Gwmess2021
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Primordial Black Holes and Cosmological Gravitational Waves

— a couple of recent topics —

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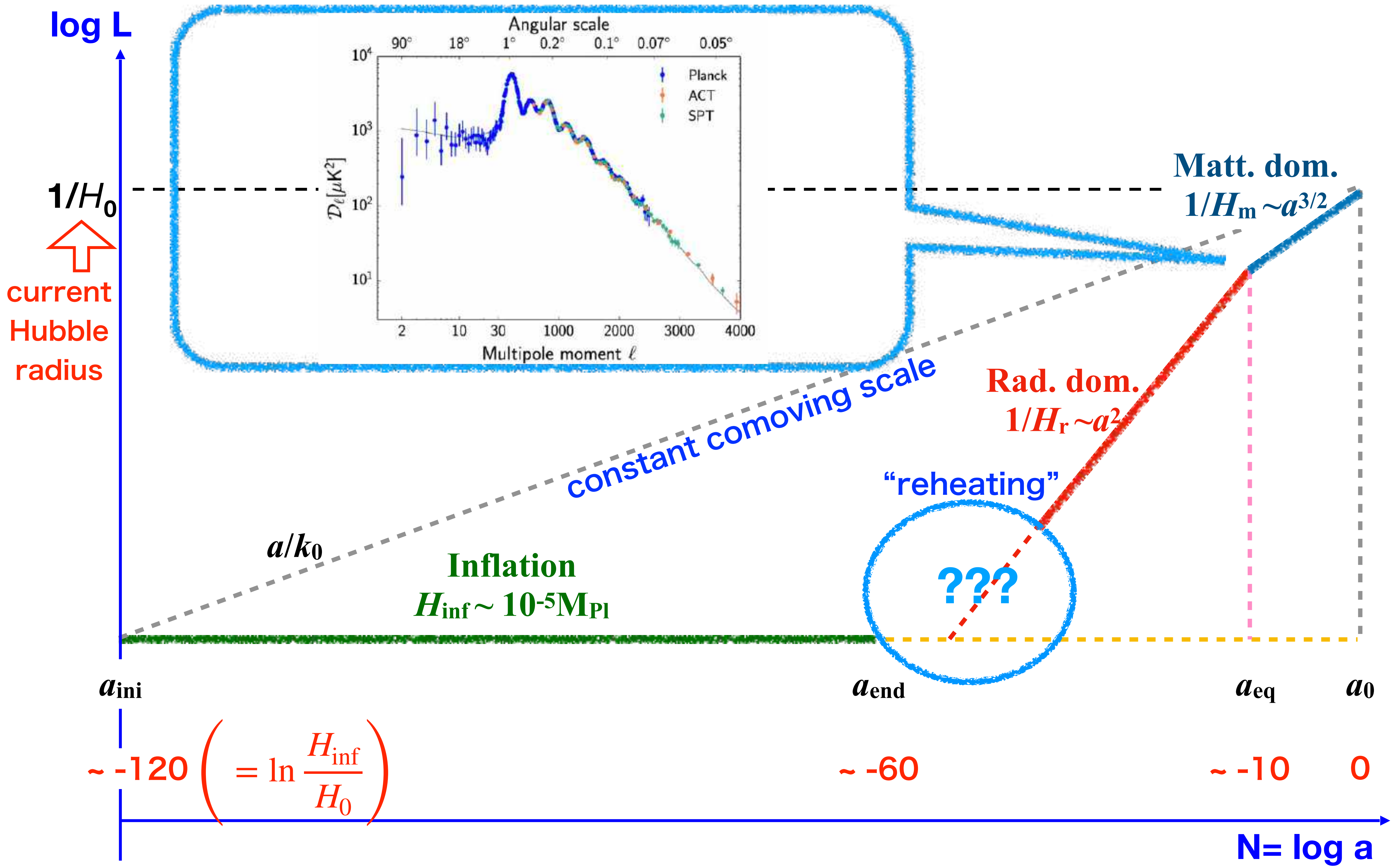
Introduction

curvature perturbation,

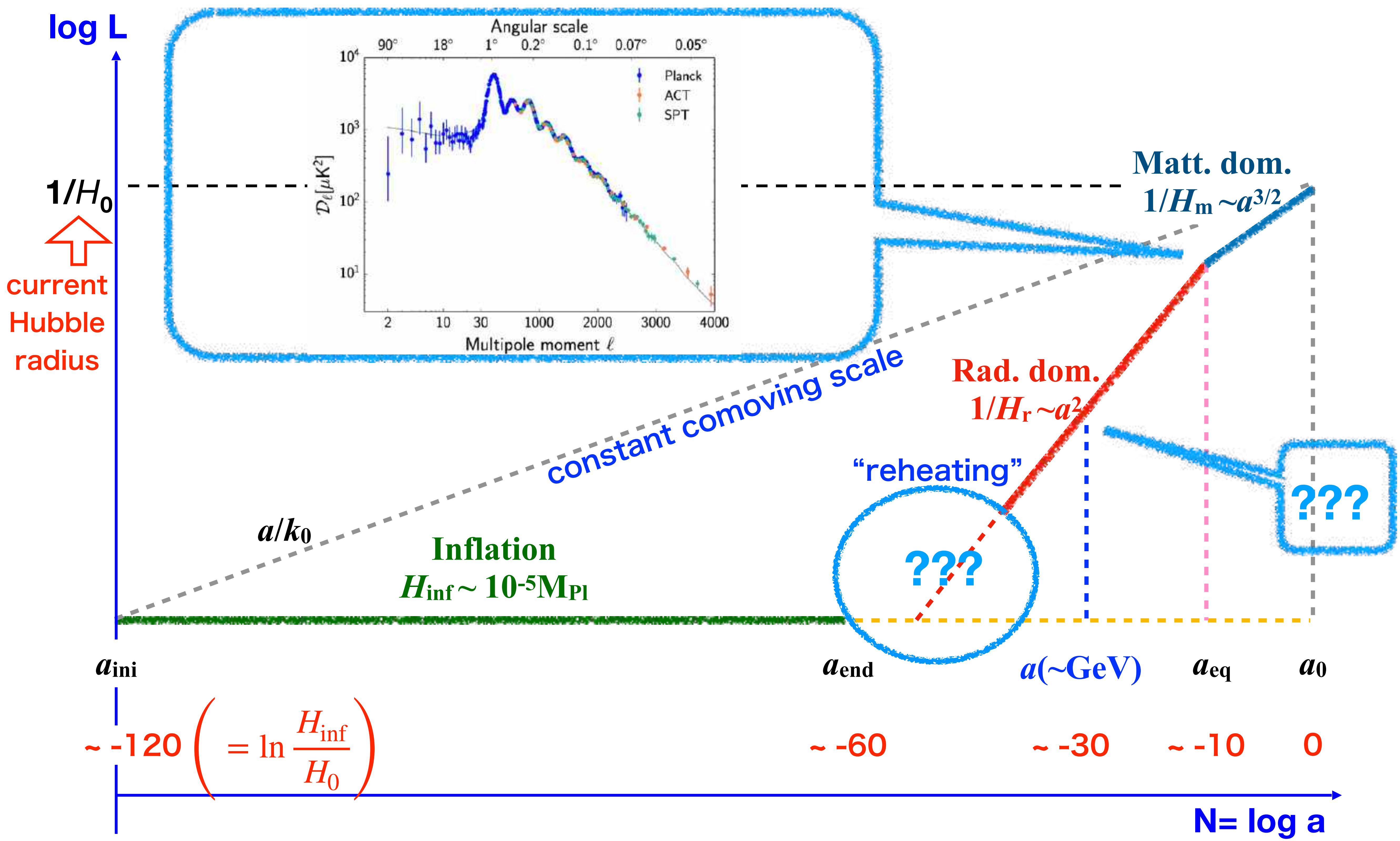
formation of PBHs,

and gravitational waves

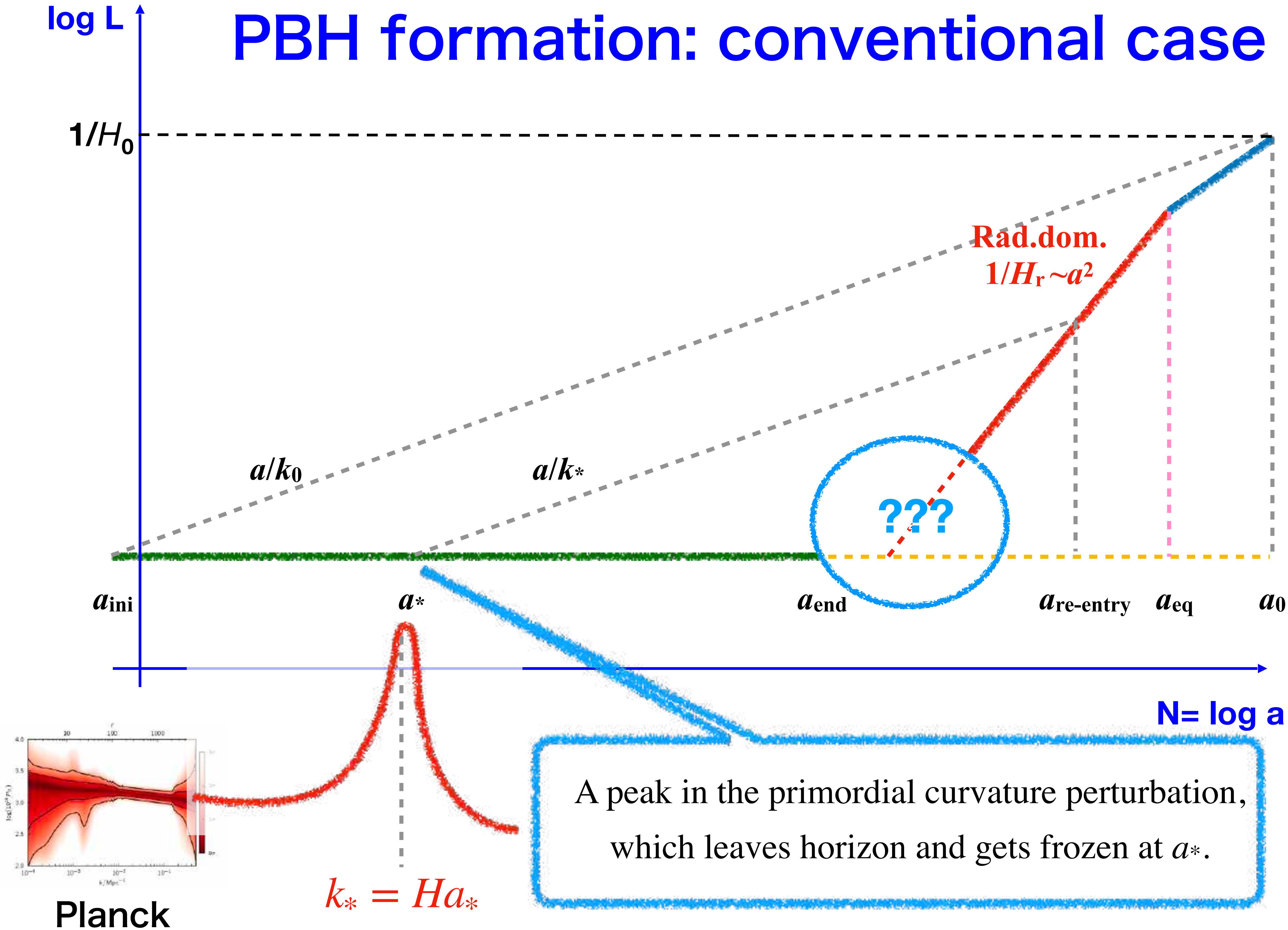
cosmic spacetime diagram



cosmic spacetime diagram



PBH formation: conventional case



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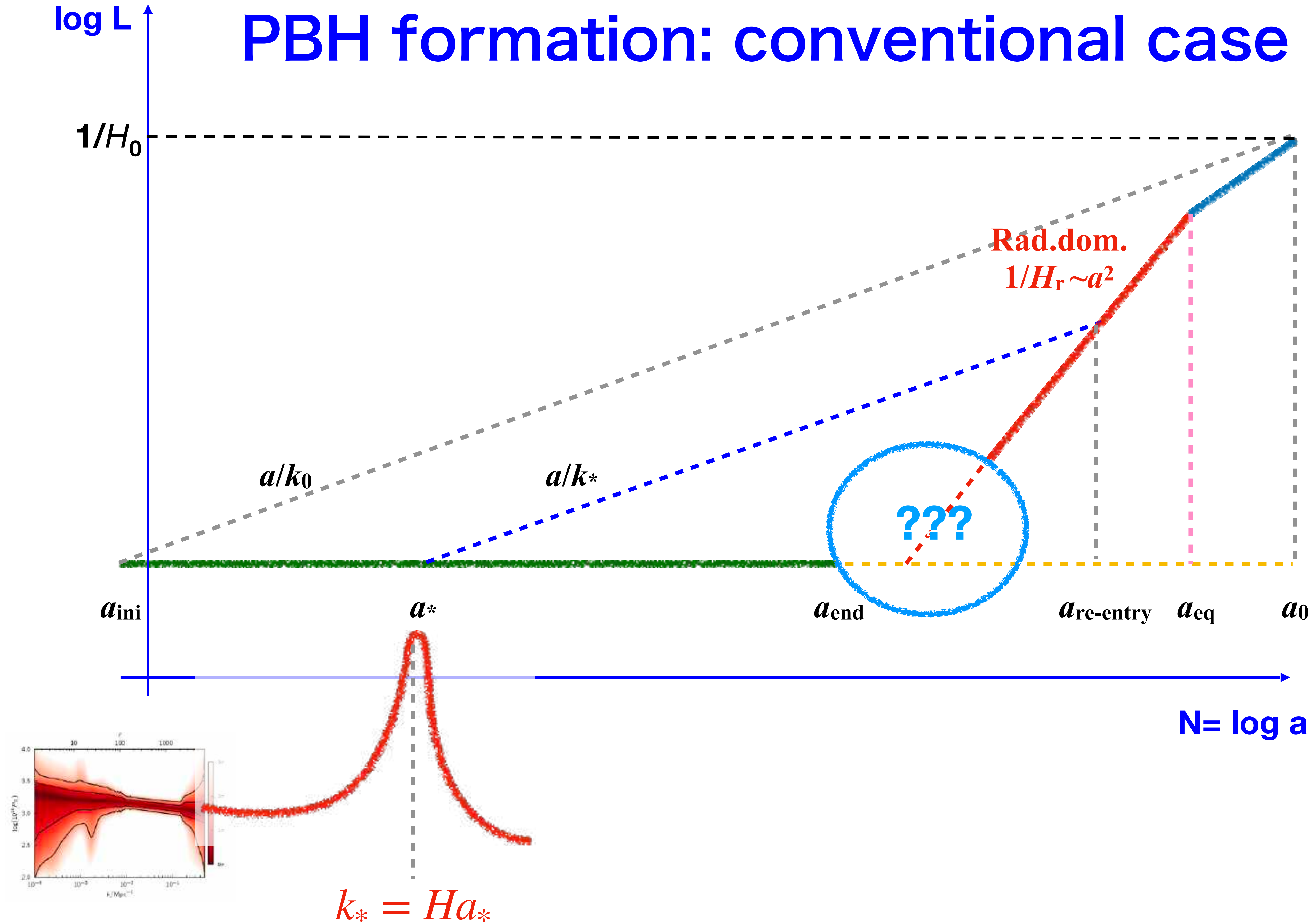
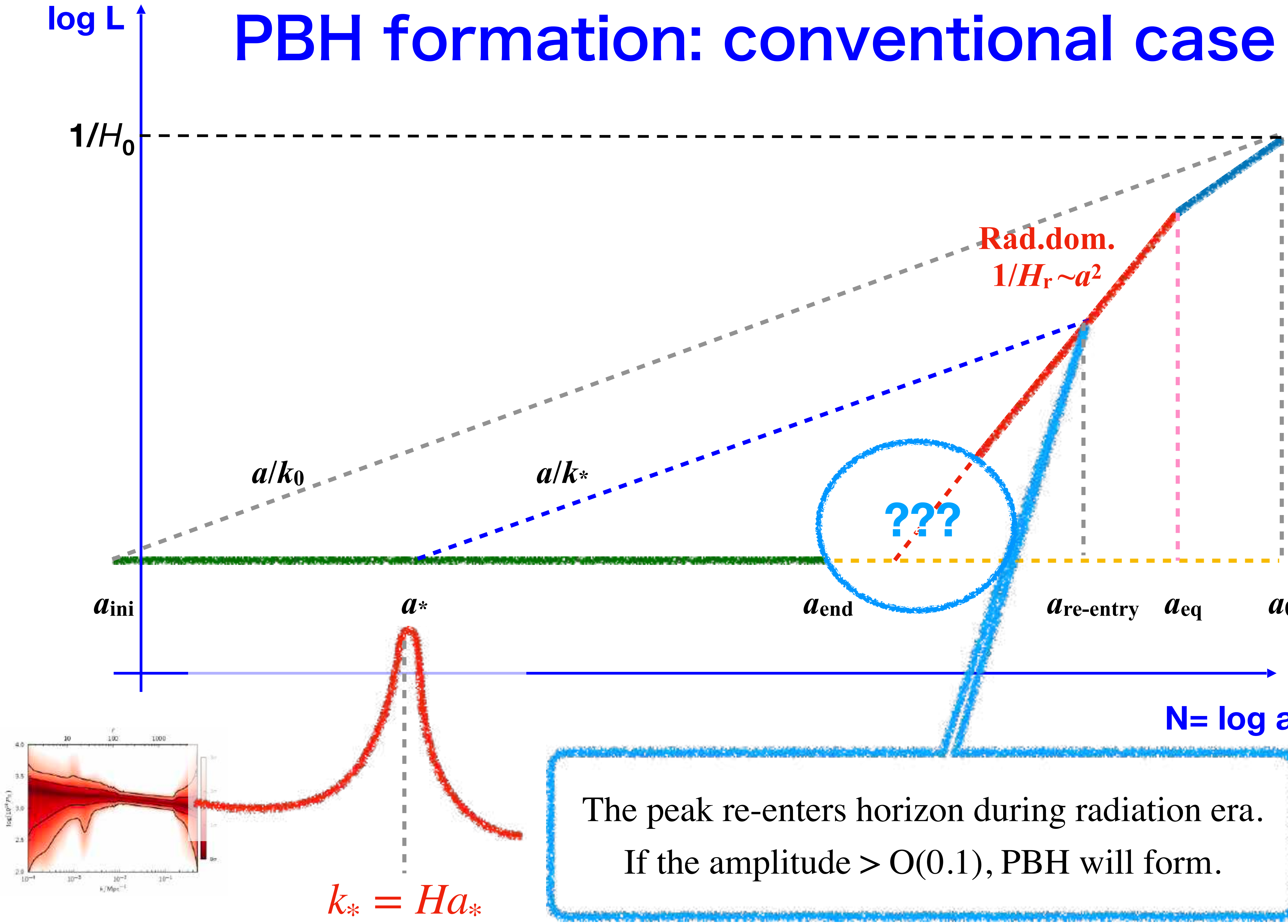


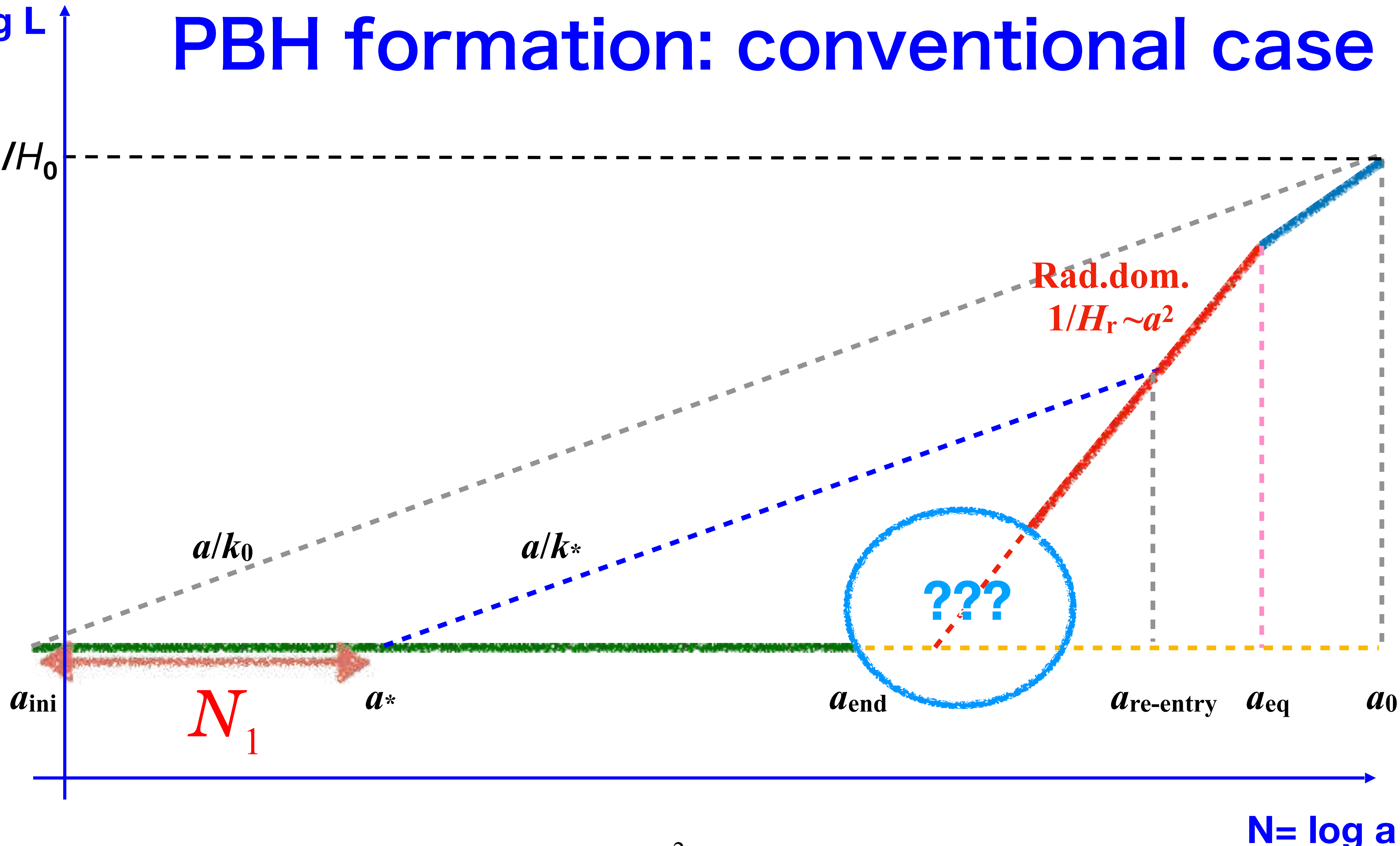
PBH formation: conventional case



The peak re-enters horizon during radiation era.

If the amplitude > O(0.1), PBH will form.

PBH formation: conventional case



PBH mass: $M_{PBH} = \gamma M_H \sim \dfrac{M_{Pl}^2}{H} = 10^{58} M_{Pl} e^{-2N_1} = M_{Pl} 10^{58-0.87N_1}$

Inverse relation: $N_1 = 44.4 - \dfrac{1}{2}\ln\left(\dfrac{M_{PBH}}{10^{16}\,\mathrm{g}}\right)$

PBH mass scale does NOT depend on the reheating physics

Curvature perturbation to PBH

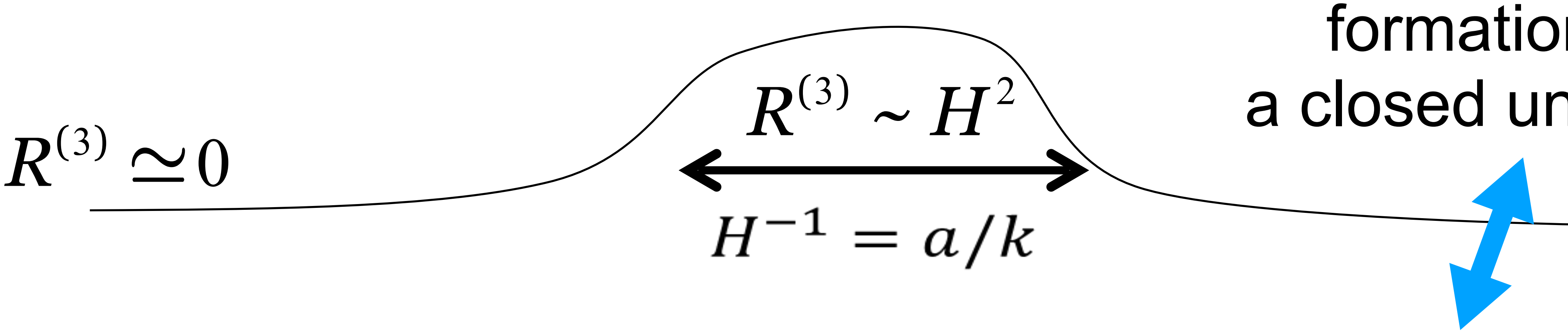
conventional (PBH formation at rad-dominance) case

➢ gradient expansion/separate universe approach

$$6H^2(t,x) + R^{(3)}(t,x) = 16\pi G\rho(t,x) + \cdots$$

Hamiltonian constraint (Friedmann eq.)

$$\Rightarrow \quad R^{(3)} \approx -\frac{4}{a^2}\nabla^2\mathscr{R}_c \approx \frac{8\pi G}{3}\delta\rho_c \quad \Rightarrow \quad \frac{\delta\rho_c}{\rho} \approx \mathscr{R}_c \text{ at } \frac{k^2}{a^2} = H^2$$



formation of a closed universe

➢ If $R^{(3)} \sim H^2$ $\left(\Leftrightarrow \delta\rho_c/\rho \sim 1\right)$, it collapses to form BH

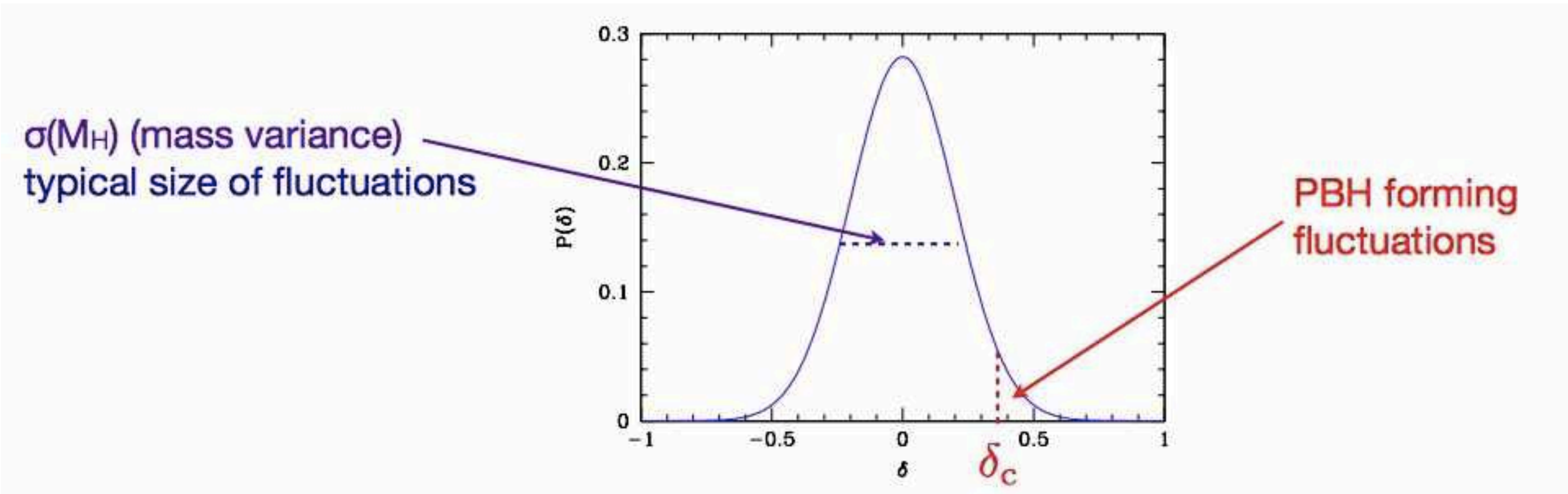
Young, Byrnes & MS 1405.7023, …

➢ Spins of PBHs are expected to be very small

De Luca et al. 1903.01179, …

fraction β that turns into PBHs

for Gaussian probability distribution



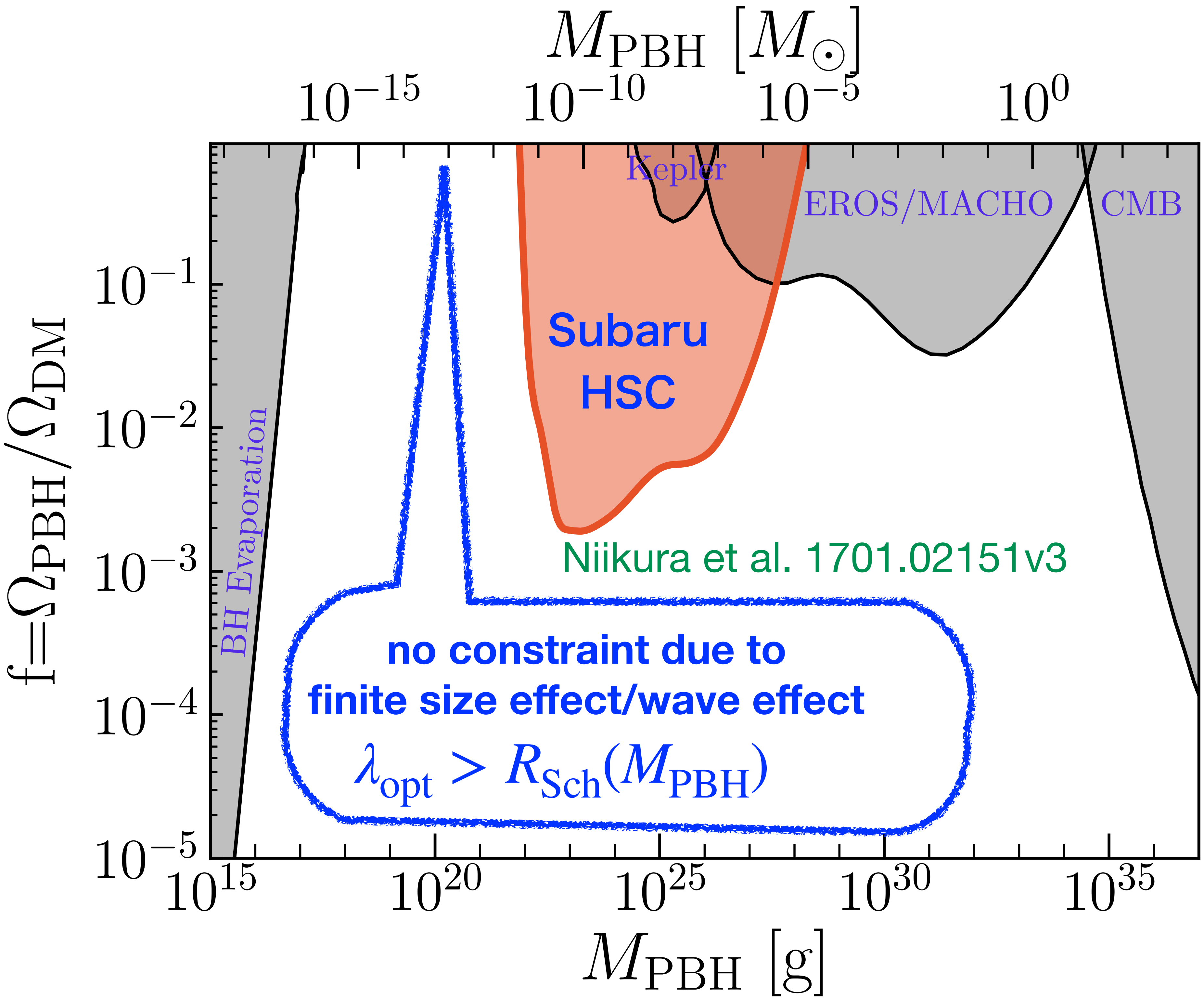
- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

$$\delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

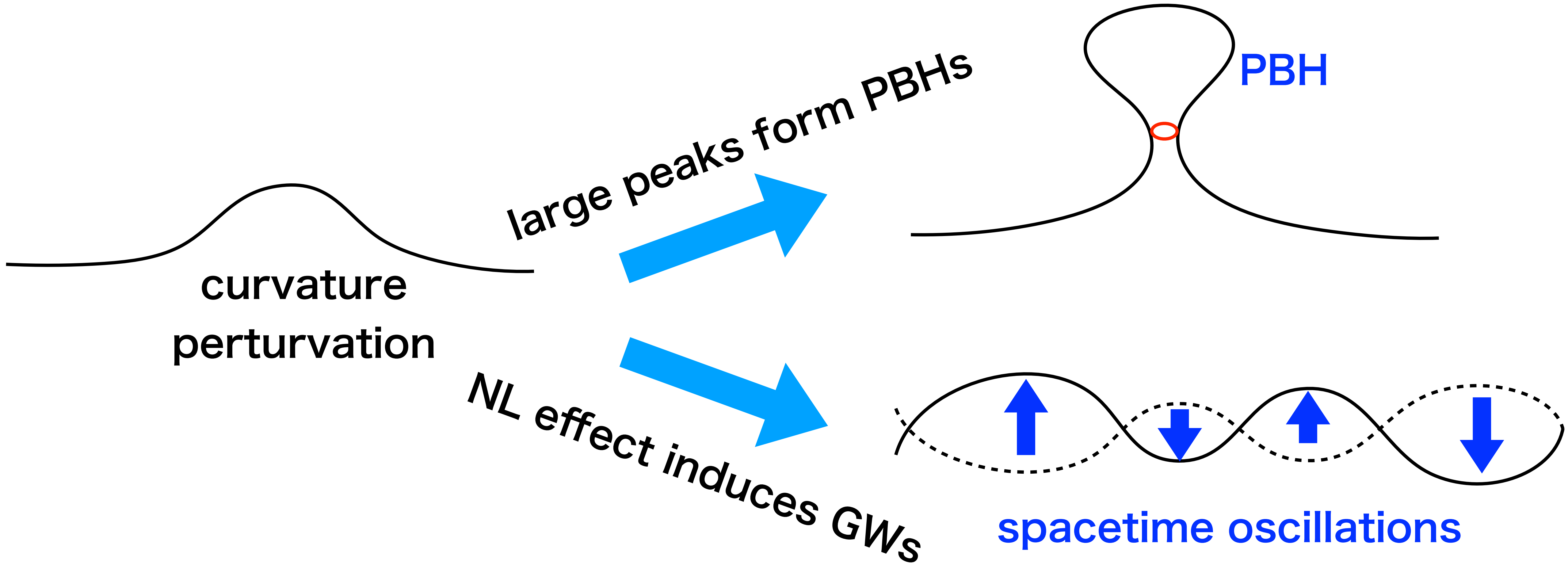
Carr '75, …

PBH constaints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$ g ⟷ $T_{\text{re-entry}} \sim 10^4$ - 10^8 GeV

GWs can capture PBHs!

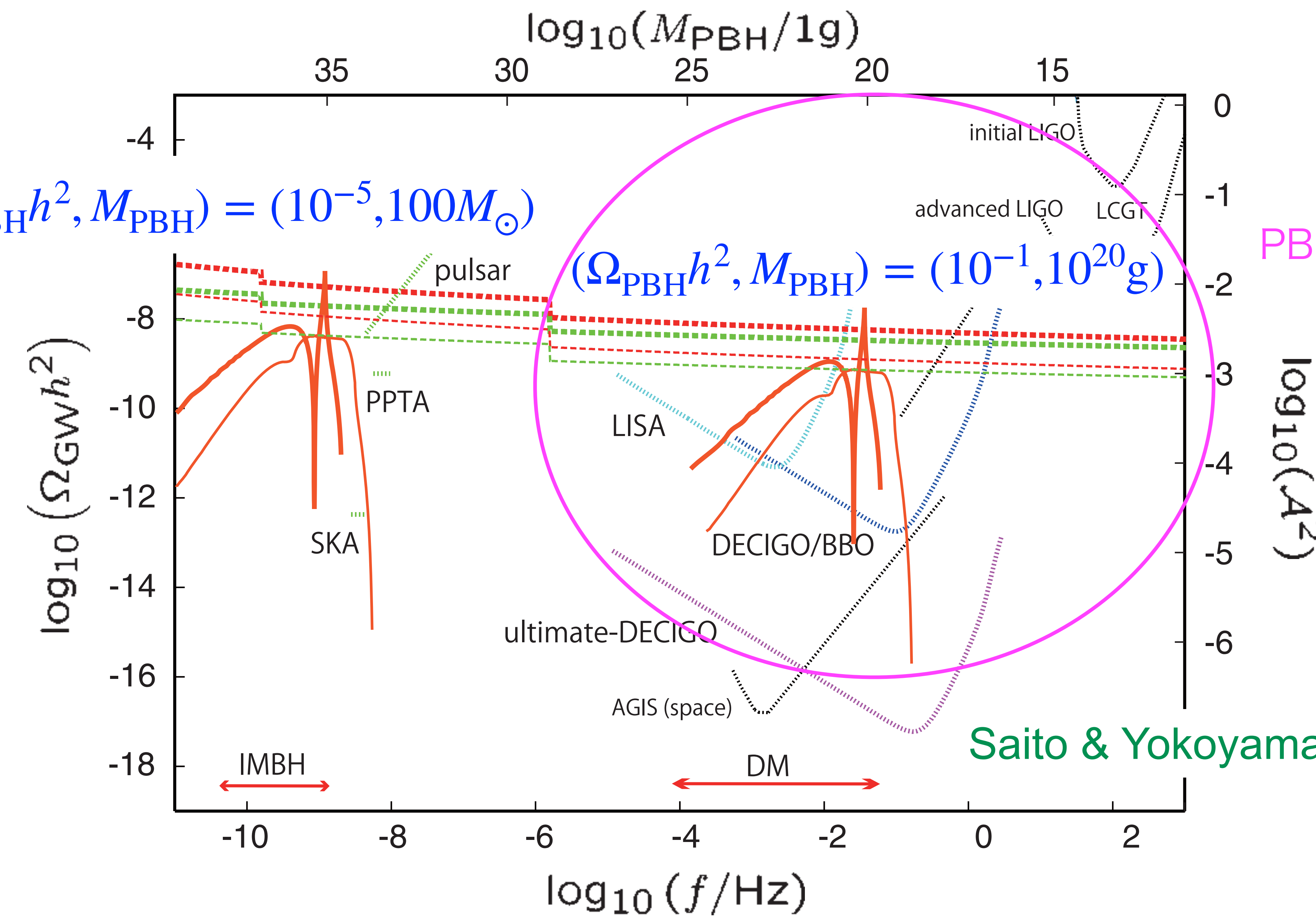


PBHs = CDM with $M_{PBH} \sim 10^{21}$g
generates GWs with f~10^{-3} Hz

⇨ Background GWs at LISA band

⇔ LIGO-Virgo :10 - 1000 Hz

GWs test PBH=DM!



$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-5}, 100M_\odot)$

$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-1}, 10^{20}{\rm g})$

PBHs = CDM?

Saito & Yokoyama 0812.4339

$$M_{\rm PBH} \sim 0.1 M_\odot \left(\frac{1{\rm GeV}}{T}\right)^2 \sim 10 M_\odot \left(\frac{1{\rm pc}^{-1}}{k}\right)^2$$

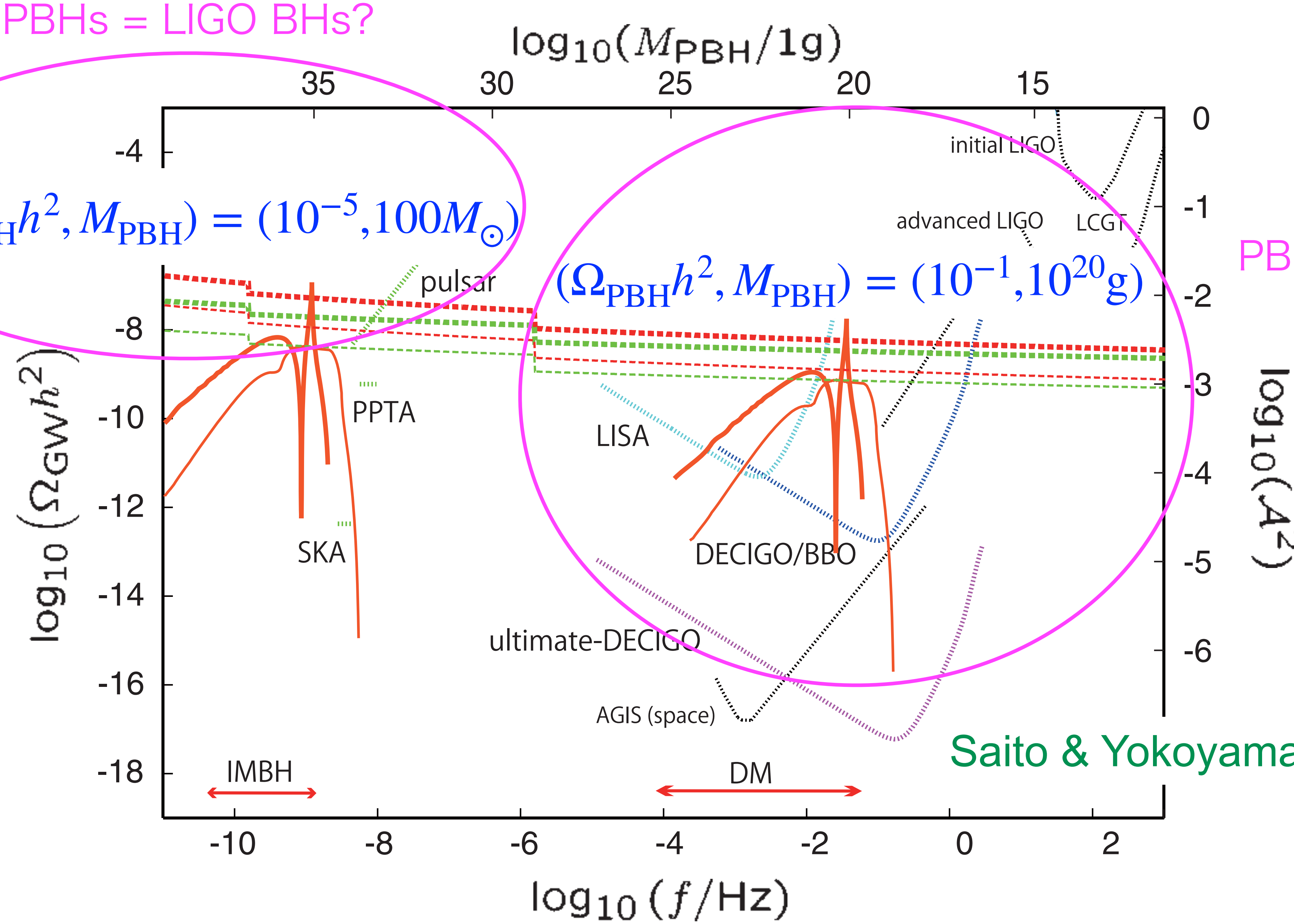
GWs test PBH=DM!

PBHs = LIGO BHs?

$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-5}, 100M_\odot)$

PBHs = CDM?

$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-1}, 10^{20}{\rm g})$



Saito & Yokoyama 0812.4339

$$M_{\rm PBH} \sim 0.1M_\odot \left(\frac{1{\rm GeV}}{T}\right)^2 \sim 10M_\odot \left(\frac{1{\rm pc}^{-1}}{k}\right)^2$$

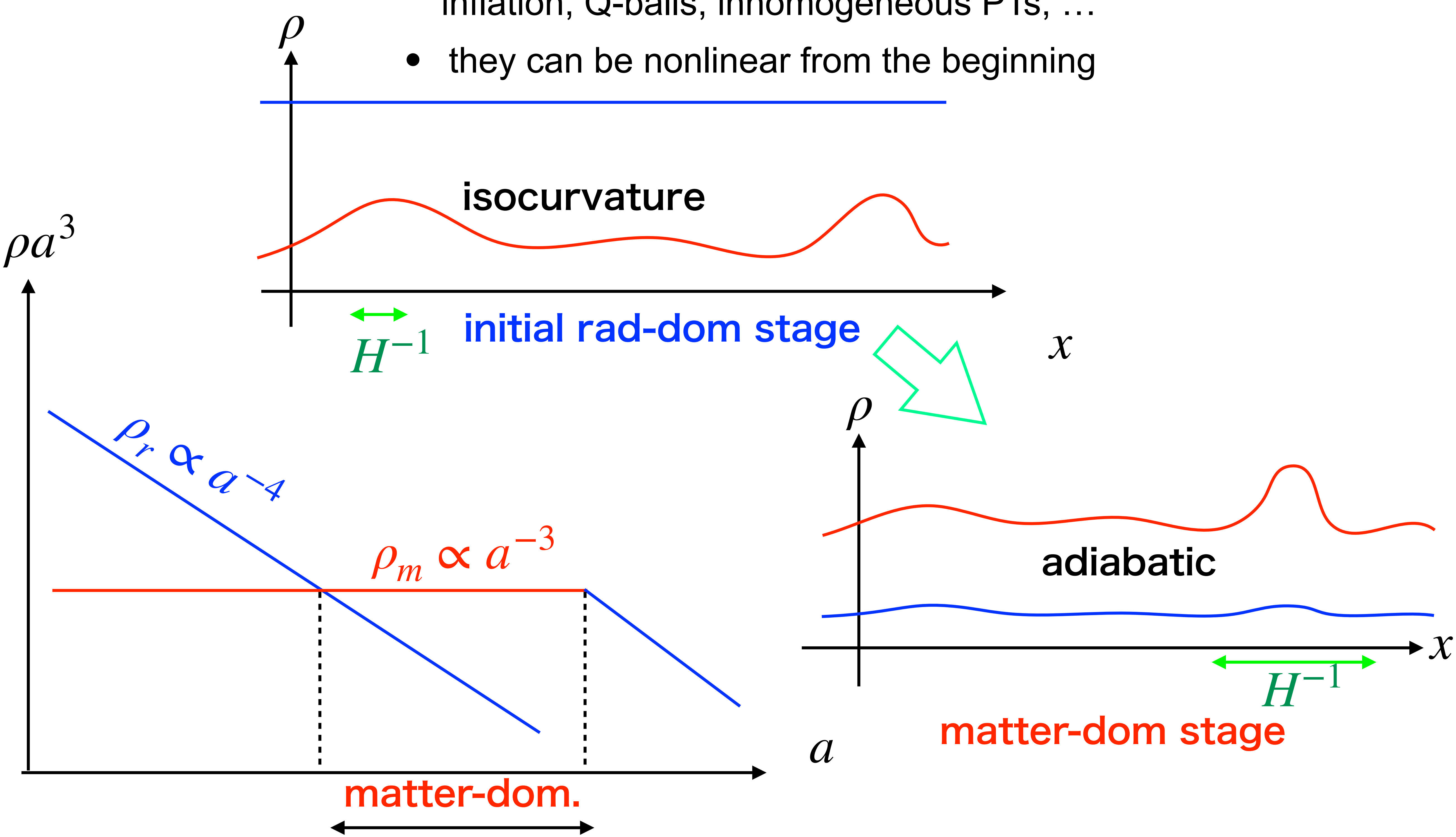
So far, most discussions have been based on primordially curvature perturbation

How about primordially isocurvature perturbations?

PBHs from Isocurvature Perterbation

eg, E. Cotner, A. Kusenko, MS & V. Takhistov,1907.10613

- isocurvature pert. may be generated from 2-field inflation, Q-balls, inhomogeneous PTs, …
- they can be nonlinear from the beginning



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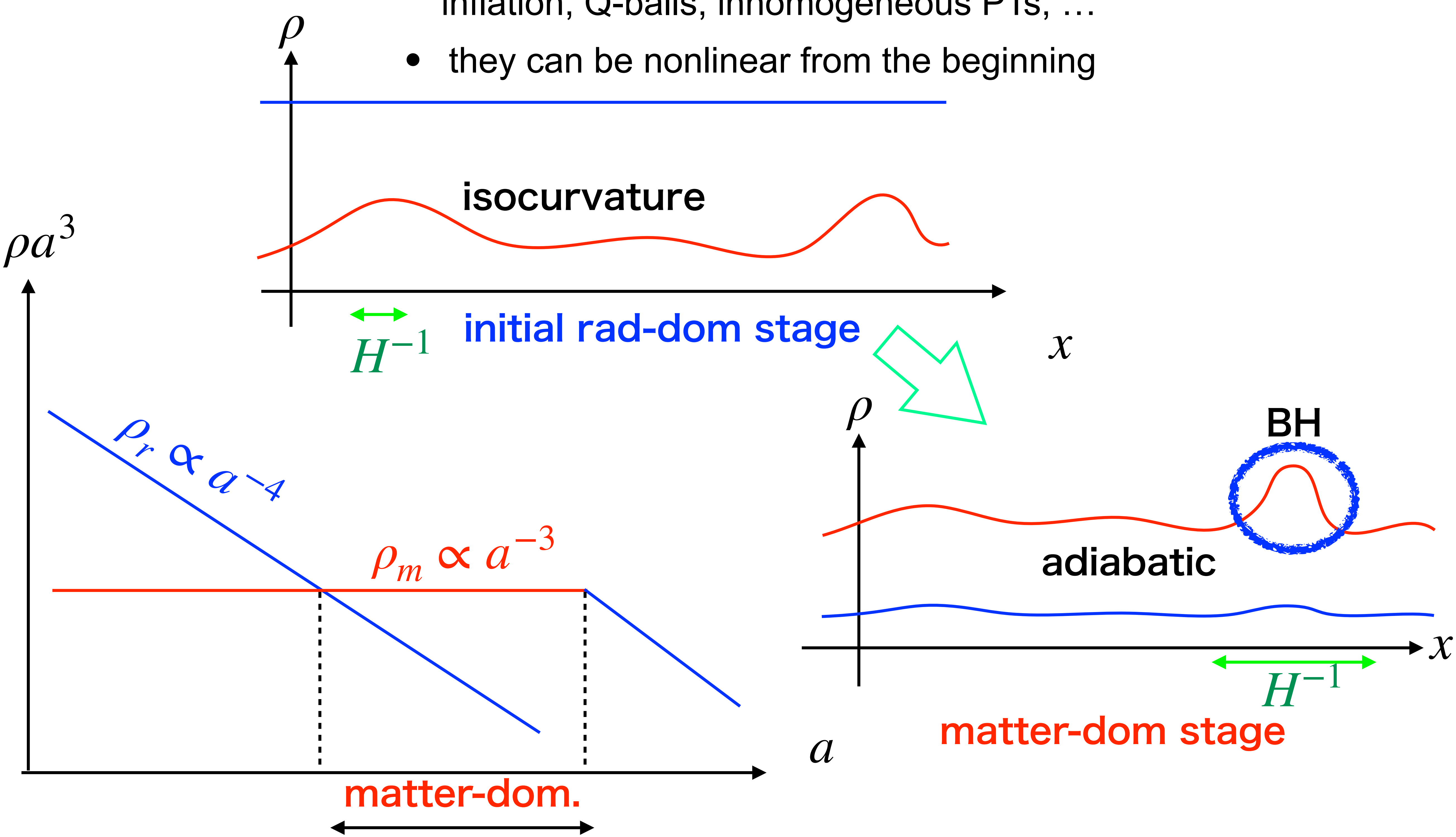

linear theory

H. Kodama & MS, IJMPA 1 (1986) 265, ibid 2 (1987) 491

matter isocurvature perturbation

$$S \equiv \delta_m - \frac{3}{4}\delta_r \to \delta_m \text{ at } a \to 0$$ (on, say, comoving slices)

evolution for $\omega \ll 1$ $\quad \omega \equiv \left(\frac{k}{Ha}\right)_{eq}$, $\quad R \equiv \frac{a}{a_{eq}}$ modes that are superhorizon at equality

$R \ll 1$

$$\begin{cases} \mathscr{R}_c = \frac{R}{4} S \quad \left(\Phi = \frac{R}{8} S\right) \\ \delta = \frac{1}{6}\omega^2 R^3 S \end{cases}$$

⇨

$1 \ll R$

$$\begin{cases} \mathscr{R}_c = \frac{1}{3} S \quad \left(\Phi = \frac{1}{5} S\right) \\ \delta = \frac{4}{15}\omega^2 R\, S \end{cases}$$

$\mathscr{R}_c$:curv pert on comoving slice

Φ : curv pert on Newton slice

horizon crossing: $\omega^2 R = \frac{1}{2}$

formation criterion: $\delta(k = aH) = \frac{2}{15} S > \delta_{\rm cr}$ **?**

linear theory continued

evolution for $\omega \gg 1$ (modes that enters horizon before equality)

$$\omega \equiv \left(\frac{k}{Ha}\right)_{eq}, \quad R \equiv a/a_{eq}$$

$R \ll \omega^{-1}$

$$\begin{cases} \mathscr{R}_c = \dfrac{R}{4} S \\ \delta = \dfrac{1}{6}\omega^2 R^3 S \end{cases}$$

⇨

$\omega^{-1} \ll R \ll 1$

$$\begin{cases} \mathscr{R}_c = \dfrac{3}{4\omega^2 R} S \\ \delta = R\, S \end{cases}$$

⇨

$1 \ll R$

$$\begin{cases} \mathscr{R}_c = \dfrac{5}{4\omega^2} S \\ \delta = \dfrac{3R}{2} S \end{cases}$$

$$\left(\Phi = \frac{3}{4\omega^2} S\right)$$

conventional growth rate at matter-dom stage

horizon crossing: $\omega R = 1/2$

$$\delta(k = aH) = \frac{1}{2\omega} S, \quad \mathscr{R}_c = \frac{3}{2\omega} S$$

linear theory continued

evolution for $\omega \gg 1$ (modes that enters horizon before equality)

$$\omega \equiv \left(\frac{k}{Ha}\right)_{eq} , \quad R \equiv a/a_{eq}$$

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horizon crossing: $\omega R = 1/2$

$$\delta(k = aH) = \frac{1}{2\omega} S , \quad \mathscr{R}_c = \frac{3}{2\omega} S$$

$\Phi = O(1)$ implies $S = O(\omega^2) \gg 1$!!

highly nonlinear initial condition

need more studies!

Isocurvature Perturbation due to inhomogeneous PBH distribution

PBH dominated early universe and GWs

G. Domenech, C. Lin & MS, 2012.08151

- PBHs may dominate the universe -> Early PBH Dominated (PBHD) Universe
- PBHs eventually evaporate to make the universe Radiation Dominated (RD)

Can we test this scenario?

Induced GWs from the curvature perturbation that produced PBHs?

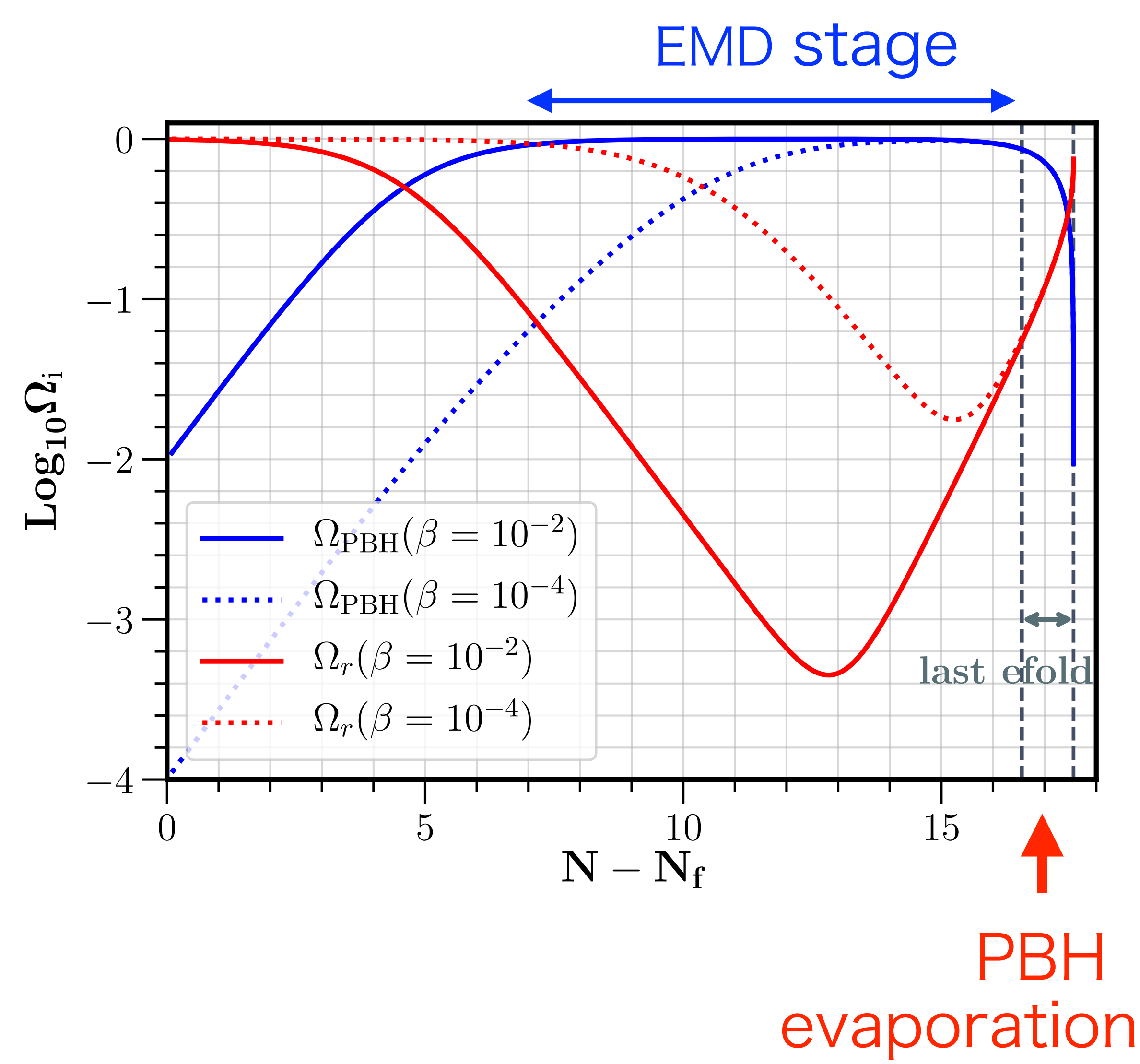
Evaporation must occur before BBN

$$T_{\rm reheat} \sim 30\,{\rm MeV}\left(\frac{M_{\rm PBH}}{10^8\,{\rm g}}\right)^{-3/2} \Rightarrow M_{\rm PBH} \lesssim 10^8\,{\rm g}$$

$$\Rightarrow f_{\rm peak} \sim 3\times 10^4\,{\rm Hz}\left(\frac{M_{\rm PBH}}{10^8\,{\rm g}}\right)^{-1/2}$$

Frequency seems too high . . .

Any other means?



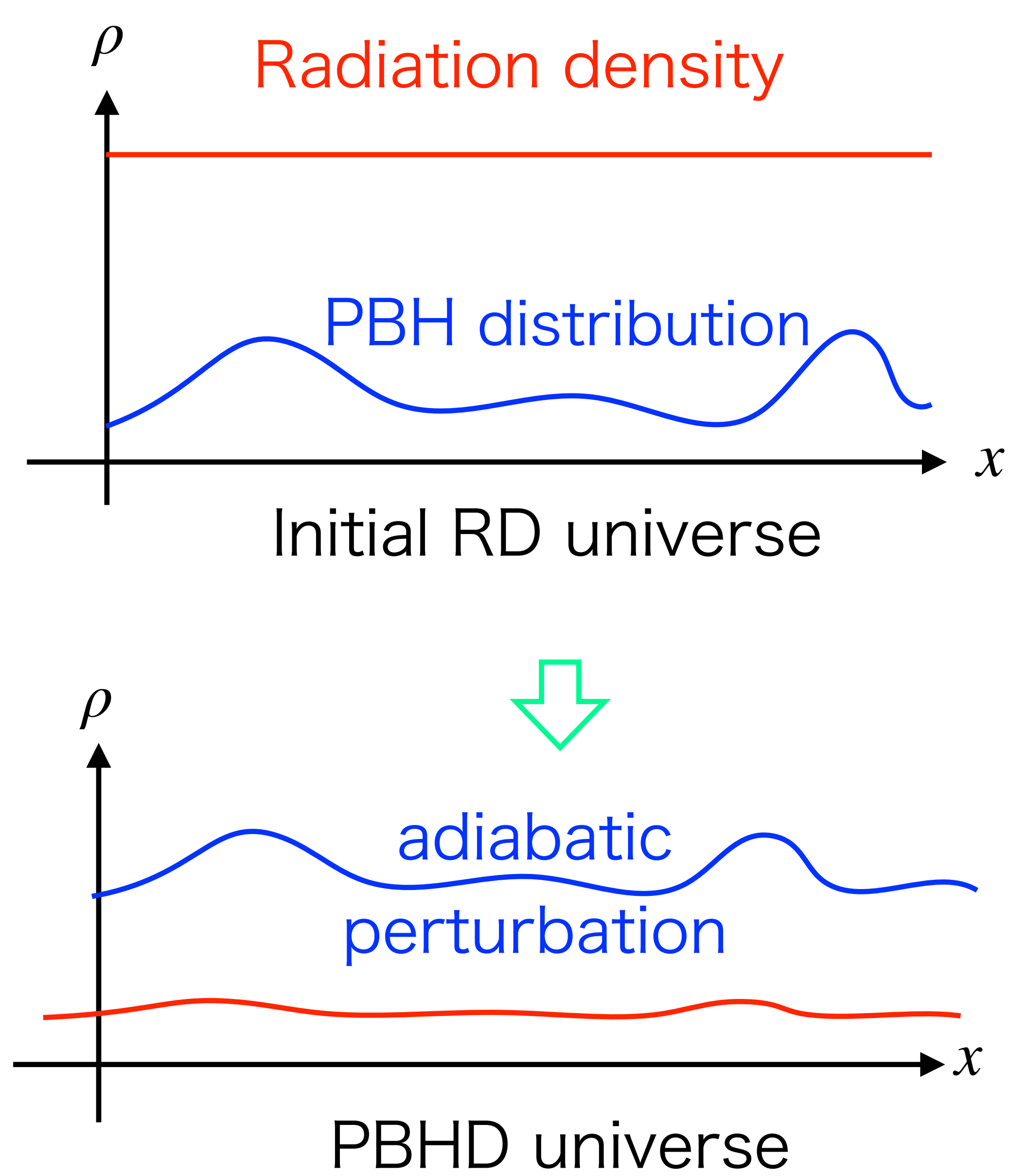
Induced GWs from inhomogeneous PBH distribution

T. Papanikolaou et al., 2010.11573

- PBHs form from rare peaks of the curvature perturbation.
- PBH distribution will be spatially inhomogeneous.
- PBH distribution is a primordially isocurvature perturbation.
- Isocurvature perturbation turns into adiabatic perturbation at PBHD stage.
- An instantaneous reheating to RD leads to strong enhancement of induced GWs on sub-horizon scales, which is the case for PBH evaporation.

K Inomata et al., 2003.10455

→ **may lead to strong constraints on early PBH dominance model**

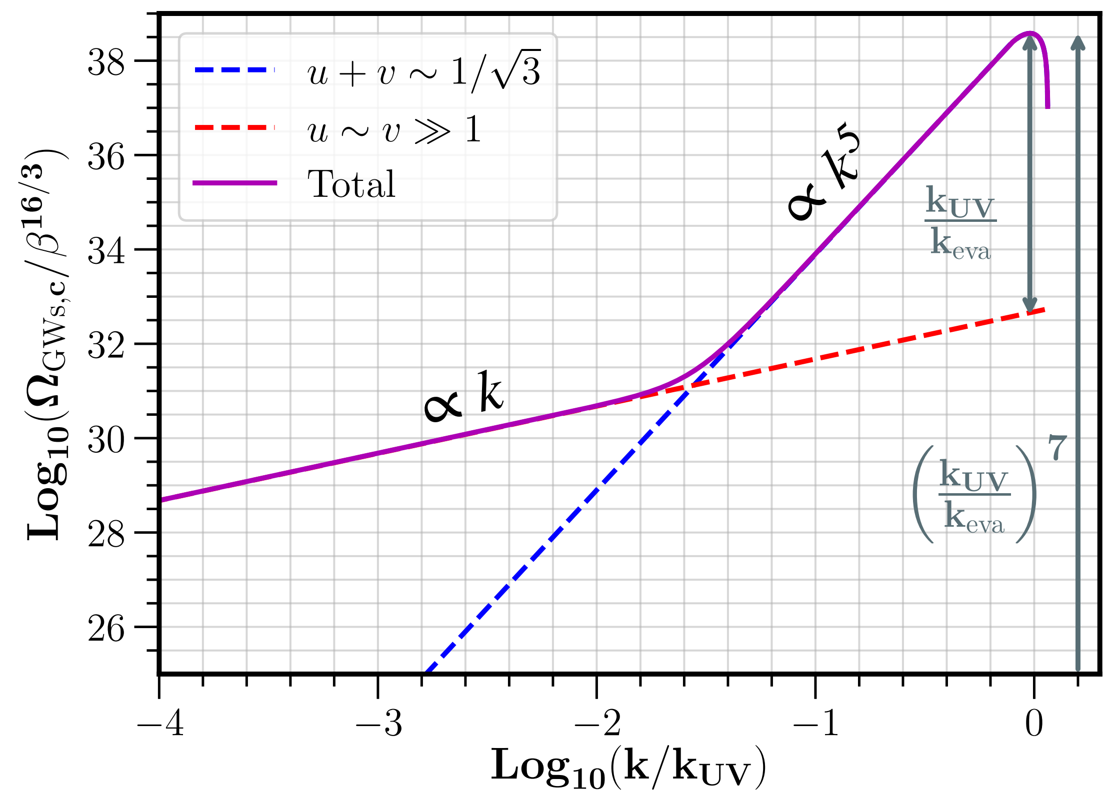


Constraints on early PBH dominated universe

Domenech, Lin & MS, 2012.08151

- **Assumptions**
  - Monochromatic mass function for PBHs.
  - Poisson distribution for $\delta n_{\rm PBH}/n_{\rm PBH}$:

$$\mathcal{P}_S(k) = \frac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$$

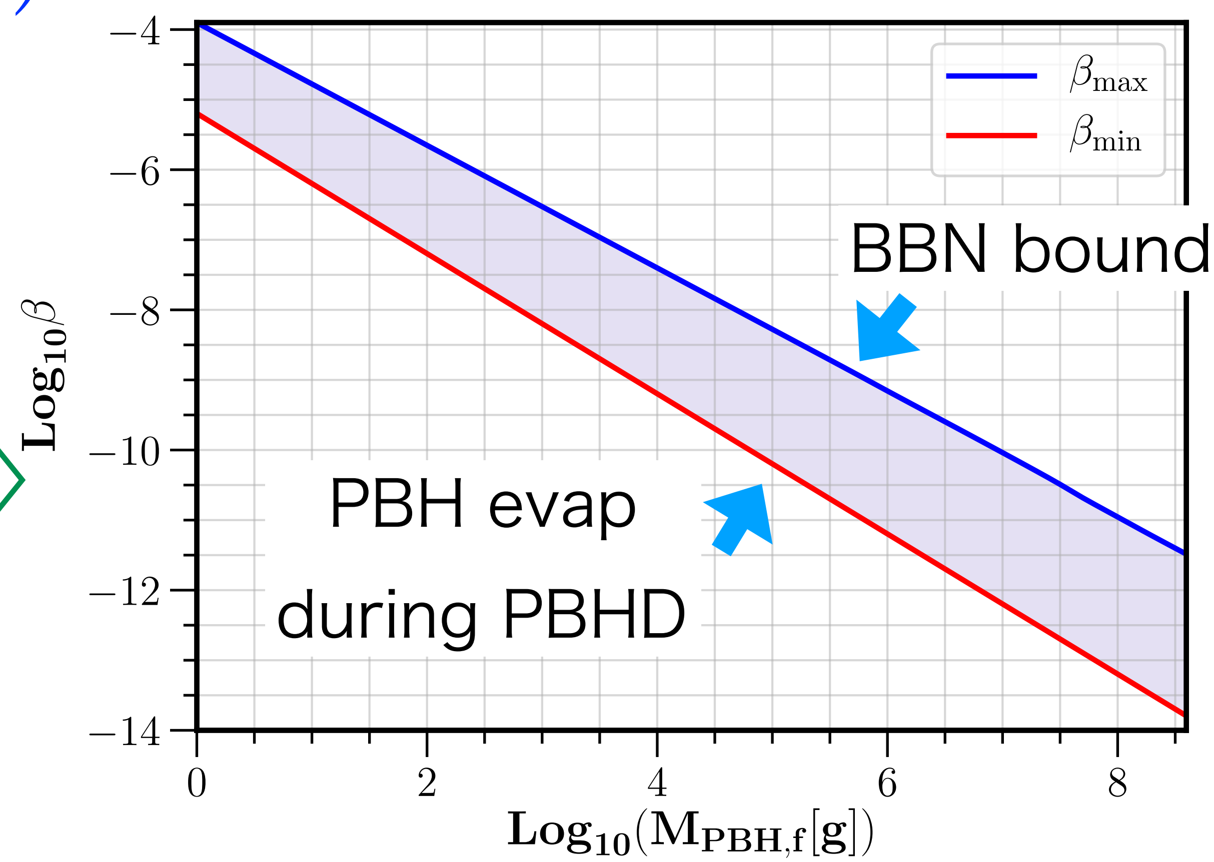


- **Peak GW amplitude**

$$\Omega_{GW,max}/\Omega_{r,0} \approx 5\times 10^{34}\,\beta^{16/3}\left(\frac{M}{10^4\,\mathrm{g}}\right)^{14/3}$$

β : PBH fraction at formation

allowable range of β

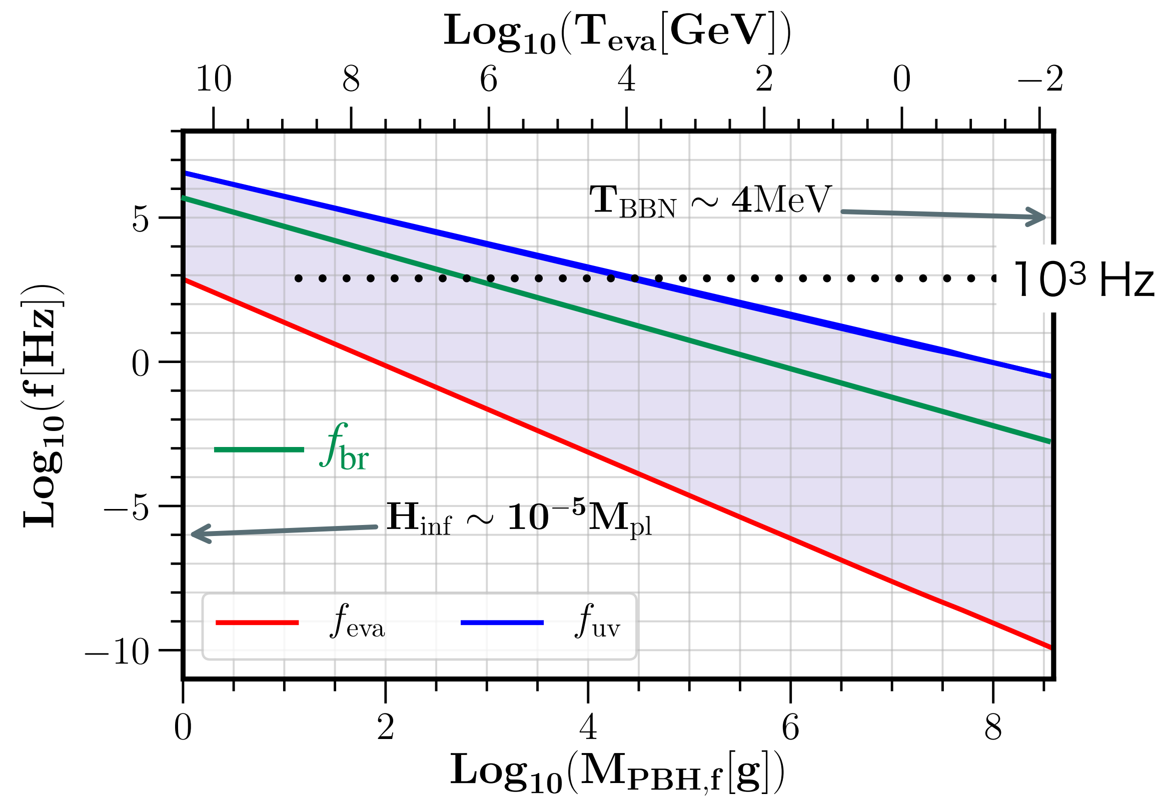


- **Frequency range**

Peak freq: $f_{\rm UV} \approx 2\times 10^3\,{\rm Hz}\left(\frac{M_{\rm PBH}}{10^4\,{\rm g}}\right)^{-5/6}$

Break freq: $f_{\rm br} \approx 70\,{\rm Hz}\left(\frac{M_{\rm PBH}}{10^4\,{\rm g}}\right)^{-1}$

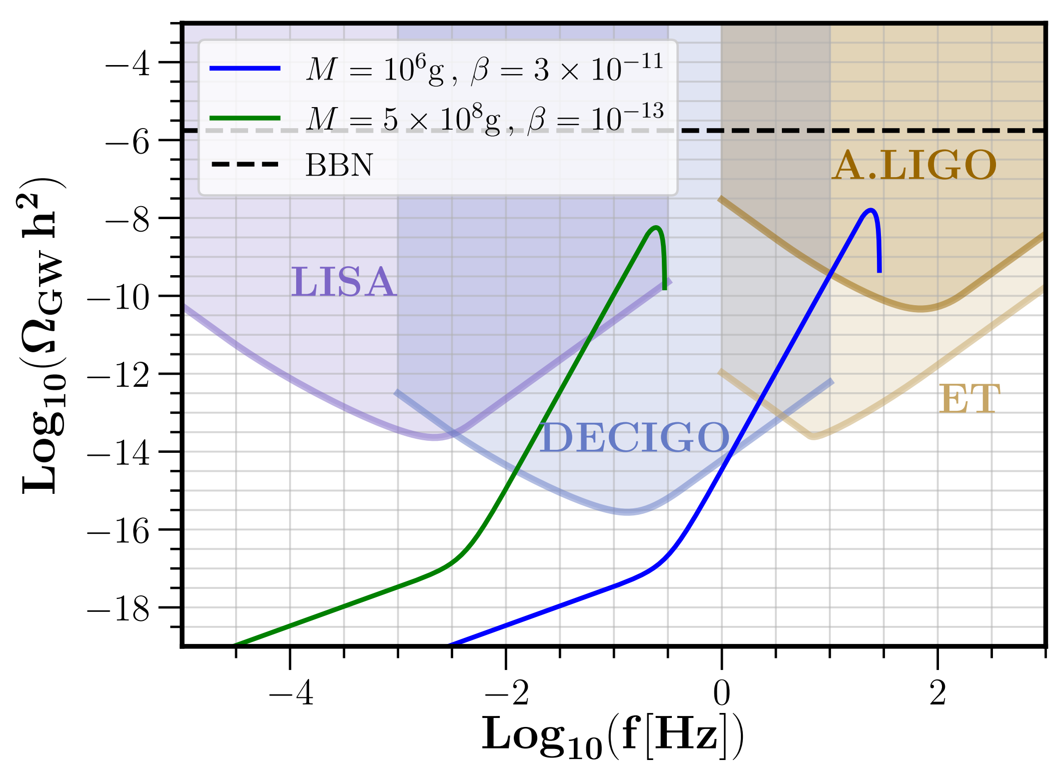
Min freq: $f_{\rm evap} \approx 5\times 10^{-3}\,{\rm Hz}\left(\frac{M_{\rm PBH}}{10^4\,{\rm g}}\right)^{-3/2}$



frequency range vs

Conclusions

- Dominant contribution comes from resonant amplification of GWs at PBH evaporation/reheating epoch
- For PBH mass in the range $10^5\,{\rm g} < M < 10^8\,{\rm g}$, isocurvature-induced GWs may be detected in the near future



GW detectors sensitivity

Caviat . . .

For the primordial isocurvature perturbation,

$$\mathscr{P}_S(k) = \frac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$$

the resulting curvature perturbation at PBH dominated Universe is

$$\Phi = \frac{3}{4}\left(\frac{k_{\rm eq}}{k}\right)^2 S \sim 0.3\left(\frac{k_{\rm eq}}{k_{\rm UV}}\right)^2\left(\frac{k}{k_{\rm UV}}\right)^{-1/2} \quad \text{for} \quad k_{\rm eq} < k < k_{\rm UV}$$

➡ The density perturbation becomes nonlinear for $k > k_{\rm NL}$:

$$\frac{\delta\rho}{\rho} = \frac{2}{3}\left(\frac{k}{aH}\right)^2 \Phi \sim 0.1\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)\left(\frac{k}{k_{\rm UV}}\right)^{3/2} \gtrsim 1$$

$$\text{for} \quad k > k_{\rm NL} \sim 5\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)^{-2/3} k_{\rm UV}$$

$$\log\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)^{2/3} \approx 2 + \frac{8}{9}\left(\log\frac{\beta}{10^{-7}} + \log\frac{M}{10^4\,{\rm g}}\right)$$

take-home message:

(Nonlinear) Isocurvature Perturbations
may play important roles in
PBH cosmology !