**

**Can this be saturated?**
MINIMALLY MODIFIED GRAVITY (MMG)
Is general relativity unique?

- **Lovelock theorem** says “yes” if we assume:
  i) 4-dimensions; ii) diffeo invariance; iii) metric only; iv) up to 2nd-order eom’s of the form $E_{ab}=0$.

- **Effective field theory** (derivative expansion) says “yes” at low energy if we assume:
  i) 4-dimensions; ii) diffeo invariance; iii) metric only.

- However, cosmological backgrounds break 4d-diffeo while keeping 3d-diffeo.

- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)
Example: simple scalar-tensor theory

- **Covariant action**

  \[ I = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ \Omega^2(\phi)^{(4)} R + P(X, \phi) \right] \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \]

- **ADM decomposition**

  \[ ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt) \]

- **Unitary gauge**

  \[ \phi = t \quad X = \frac{1}{2} \frac{1}{N^2} \quad g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^i}{N^2} \\ \frac{N^j}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix} \]

  This is a good gauge iff derivative of \( \phi \) is timelike.

- **Action in unitary gauge**

  \[ I = \int dt d^3 \bar{x} \sqrt{h} \left\{ f_1(t) \left[ K^{ij} K_{ij} - K^2 + (3)^R \right] + \frac{2}{N} \dot{f}_1(t) K + f_2(N, t) \right\} \]

  \[ \Omega^2(\phi) = f_1(t) \quad P(X, \phi) = f_2(N, t) \]
Is general relativity unique?

- Lovelock theorem says “yes” if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only; (iv) up to 2\textsuperscript{nd}-order eom’s of the form $E_{ab}=0$.

- Effective field theory (derivative expansion) says “yes” at low energy if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only.

- However, cosmological backgrounds break 4d-diffeo while keeping 3d-diffeo.

- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)

- Is GR unique when we assume: (i) 4-dimensions; (ii) 3d-diffeo invariance; (iii) metric only; (iv) 2 local physical d.o.f. (= 2 polarizations of TT gravitational waves)?

- Answer is “no” \(\rightarrow\) Minimally modified gravity (MMG)
EXAMPLES OF TYPE-I & TYPE-II MMG THEORIES
Type-I & type-II modified gravity

- Jordan (or matter) frame

\[ I = \frac{1}{2} \int d^4x \sqrt{-g^J} \left[ \Omega^2(\phi) R[g^J] + \cdots \right] + I_{\text{matter}}[g^J_{\mu\nu}; \text{matter}] \]

- Einstein-frame

\[ g^E_{\mu\nu} = \Omega^2(\phi)g^J_{\mu\nu} \quad \text{K. Maeda (1989)} \]

\[ I = \frac{1}{2} \int d^4x \sqrt{-g^E} \left[ R[g^E] + \cdots \right] + I_{\text{matter}}[\Omega^{-2}(\phi)g^E_{\mu\nu}; \text{matter}] \]

- Do we call this GR? No. This is a modified gravity because of non-trivial matter coupling \( \rightarrow \) **type-I**

- There are more general scalar tensor theories where there is no Einstein frame \( \rightarrow \) **type-II**
Type-I & type-II modified gravity

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM
and Michele Oliosi, JCAP 01 (2019) 017

• **Type-I:**
  There exists an Einstein frame
  Can be recast as GR + extra d.o.f. + **matter, which couple(s) non-trivially**, by change of variables

• **Type-II:**
  No Einstein frame
  Cannot be recast as GR + extra d.o.f. + matter by change of variables
Type-I minimally modified gravity (MMG)

Katsuki Aoki, Chunshan Lin and SM, PRD98 (2018) 044022

• # of local physical d.o.f. = 2
• There exists an Einstein frame
• Can be recast as GR + matter, which couple(s) non-trivially, by change of variables
• The most general change of variables = canonical tr.
• Matter coupling just after canonical tr. \( \rightarrow \) breaks diffeo \( \rightarrow \) 1\textsuperscript{st}-class constraint downgraded to 2\textsuperscript{nd}-class \( \rightarrow \) leads to extra d.o.f. in phase space \( \rightarrow \) inconsistent
• Gauge-fixing after canonical tr. \( \rightarrow \) splits 1\textsuperscript{st}-class constraint into pair of 2\textsuperscript{nd}-class constraints
• Matter coupling after canonical tr. + gauge-fixing \( \rightarrow \) a pair of 2\textsuperscript{nd}-class constraints remain \( \rightarrow \) consistent
A type-I MMG fitting Planck data better than $\Lambda$CDM

- $f(\mathcal{H})$ theory with $f'(C) = f_C$, $\mathcal{H} < 0$
  
  $$f_C = 1 + \frac{1}{2}a_1 - \frac{1}{2}a_1 \tanh \left[ \frac{1}{a_3} \left( \frac{C}{H_0^2} + a_2 \right) \right]$$

- 3 additional parameters
- $\Delta \chi^2 = 16.6$ improvement

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### Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>95% limits</th>
</tr>
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<tbody>
<tr>
<td>$a_1$</td>
<td>$0.0028^{+0.0006}_{-0.0002}$</td>
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<td>$\log_{10} a_2$</td>
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<td>$\log_{10} \beta$</td>
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<tr>
<td>$10^2 \omega_b$</td>
<td>$2.284^{+0.019}_{-0.036}$</td>
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<tr>
<td>$\tau_{reio}$</td>
<td>$0.052^{+0.013}_{-0.015}$</td>
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<tr>
<td>$n_s$</td>
<td>$0.9778^{+0.0058}_{-0.0092}$</td>
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<td>$H_0$</td>
<td>$69.19^{+0.67}_{-0.90}$</td>
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<tr>
<td>$\Omega_m$</td>
<td>$0.2952^{+0.0104}_{-0.0090}$</td>
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### Data sets

- Planck highl TTTTEEE
- Planck lowl EE
- Planck lowl TT
- JLA
- bao boss dr12
- bao smallz 2014
- HST

<table>
<thead>
<tr>
<th>Data sets \downarrow</th>
<th>$\chi^2$ for bestfit of $\Lambda$CDM</th>
<th>$\chi^2$ for bestfit of kink model</th>
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<td>Planck highl TTTTEEE</td>
<td>2351.98</td>
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<td>Planck lowl EE</td>
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<td>bao boss dr12</td>
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<td>bao smallz 2014</td>
<td>2.41</td>
<td>2.38</td>
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<tr>
<td>HST</td>
<td>13.03</td>
<td>11.63</td>
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<tr>
<td>All chosen data sets: in total</td>
<td>$\chi^2 = 3473.27$</td>
<td>in total $\chi^2 = 3456.67$</td>
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</table>
Type-II minimally modified gravity (MMG)

- # of local physical d.o.f. = 2
- No Einstein frame
- Cannot be recast as GR + matter by change of variables
- Is there such a theory? Yes!
- Example: Minimal theory of massive gravity
- Another example:
  arXiv 2004.12549 w/ Antonio De Felice and Andreas Doll
VCDM: a theory of type-II MMG
Antonio De Felice, Andreas Doll and Shinji Mukohyama [arXiv 2004.12549]

• Simple construction with a free function $V(\phi)$
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (including CDM)

$$\mathcal{L} = N \sqrt{-\gamma} \left[ \frac{M_P^2}{2} \left( R + K_{ij} K^{ij} - K^2 - 2V(\phi) \right) - \frac{\lambda_{gf}^i}{N} M_P^2 \partial_i \phi - \frac{3M_P^2 \lambda^2}{4} - M_P^2 \lambda (K + \phi) \right]$$

• No Einstein frame, equivalent to cuscuton
  [arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

• $V(\phi)$ reconstructed from FLRW background

• $c_{GW} = 1$, no extra dof

• Can reduce $H_0$ tension
  [arXiv: 2009.08718 w/ Antonio De Felice & Masroor C. Pookkillath]

• Extension to address both $H_0$ & $S_8$ tensions? [arXiv:2011.04188 w/ Antonio De Felice]
**Refined classification**

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

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<thead>
<tr>
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<tbody>
<tr>
<td><strong>Type-Ia</strong></td>
<td><strong>Type-IIa</strong></td>
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<tr>
<td>$c_T^2(k^2) = 1$</td>
<td></td>
</tr>
<tr>
<td>$g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$</td>
<td></td>
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<table>
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<th><strong>Type-Ib</strong></th>
<th><strong>Type-IIb</strong></th>
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<td>$c_T^2(k^2) \neq 1$</td>
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</tr>
<tr>
<td>$g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$</td>
<td>$\omega^2 = k^2 + m^2$</td>
</tr>
<tr>
<td></td>
<td>$4\text{DEGB: } \omega^2 = k^2 + k^4/\Lambda^2$</td>
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**Proof of the absence of Einstein frame in cuscuton/VCDM**

1. GWs → cuscuton/VCDM is of type-Ia or type-IIa
2. GR + conformal-type canonical tr. → most general type-Ia MMG
3. Vacuum Bianchi-I universes → cuscuton/VCDM is not of type-Ia
4. 1 & 3 → cuscuton/VCDM is of type-IIa, thus no Einstein frame
# Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

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<td><strong>Type-Ia</strong></td>
<td><strong>Type-IIa</strong></td>
</tr>
<tr>
<td>$c_T^2(k^2) = 1$</td>
<td>Cuscution/VCDM:</td>
</tr>
<tr>
<td>$g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$</td>
<td>$\omega^2 = k^2$</td>
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<tr>
<td><strong>Type-Ib</strong></td>
<td><strong>Type-IIb</strong></td>
</tr>
<tr>
<td>$c_T^2(k^2) \neq 1$</td>
<td>MTMG:</td>
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<tr>
<td>$g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$</td>
<td>$\omega^2 = k^2 + m^2$</td>
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<td></td>
<td>4DEGB:</td>
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**Proof of the absence of Einstein frame in cuscution/VCDM**

1. GWs $\rightarrow$ cuscution/VCDM is of type-Ia or type-IIa
2. GR + conformal-type canonical tr. $\rightarrow$ most general type-Ia MMG
3. Vacuum Bianchi-I universes $\rightarrow$ cuscution/VCDM is not of type-Ia
4. 1 & 3 $\rightarrow$ cuscution/VCDM is of type-IIa, thus no Einstein frame
Weaker gravity for DM: VCCCDM

Antonio De Felice and Shinji Mukohyama [arXiv 2011.04188]

- Simple construction with **free functions** \( f_0(\phi) \) & \( f_1(\phi) \)
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const & **dark matter** in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (no dark matter here)

\[
\mathcal{L} = N \sqrt{-\gamma} \left[ \frac{M_P^2}{2} \left( R + K_{ij} K^{ij} - K^2 - 2V(\phi) \right) - \frac{\lambda_{gf}}{N} M_P^2 \partial_i \phi - \frac{3M_P^2 \lambda^2}{4} - M_P^2 \lambda (K + \phi) \right]
\]

**SM metric:**

\[ g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij} (dx^i + N^i dt)(dx^j + N^j dt) \]

**DM metric:**

\[ g_{\mu\nu}^{\text{eff}} dx^\mu dx^\nu = -\frac{N^2}{f_1^2} dt^2 + \frac{\gamma_{ij}}{f_0} (dx^i + N^i dt)(dx^j + N^j dt) \]

- \( f_0(\phi) \) & \( f_1(\phi) \) reconstructed from \( H(z) \) & \( G_{DM}(z)/G_N \)
- \( c_{GW} = 1, \ G_{SM} = G_N \), no extra dof
- \( V(\phi) \equiv \frac{\Lambda}{f_1 f_0^{3/2}} \)
- May reduce \( H_0 \) & \( S_8 \) tensions
D→4 EGB GRAVITY WITH 2 DOF

EGB theory and $D \to 4$

$$S_{\text{EGB}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[ \mathcal{R} - 2\Lambda + \alpha \mathcal{R}_{\text{GB}}^2 \right]$$

$$\mathcal{R}_{\text{GB}}^2 = \mathcal{R}^2 - 4\mathcal{R}^\mu{}_{\nu\rho\sigma} \mathcal{R}^\mu{}_{\nu\rho\sigma} + \mathcal{R}_{\mu\nu\rho} \mathcal{R}_{\mu\nu\rho}$$

- For $D=4$, the GB term is total derivative and thus does not contribute to eom's.
- $D \to 4$ with $\tilde{\alpha} = (D - 4) \alpha$ kept fixed? $0/0 = \text{finite?}$
  - [Glavan&Lin, PRL124, 081301 (2020)]
- Maybe yes, but requires either extra dof. or Lorentz violation due to Lovelock theorem
- The best we can do without extra d.o.f. is to keep $3d$ diffeo $\to$ MMG framework
Hamiltonian of 4D theory with 2 dof

\[ H^{4D}_{EGB} = \int d^3x (N^3\mathcal{H}_0 + N^i \mathcal{H}_i + \lambda^0 \pi_0 + \lambda^i \pi_i + \lambda_{G\mathcal{F}}^3 \mathcal{G}) \]

\[ 3\mathcal{H}_0 = \frac{\sqrt{\gamma}}{2\kappa^2} \left[ 2\Delta - M + \tilde{\alpha} \left( 4M_{ij}M^{ij} - \frac{3}{2} M^2 \right) \right] \]

\[ \mathcal{H}_i = -2\sqrt{\gamma} \gamma_{ik} D_j \left( \frac{\pi^{jk}}{\sqrt{\gamma}} \right) \]

\[ M_{ij} := R_{ij} + \mathcal{K}^k_k \mathcal{K}_{ij} - \mathcal{K}_{ik} \mathcal{K}^k_j \]

\[ \pi^i_j = \frac{\sqrt{\gamma}}{2\kappa^2} \left[ \mathcal{K}^i_j - \mathcal{K} \delta^i_j - \frac{8}{3} \tilde{\alpha} \delta^{ikl}_{jrs} \mathcal{K}^r_k \left( R_i^s - \frac{1}{4} \delta^s_i R + \frac{1}{2} (\mathcal{M}_i^s - \frac{1}{4} \delta^s_i \mathcal{M}) \right) \right] \]

- **1\textsuperscript{st} class x 6**
  \[ \pi_i \approx 0, \quad \mathcal{H}_i \approx 0 \]

- **2\textsuperscript{nd} class x 4**
  \[ \pi_0 \approx 0, \quad 3\mathcal{H}_0 \approx 0, \quad 3\mathcal{G} \approx 0, \quad 3\dot{\mathcal{G}} \approx 0 \]

- **10x2 – 6x2 – 4 = 4 \Rightarrow 2 \text{ dof}**
5 properties of 4D theory

4D theory is unique up to a choice of $3\mathcal{G}$.

i. 3D spatial diffeo invariance is respected

ii. # of dof = 2

iii. Reduces to GR when $\tilde{\alpha} = 0$

iv. Correction terms are 4th-order in derivatives

v. If the Weyl tensor of the spatial metric and the Weyl part of $K_{ik}K_{jl} - K_{il}K_{jk}$ vanish for a solution of (d+1)-dim EGB, then the $d \rightarrow 3$ limit of the solution satisfies eoms of 4D theory.

A consistent theory of $D \rightarrow 4$ EGB gravity
Lagrangian of 4D theory with 2 dof

\[ \mathcal{L}_{4D}^{\text{EGB}} = \frac{1}{2\kappa^2} \left( -2\Lambda + \mathcal{K}_{ij}\mathcal{K}^{ij} - \mathcal{K}_i^i\mathcal{K}_j^j + R + \tilde{\alpha}R_{4\text{DGB}}^2 \right) \]
\[ R_{4\text{DGB}}^2 = -\frac{4}{3} \left( 8R_{ij}R^{ij} - 4R_{ij}\mathcal{M}^{ij} - \mathcal{M}_{ij}\mathcal{M}^{ij} \right) + \frac{1}{2} \left( 8R^2 - 4RM - \mathcal{M}^2 \right) \]
\[ \mathcal{K}_{ij} = K_{ij} - \frac{1}{2N}\gamma_{ij}D^2\lambda_{\text{GF}} \]
\[ \mathcal{M}_{ij} := R_{ij} + K_k^k\mathcal{K}_{ij} - K_{ik}K_{ij}^k \]

- Valid for specific choice: \[ ^3G = \sqrt{\gamma}D_kD^k(\pi^{ij}\gamma_{ij}/\sqrt{\gamma}) \]
  compatible with cosmology & static sol
- d→3 limit of any solutions of (d+1)-dim EGB with conformally flat spatial metric and vanishing Weyl part of \( K_{ik}K_{ji} - K_{il}K_{jk} \) are solutions (e.g. FLRW & spherical sol of Glavan&Lin)
Constraints

• Stability of scalar perturbation
  \[ \dot{H} < 0 \]

• Stability of tensor perturbation
  \[ \tilde{\alpha} > 0 \]

• Propagation of gravitational waves
  \[ \tilde{\alpha} \lesssim \mathcal{O}(1) \text{ eV}^{-2} \]

• Properties of neutron stars
  \[ \tilde{\alpha} \lesssim \mathcal{O}(1) \text{ eV}^{-2} \]
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Minimalism in modified gravity

- Minimal # of d.o.f. in modified gravity = 2 can be saturated $\rightarrow$ **minimally modified gravity (MMG)**

- **Type-I MMG**: $\exists$ Einstein frame
  - Type-II MMG: no Einstein frame

- Examples of type-I MMG
  - GR + canonical tr. + gauge-fixing + adding matter
  - Rich phenomenology: $w_{DE}$, $G_{\text{eff}}$, etc.
  - $f(H)$ theory can fit Planck data better than $\Lambda$CDM

- An example of type-II MMG
  - Minimal theory of massive gravity (MTMG)

- Another example of type-II MMG: cuscuton/VCDM
  - GR + canonical tr. + cc + gauge-fixing + inverse canonical tr.
  - $V(\phi)$ reconstructed from FLRW background
  - May reduce $H_0$ tension
  - Extension (VCCDM) may address both $H_0$&$S_8$ tensions
D→4 Einstein Gauss-Bonnet gravity

• We proposed a consistent theory of D→4 EGB gravity with 2 dofs in the framework of type-II MMG.

• Under a set of reasonable assumptions (i)-(v), the consistent theory is unique up to a choice of a constraint that stems from a temporal gauge condition.

• D→4 limit of any solutions of D-dim EGB with conformally flat spatial metric and vanishing Weyl part of $K_{ik}K_{ji} - K_{il}K_{jk}$ are solutions

• Interesting phenomenology such as the $k^4$ term in the dispersion relation of GWs

• Constraints: $\dot{H} < 0$, $\tilde{\alpha} > 0$, $\tilde{\alpha} \lesssim \mathcal{O}(1) \text{ eV}^{-2}$

• This is the unique theory (up to a choice of a constraint)
Thank you!