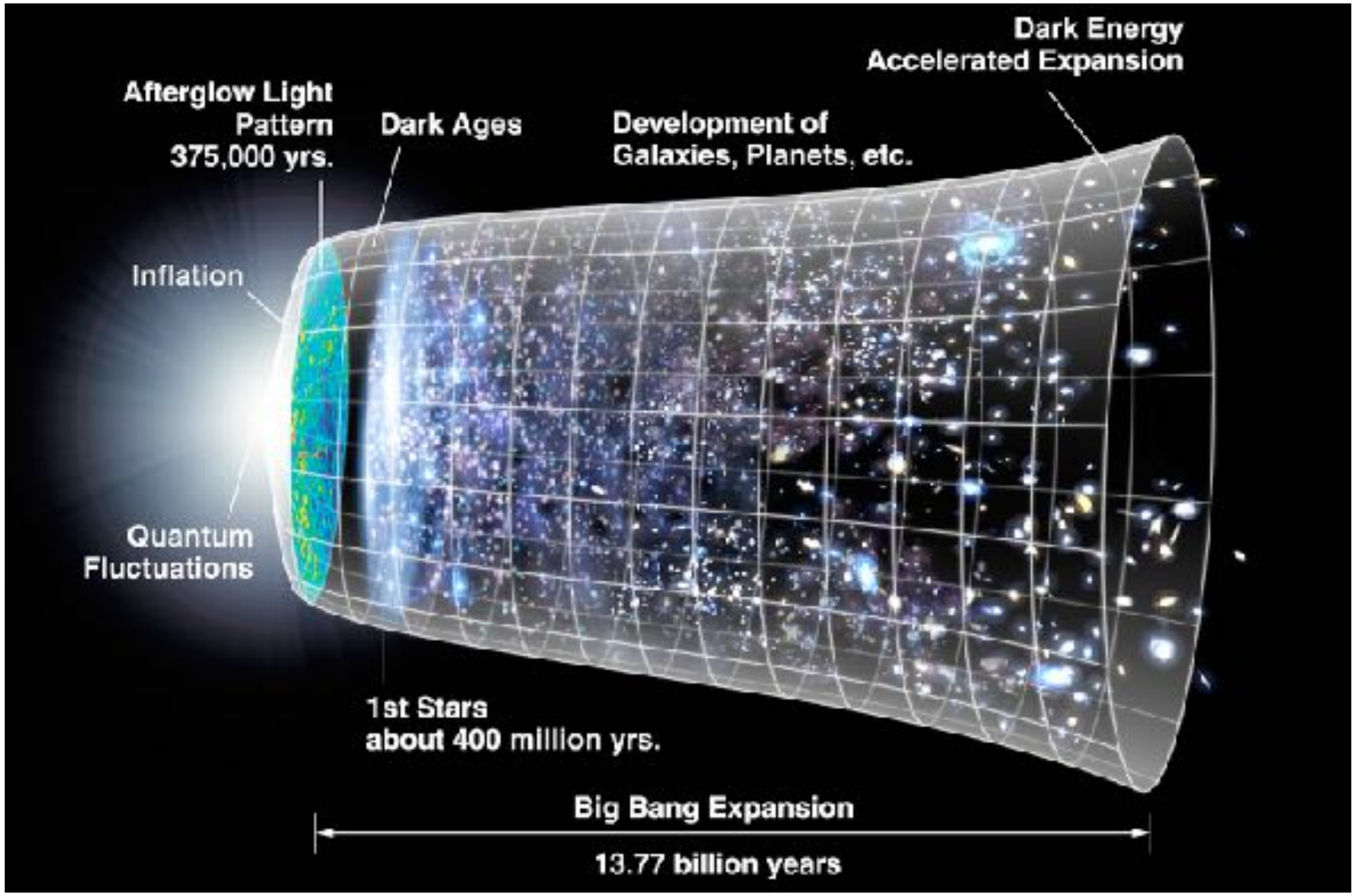
Gravitational waves and cosmology



Danièle Steer
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Gravitational waves and cosmology

late-time universe

Individual sources
and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star- black-hole binary (NS-BH)…

– Expansion rate $H(z)$
– H_0, Hubble constant
– Ω_m
– beyond $\Lambda\mathrm{CDM}$
dark energy $w(z)$ and dark matter
– modified gravity (modified GW propagation)
– astrophysics; eg BH populations, PISN mass gap?

Very early universe
$t \gtrsim t_{Pl}$

Stochastic background
of GWs of cosmological origin

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- GW observation of binary gives luminosity distance. To extract redshift, different methods: direct EM counterparts, galaxy catalogues, and/or through BBH, BNS populations.

- In near future: expect important impact on measurements of cosmological parameters, resolving (or not) the Hubble tension

- Number of effects to consider: overlapping sources and parameter estimation; higher order modes; precessing spins; waveform accuracy ? etc

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More speculative. Physics beyond GR, early universe sources beyond the standard model of particle physics!

Part 2) probing the very early universe with GWs

- SGWB: superposition of GWs arriving at random times and from random directions, overlapping so much that individual waves not detectable

- Two components $\nearrow$ astrophysical SGWB, $\searrow$ Cosmological SGWB,

astrophysical SGWB: superposition of a large number of unresolved sources since the beginning of the stellar activity

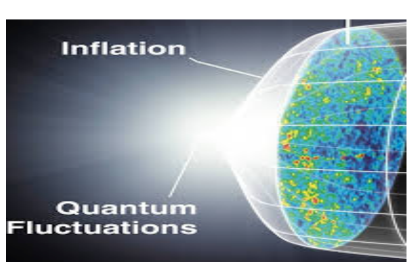
- black-hole mergers
- core-collapse of neutron stars
- Pulsars
- ….

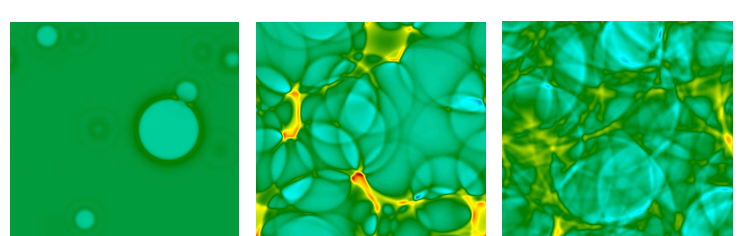
distribution of galaxies up to 100 Mpc is not isotropic but strongly concentrated in the direction of the VIRGO cluster + Great attractor;

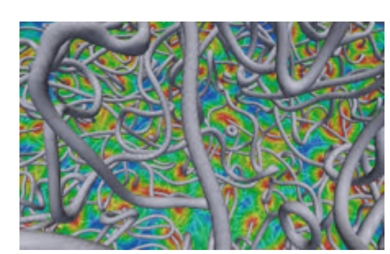
Cosmological SGWB: from events very early in the history of the universe

– quantum processes during inflation
– Phase transitions in Early universe
– topological defects, eg cosmic strings
– primordial black holes
– ……..

Expected to be nearly homogeneous and isotropic; unpolarised; gaussian





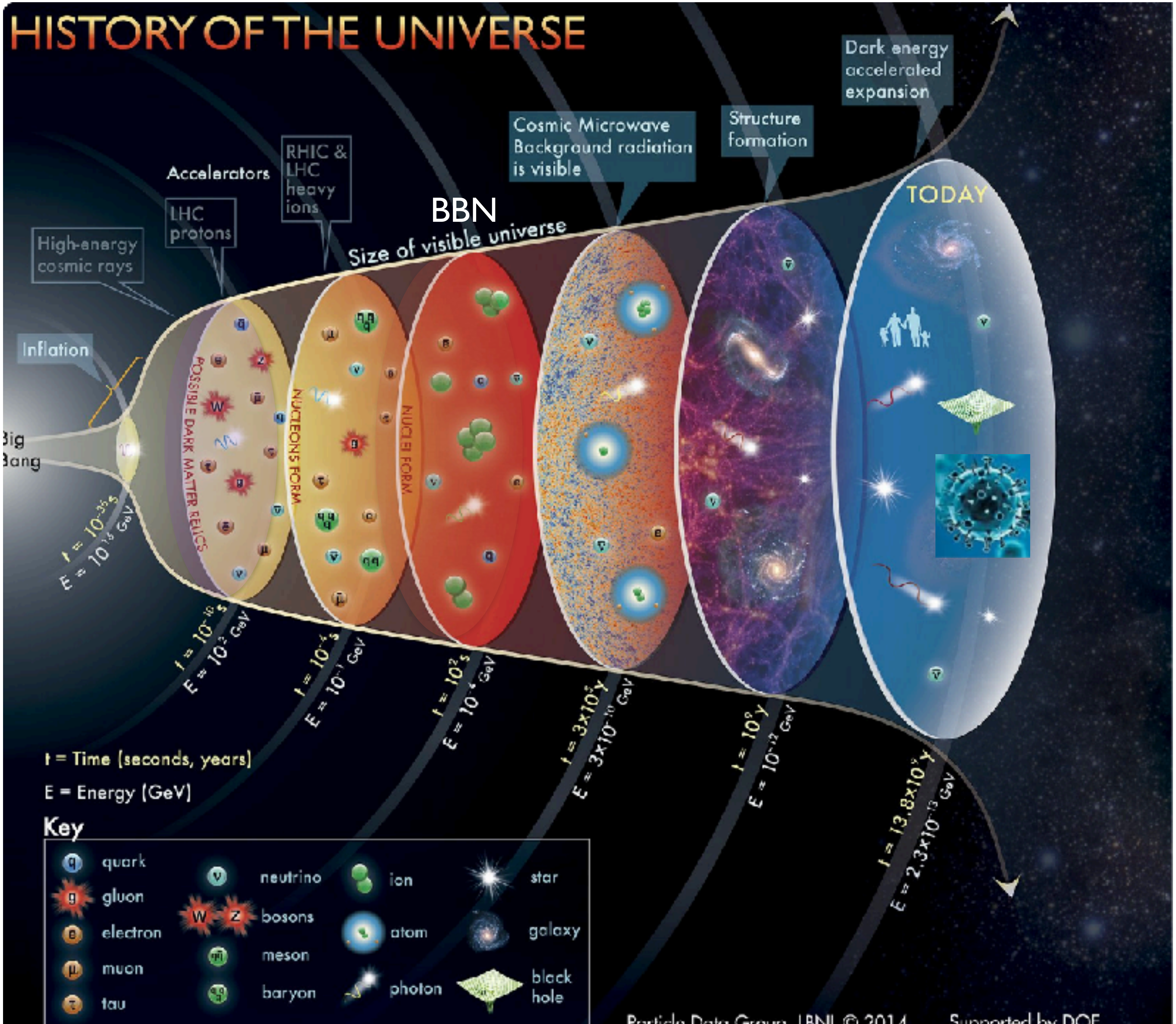


• Analogue of the CMB of photons, but crucial difference: gravitons decoupled below Planck scale!

=> Direct access to earliest stages in the evolution of the universe, which we cannot access through EM radiation.

=> Predictions based on physics beyond the standard model of particle physics (and possibly beyond GR)

=> Corollary: 1) predictions uncertain (based on untested physics)
2) if SGWB of cosmological origin detected, then huge discovery potential



Tested cosmology,

$$t \gtrsim 10^{-3}\text{sec}$$

$$T \lesssim 100\,\text{MeV}$$

Tested particle physics

$$T \lesssim 1\,\text{TeV}$$

New physics at higher energy scales.

SGWB of cosmological origin a fantastic way of probing this physics

but..predictions uncertain

Plan

Part 1) Late time cosmology with GWs

Part 2) probing the very early universe with GWs

1) generalities: on the source, and why a SGWB?
2) Characterisation of the SGWB (frequency, amplitude etc)
3) Brief summary of experimental bounds.
4) Introducing the source
5) universe sources
- Bubble collisions from Electroweak phase transition.
- Cosmic strings
- Inflation

very much based on [C.Caprini and D.Figueroa 1801.04268], and different lectures by C.Caprini

Generalities: a) on the sources

- Different sources for the SGWB can have very different characteristics properties.

- To discuss them (and at the same time recall some basic notation), useful to go back to basics

- Unperturbed FLRW metric

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$$

with $a(t) \sim t^p$ $\quad p = \frac{1}{2}$ (radiation era) $\quad p = \frac{2}{3}$ (matter era) $\quad p > 1$ (inflation)

Hubble radius $\frac{1}{H} = \frac{a}{\dot{a}}$ and comoving Hubble radius $\frac{1}{aH} = \frac{1}{\dot{a}}$

$q = \frac{p}{1-p}$

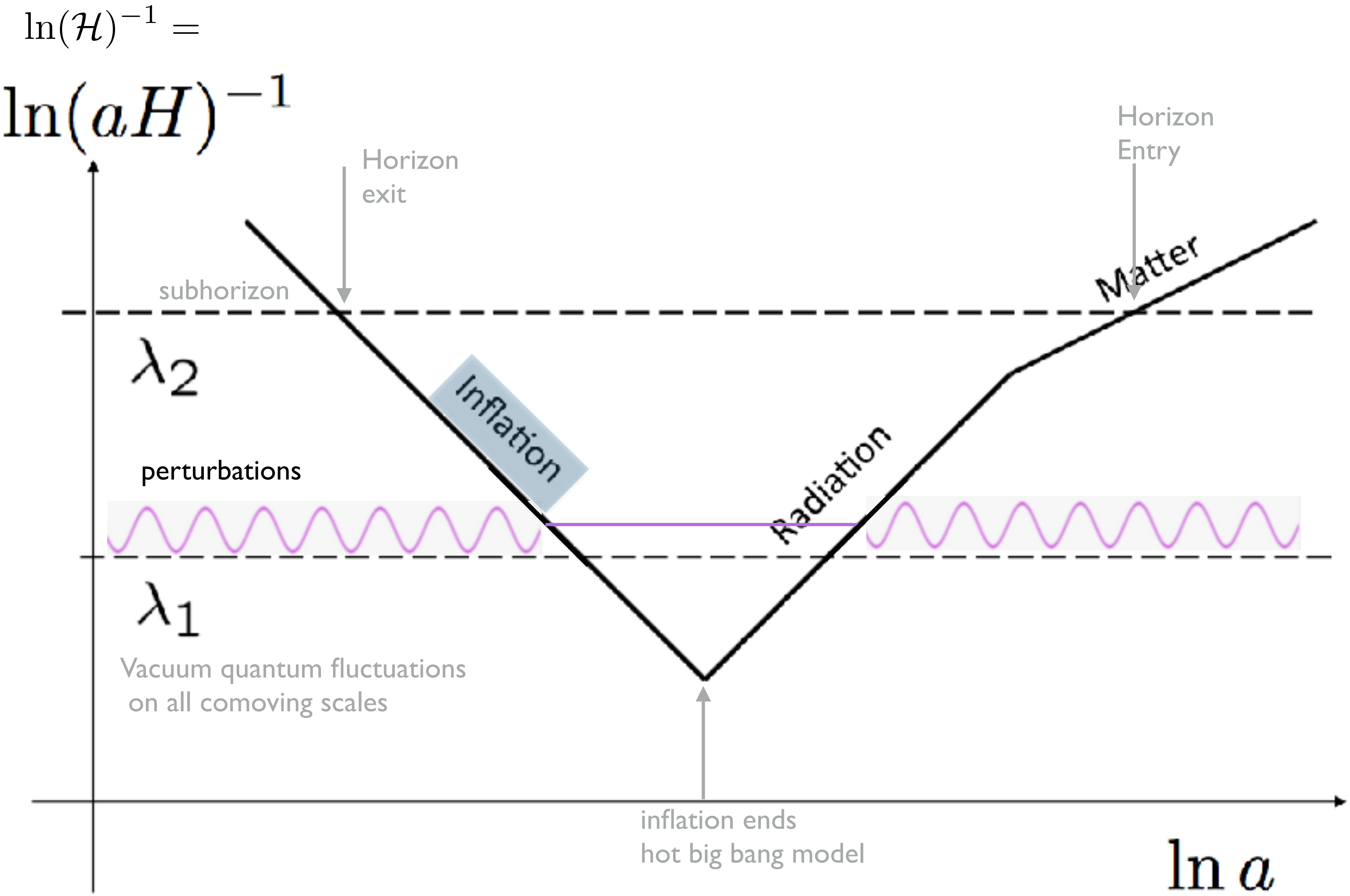
- In terms of conformal time $\eta = \int \frac{dt}{a(t)}$

$a(\eta) \sim \eta^q$ $\quad q = 1$ (radiation era) $\quad q = 2$ (matter era) $\quad q < 0$ (inflation)

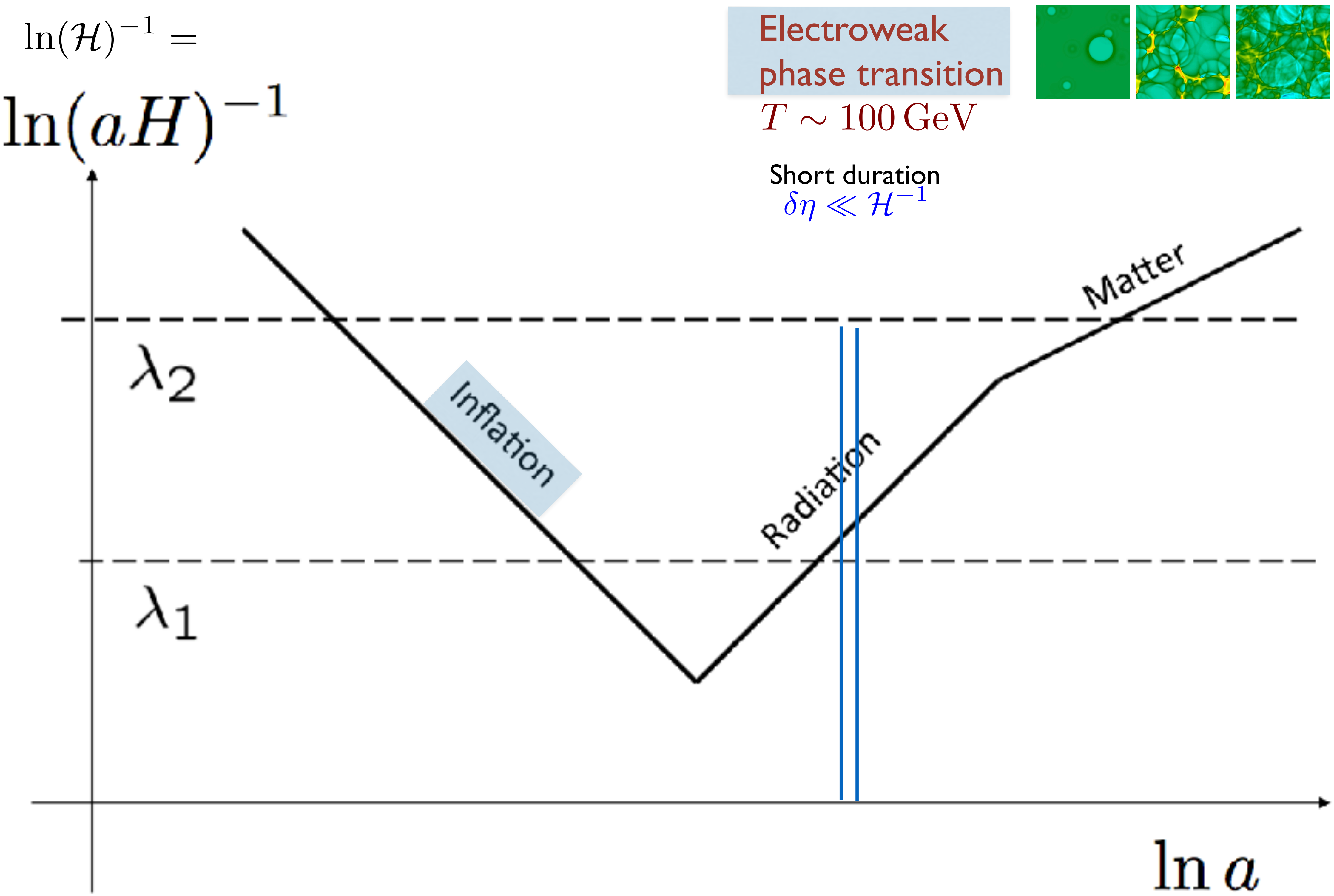
comoving Hubble radius $\frac{1}{aH} = \frac{1}{\dot{a}} = \frac{a}{a'} = \frac{1}{\mathcal{H}}$

- Hence $\mathcal{H}^{-1} \sim a^{1/q}$ so that $\boxed{\ln(\mathcal{H}^{-1}) \sim \frac{1}{q} \ln a}$

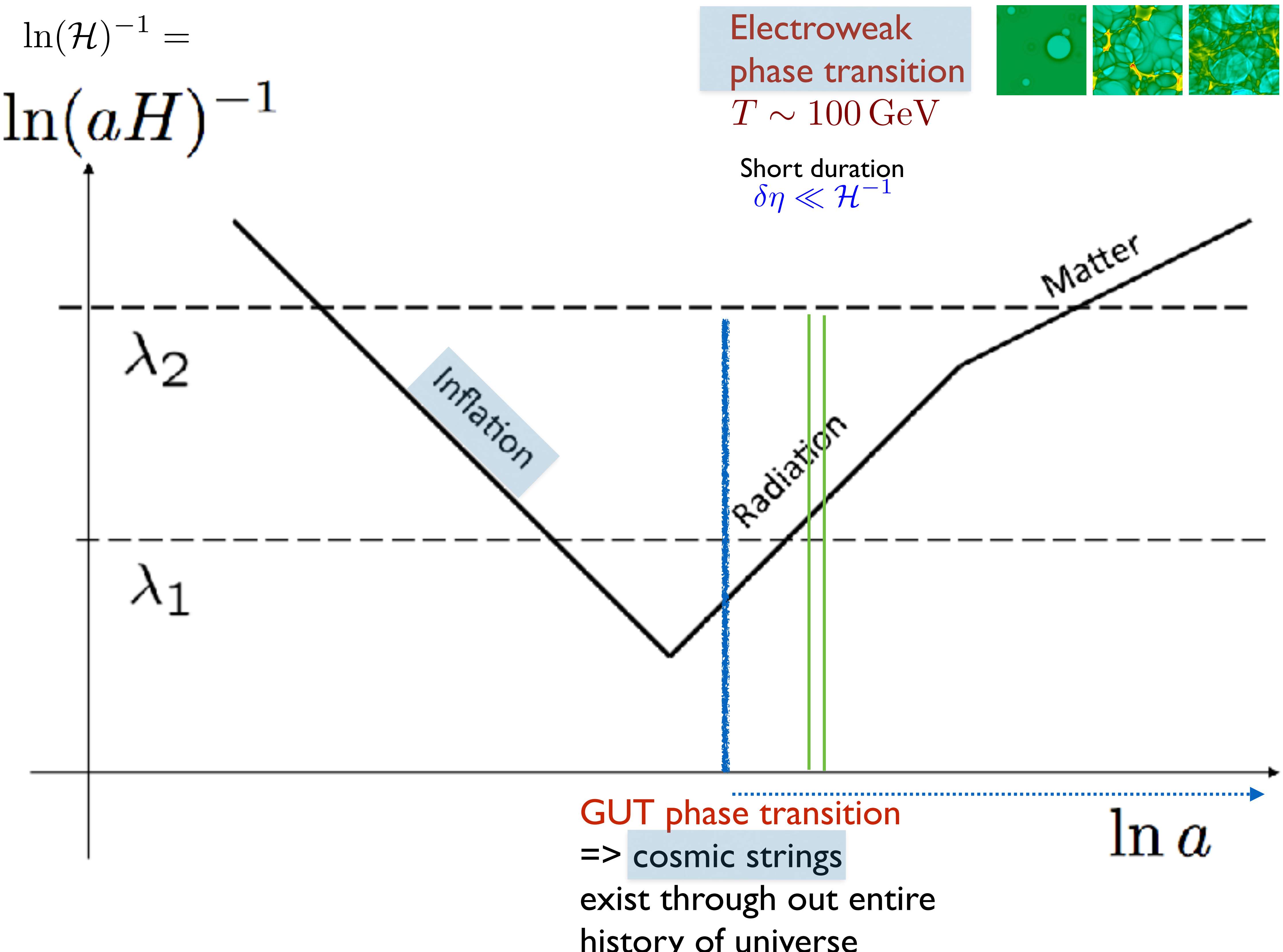
Generalities: a) on the sources



Generalities: a) on the sources

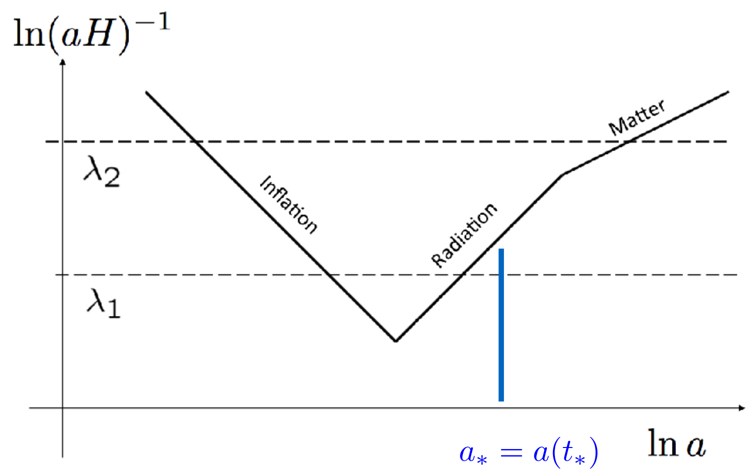


Generalities: a) on the sources



Generalities: b) why a SGWB?

[standard phase of cosmic expansion]



- assume the source operates at some time $t = t_\star$
  Why does it generate a SGWB?

- Causality: produced signal correlated on length/time scales at most as large as the causal horizon at that time

$$\ell_* \leq H^{-1}(t_*)$$
$$\Delta t_* \leq H^{-1}(t_*)$$

ℓ_* characteristic length-scale of the source
(typical size of variation of the tensor anisotropic stresses)

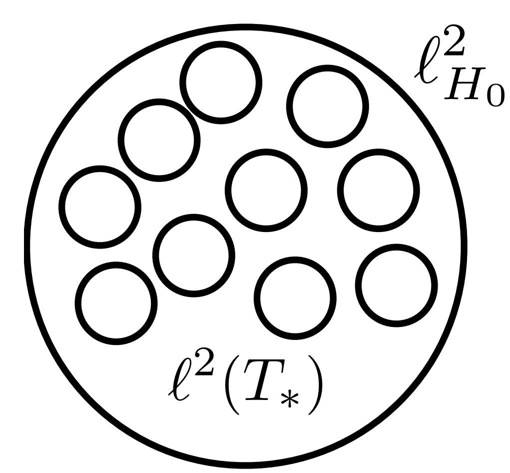
- Length scale redshifted to today corresponds to scale $\ell_*^0 = \ell_* \left(\frac{a(t_0)}{a(t_*)}\right)$

- Compared to the Hubble radius today. Using the Friedmann equations

(in the radiation era, neglecting changes in effective number of relativistic d of f, and saturating the inequality)

$$\frac{\ell_*^0}{H^{-1}(t_0)} \sim 10^{-11} \left(\frac{\text{GeV}}{T_*^4}\right)$$

correlation length tiny compared to present size of universe today.

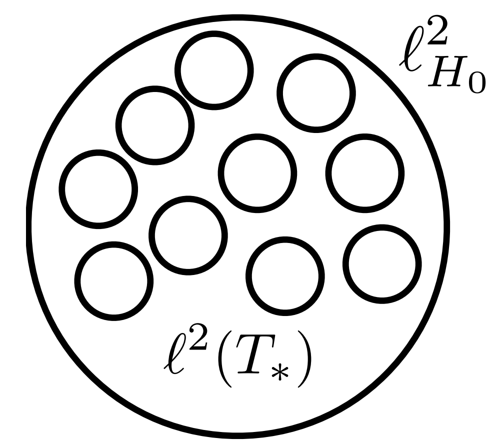


- Angular size of that patch on the sky today?

Generalities: b) why a SGWB?
[standard phase of cosmic expansion]

• Angular size of that patch on the sky today $\Theta_* = \dfrac{\ell_*}{d_A(z_*)}$ $\ell_* \leq H^{-1}(t_*)$

$$d_A(z_*) = \frac{1}{H_0(1+z_*)} \int_0^{z_*} \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_r(1+z')^4 + \Omega_\Lambda}}$$



• number of uncorrelated regions today from which we are receiving independent GW signals $\sim \Theta_*^{-2}$

EW scales $\Theta(T_* = 100\,\mathrm{GeV}) \simeq 10^{-12}\mathrm{deg}$

equality $\Theta(z_* = 1090) \simeq 0.9\,\mathrm{deg}$

GW signal (as received today on Earth) due to a causal process operating at the EW epoch, is composed by the superposition of independent signals emitted by at least 10^{24} uncorrelated regions

• a GW signal from the early universe cannot possibly be resolved beyond its stochastic nature

$\Theta(z_* \lesssim 17) \lesssim 10\,\mathrm{deg}$
(GW detector angular resolution)

– Can only the statistical properties of the signal
– Must treat $h_{ij}(\vec{x}, t)$ as a random variable

• By observing large enough regions of the Universe today (or a given region for long enough time), have access to many realisations of the system: replace *ensemble averages* with *volume/time averages*
(over a length scale much larger than the typical GW wavelength, and much smaller than the Horizon)

[exception in the case of inflation]

2) Characterisation of the SGWB

- Perturbed FRWL metric (ignoring scalars and vectors):

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$|h_{ij}| \ll 1$$
$$h^i_i = \partial_j h^j{}_i = 0$$

- from Einstein equations

$$\ddot{h}_{ij}(\vec{x},t) + 3H\dot{h}_{ij}(\vec{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\vec{x},t) = 16\pi G \Pi^{TT}_{ij}(\vec{x},t)$$

source: tensor anisotropic stress

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

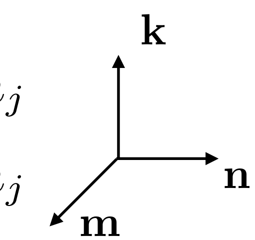
- Fourier transform, and polarisation components +, x

$$h_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, h_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

$$\Pi_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \Pi_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

k= comoving wave number

$$e^+_{ij}(\hat{\mathbf{k}}) = \hat{m}_i\,\hat{m}_j - \hat{n}_i\,\hat{n}_j$$
$$e^\times_{ij}(\hat{\mathbf{k}}) = \hat{m}_i\,\hat{n}_j + \hat{n}_i\,\hat{m}_j$$
$$e^r_{ij}(\hat{\mathbf{k}})\, e^{r'}_{ij}(\hat{\mathbf{k}}) = 2\,\delta_{rr'}$$

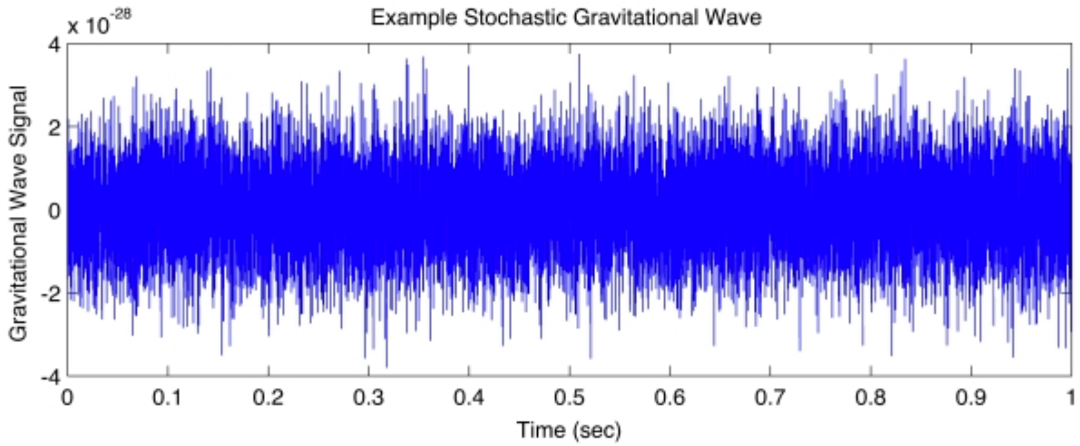


- The equation decouples for each polarisation mode. In terms of conformal time

$$h''_r(\mathbf{k},\eta) + 2\mathcal{H}\, h'_r(\mathbf{k},\eta) + k^2 h_r(\mathbf{k},\eta) = 16\pi G a^2 \Pi_r(\mathbf{k},\eta)$$

Note from Friedmann equation: $\mathcal{H}^2 = H^2 a^2 = \frac{8\pi G}{3} a^2 \bar{\rho}$

Statistical properties



- In general the SGWB is

– *homogenous and isotropic*
(inherited from FLRW universe)

$$\langle h_{ij}(\mathbf{x},\eta_1)\, h_{lm}(\mathbf{y},\eta_2)\rangle = \xi_{ijlm}(|\mathbf{x}-\mathbf{y}|,\eta_1,\eta_2)$$

– *unpolarised*
(absence of significant source of parity violation
in the universe)

$$\langle h_+(\mathbf{k},\eta)h_\times(\mathbf{k},\eta)\rangle = 0.$$

– *gaussian*
(formed by emission from many uncorrelated regions)

therefore characterized by the 2-point function

- In terms of the Fourier amplitudes $h_r(\mathbf{k},\eta)$:

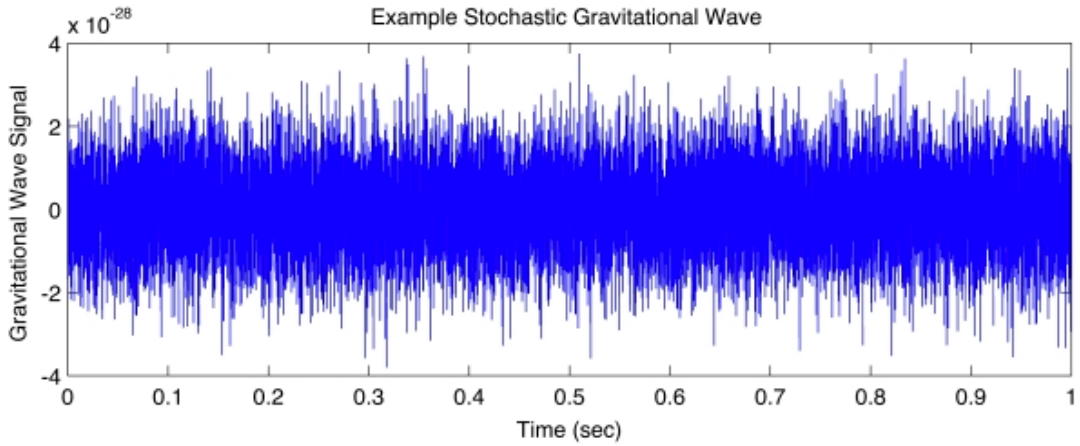
$$h_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, h_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

Statistical properties

- the 2-point function

homogeneity and isotropy, unpolarised

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,h_c^2(k,\eta)$$



$$h_{ij}(\mathbf{x},t) = \sum_{r=+,\times}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\, h_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

- Hence

$$\langle h_{ij}(\mathbf{x},\eta)\, h_{ij}(\mathbf{x},\eta)\rangle = 2\int_0^{+\infty}\frac{dk}{k}\,h_c^2(k,\eta)$$

characteristic GW amplitude per logarithmic wave-number interval and per polarization state, at time η

- In terms of which can express GW energy density, given by

$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}(\mathbf{x},t)\,\dot{h}_{ij}(\mathbf{x},t)\rangle}{32\pi G} = \frac{\langle h'_{ij}(\mathbf{x},\eta)\,h'_{ij}(\mathbf{x},\eta)\rangle}{32\pi G\,a^2(\eta)} \equiv \int_0^{+\infty}\frac{dk}{k}\,\frac{d\rho_{\rm GW}}{d\log k}$$

spectrum of GW energy density per logarithmic wave-number interval

- GW energy density parameter. In terms of present day physical frequency $f = k/(2\pi\, a_0)$

$$\Omega_{\rm gw}(t_0,f) = \frac{f}{\rho_c}\,\frac{d\rho_{\rm gw}}{df}(t_0,f)$$

$$\rho_c = 3H_0^2/8\pi G$$

critical energy density of universe.

Spectrum of GW energy density per logarithmic wave-number interval for free waves $\frac{d\rho_{\rm GW}}{d\log k}$

$$h_r''(\mathbf{k},\eta) + 2\mathcal{H}\, h_r'(\mathbf{k},\eta) + k^2 h_r(\mathbf{k},\eta) = 16\pi G a^2 \Pi_r(\mathbf{k},\eta)$$

Solution of the homogeneous equation

$$H_r(\mathbf{k},\eta) = a(\eta) h_r(\mathbf{k},\eta) \quad \Rightarrow \quad H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = 0$$

where for $a \sim \eta^q$ have $\frac{a''}{a} \sim \mathcal{H}^2$

– *Sub Hubble modes,*

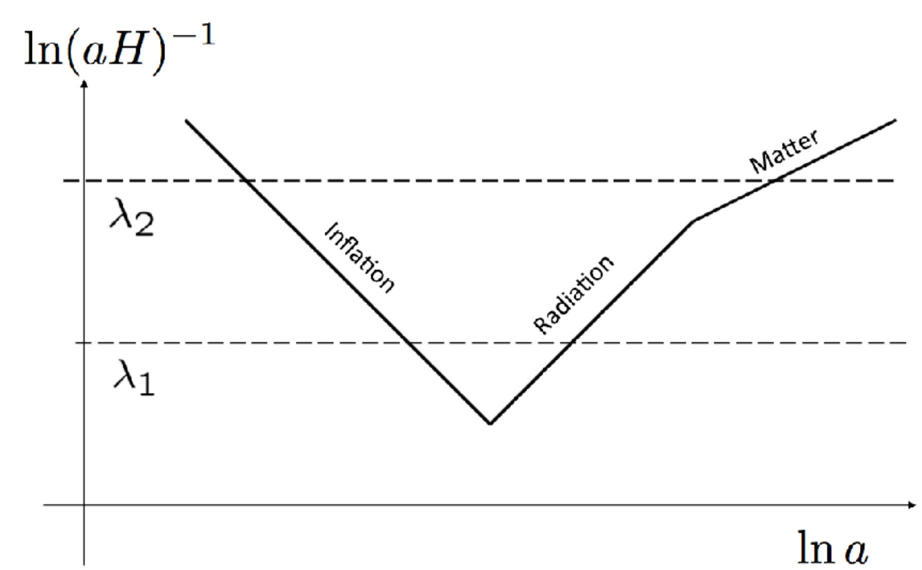
$$k^2 \gg \mathcal{H}^2 \qquad h_r(\vec{k},\eta) = \frac{A_r(\vec{k})}{a(\eta)} e^{ik\eta} + \frac{B_r(\vec{k})}{a(\eta)} e^{-ik\eta}$$

plane waves with redshifting amplitude.

– *Super-Hubble modes*, relevant for inflationary tensor perturbations

$$k^2 \ll \mathcal{H}^2 \qquad h_r(\vec{k},\eta) = A_r(\vec{k}) + B_r(\vec{k}) \int^{\eta} \frac{d\eta'}{a^2(\eta)}$$

modes frozen



$$k^2 \gg \mathcal{H}^2 \qquad h_r(\vec{k},\eta) = \frac{A_r(\vec{k})}{a(\eta)} e^{ik\eta} + \frac{B_r(\vec{k})}{a(\eta)} e^{-ik\eta}$$

$$\int e^{i(k-q)\eta} d\eta \sim \delta(k-q)$$

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, h_c^2(k,\eta)$$

- For sub-Hubble modes (no source)

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{1}{a^2(\eta)} [\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle] .$$

$$\updownarrow$$

$$h_c(k,\eta) \propto \frac{1}{a(\eta)}$$

- For the energy density in GWs, need the correlator $\langle h'_{ij}(\mathbf{x},\eta)\, h'_{ij}(\mathbf{x},\eta)\rangle$

$$\langle h'_r(\mathbf{k},\eta) h'_p(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^4} \delta^{(3)}(\mathbf{k}-\mathbf{q}) \delta_{rp} X(k,\eta)$$

$$\rho_{\rm GW} = \frac{\langle h'_{ij}(\mathbf{x},\eta)\, h'_{ij}(\mathbf{x},\eta)\rangle}{32\pi G\, a^2(\eta)} = \int_0^{+\infty} \frac{dk}{k} \frac{d\rho_{\rm GW}}{d\log k}$$

$$X(k,\eta) \sim k^2\, h_c^2(k,\eta) \qquad k \gg \mathcal{H}$$

- Hence $\boxed{\rho_{GW} \propto \frac{1}{a^4}}$

(GW energy density diluted like radiation for freely propagating subHubble modes; contributes to RHS of Friedmann equation)

- Finally, spectrum $\dfrac{d\rho_{\rm GW}}{d\log k} = \dfrac{k^2\, h_c^2(k,\eta)}{16\pi G\, a^2(\eta)}$

Characteristic frequency for *causal sources?*

- Depends on:

– production mechanism (model-dependent)
– kinematical (depending on the redshift from the production era)

- GWs produced with frequency $f_\star$ at $t = t_\star$ have characteristic frequency today of

(assuming standard thermal history and radiation era)

$$f_c = f_\star \left(\frac{a_\star}{a_0}\right) = 2 \times 10^{-5} \left(\frac{f_\star}{H_\star}\right) \left(\frac{T_\star}{1\text{TeV}}\right)$$

temperature (energy density) of the universe at the source time

- But expect $f_* \sim \ell_*^{-1} \geq H(t_*)$

- Then

$$f_c = f_* \frac{a_*}{a_0} \simeq \frac{10^{-7}}{\epsilon_*} \frac{T_*}{\text{GeV}} \text{ Hz}$$

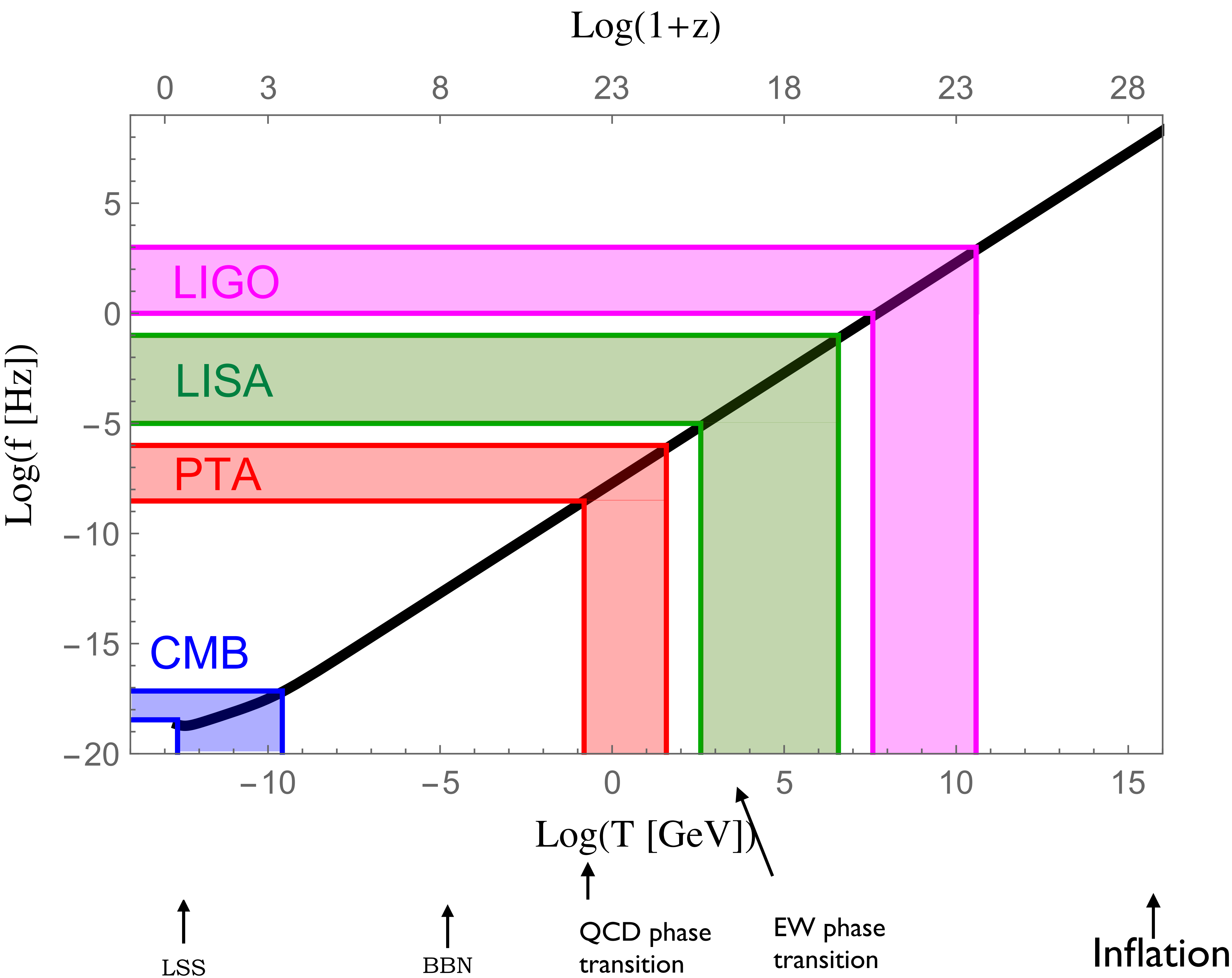
$\epsilon_* = \ell_* H_* \leq 1$

production mechanism (model-dependent)

$$f_c = f_* \frac{a_*}{a_0} \simeq \frac{10^{-7}}{\epsilon_*} \frac{T_*}{\mathrm{GeV}} \ \mathrm{Hz}$$

e.g., for electroweak scales $T_* \simeq 1\,\mathrm{TeV} \longrightarrow f_c \simeq \mathrm{mHz}$

LISA band



Can GWs probe the very high energy regime?

$$f_c = f_* \frac{a_*}{a_0} \simeq \frac{10^{-7}}{\epsilon_*} \frac{T_*}{\mathrm{GeV}}\ \mathrm{Hz}$$

~GUT scales; inflationary scale $T_\star \sim 10^{16}\mathrm{GeV} \Longrightarrow f_c \sim 2\mathrm{GHz}$

~ Planck scales (q-gravity?) $T_\star \sim 10^{18}\mathrm{GeV} \Longrightarrow f_c \sim 100\mathrm{GHz}$

• seem totally inaccessible to ground based interferometers and LISA

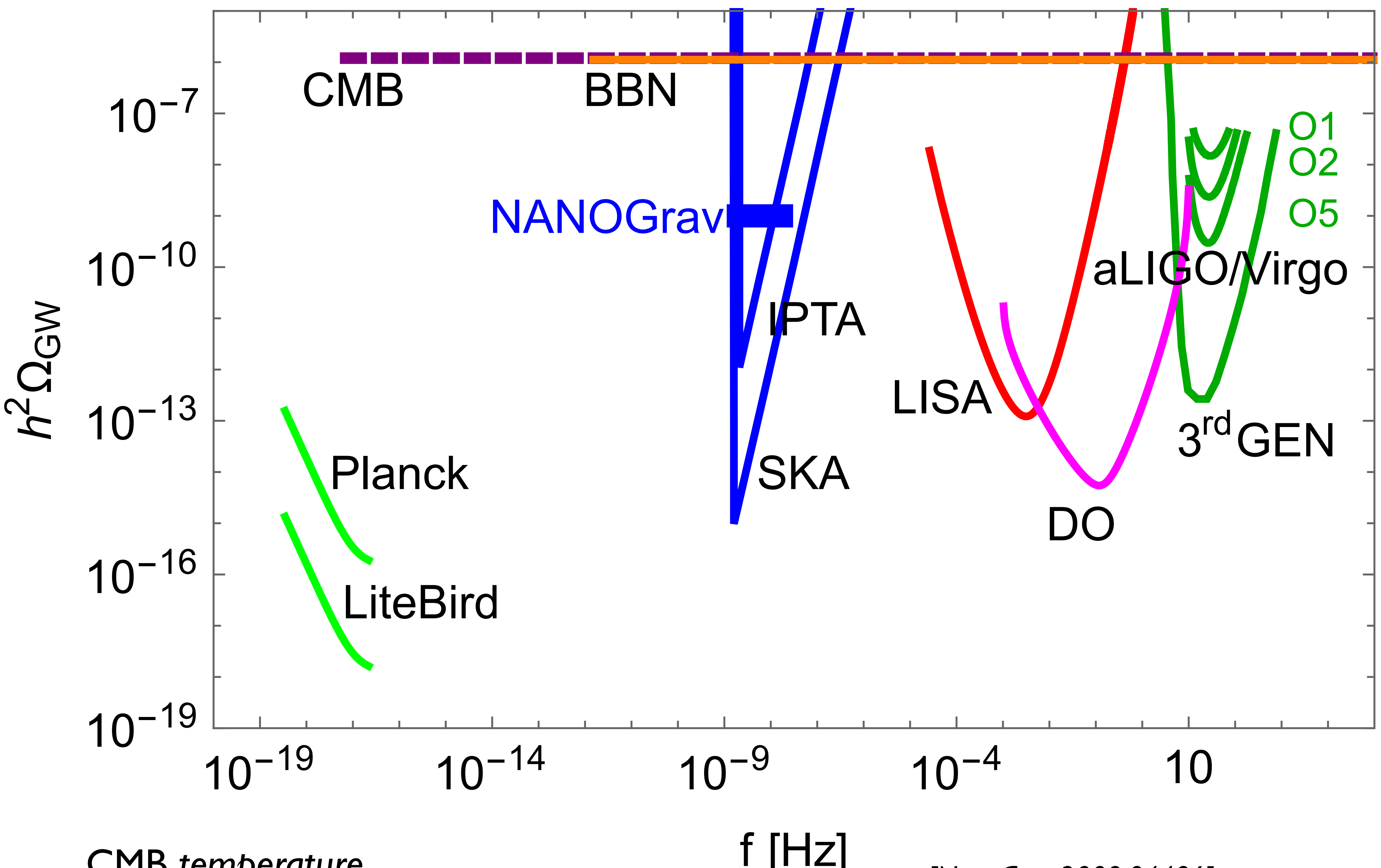
• BUT, whilst it's true that

$$\Omega(f > f_c) \to 0$$

the spectrum for **lower** frequencies is **not** fixed by these arguments:
– *if* for e.g. it is flat when $f < f_c$ and with a sufficiently high amplitude, then it *could* be seen at lower frequencies

• As our examples will show, in many cases the spectrum is (nearly) flat over a large range of frequencies

Experimental bounds on $h^2\Omega_{\rm GW}(f)$



[LIGO-Virgo 2101.12130]

$\Omega_{\rm GW} \leq 5.8 \times 10^{-9}$

[NanoGrav 2009.04496]

CMB *temperature*

CMB *B-modes*

$10^{-18}\,\mathrm{Hz} < f < 10^{-16}\,\mathrm{Hz}$ $\quad \frac{H_0}{2\pi} < f < \frac{H_{dec}}{2\pi}\frac{a_{dec}}{a_0}$

Cosmic microwave background

$$10^{-18}\,\mathrm{Hz} < f < 10^{-16}\,\mathrm{Hz}$$

$$\frac{H_0}{2\pi} < f < \frac{H_{dec}}{2\pi}\frac{a_{dec}}{a_0}$$

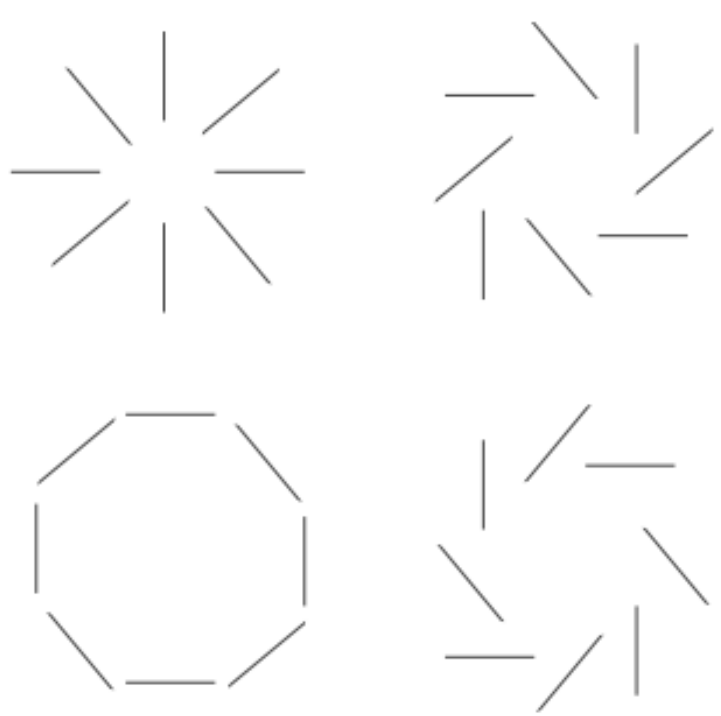
• fluctuations in CMB *temperature* (constrained by Planck data) GWs stretch and compress space, in which the decoupled CMB photons travel

$$\frac{\Delta T}{T} = -\int_i^f \dot{h}_{ij} n^i n^j d\lambda$$

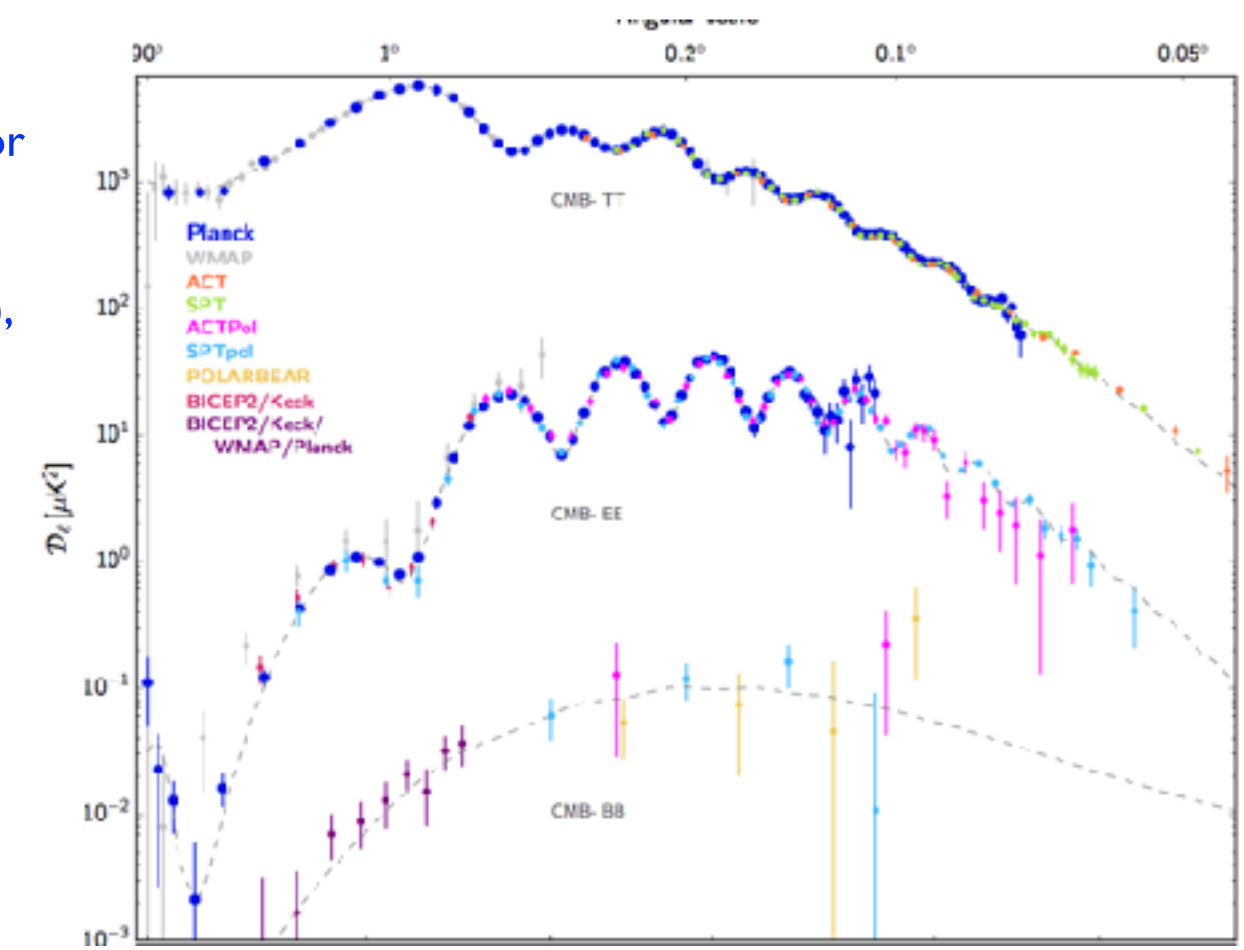
Sachs Wolfe effect

• fluctuations in CMB *polarisation*
(constrained by LiteBird and other experiments)

E-mode *B-mode*



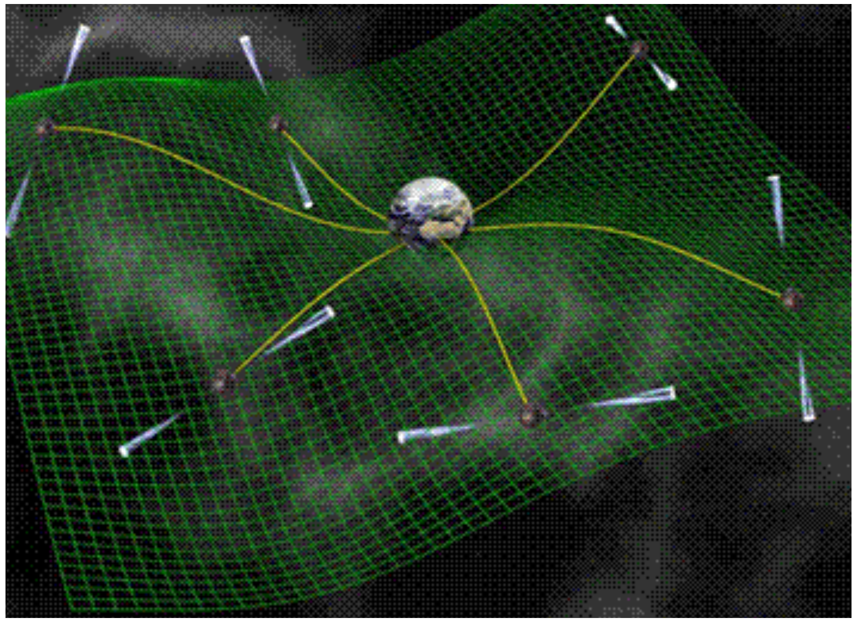
Sourced by primordial tensor perturbations at last scattering (and possible vector modes), or by foregrounds and gravitational lensing



[Planck collaboration, 1807.06205]

PTA = Pulsar Timing array

$$10^{-9}\mathrm{Hz} < f < 10^{-7}\mathrm{Hz}$$



• Pulsars: rapidly rotating and highly magnetized neutron stars. Emit beam of EM radiation in direction of rotating magnetic axis, leading to regular train of pulsed radiation reaching the earth each time the beam crosses observers line of sight.

• Arrival times predicted very accurately over long time scales —> stable clocks.

• Used as direct detectors of GWs, through fluctuation generated in the time of arrival of the pulse due to the GWs. *Pulsar timing.*

• Observations on time scales of a year

– Recent NanoGRAV result?

[NanoGrav 2009.04496]

$$h_c(f) = A\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}$$

$$\Omega(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}$$

$$\Omega_{yr} = \frac{2\pi^2}{3H_0^2} A^2 f_{yr}^2 \qquad \gamma = 3 - 2\alpha$$

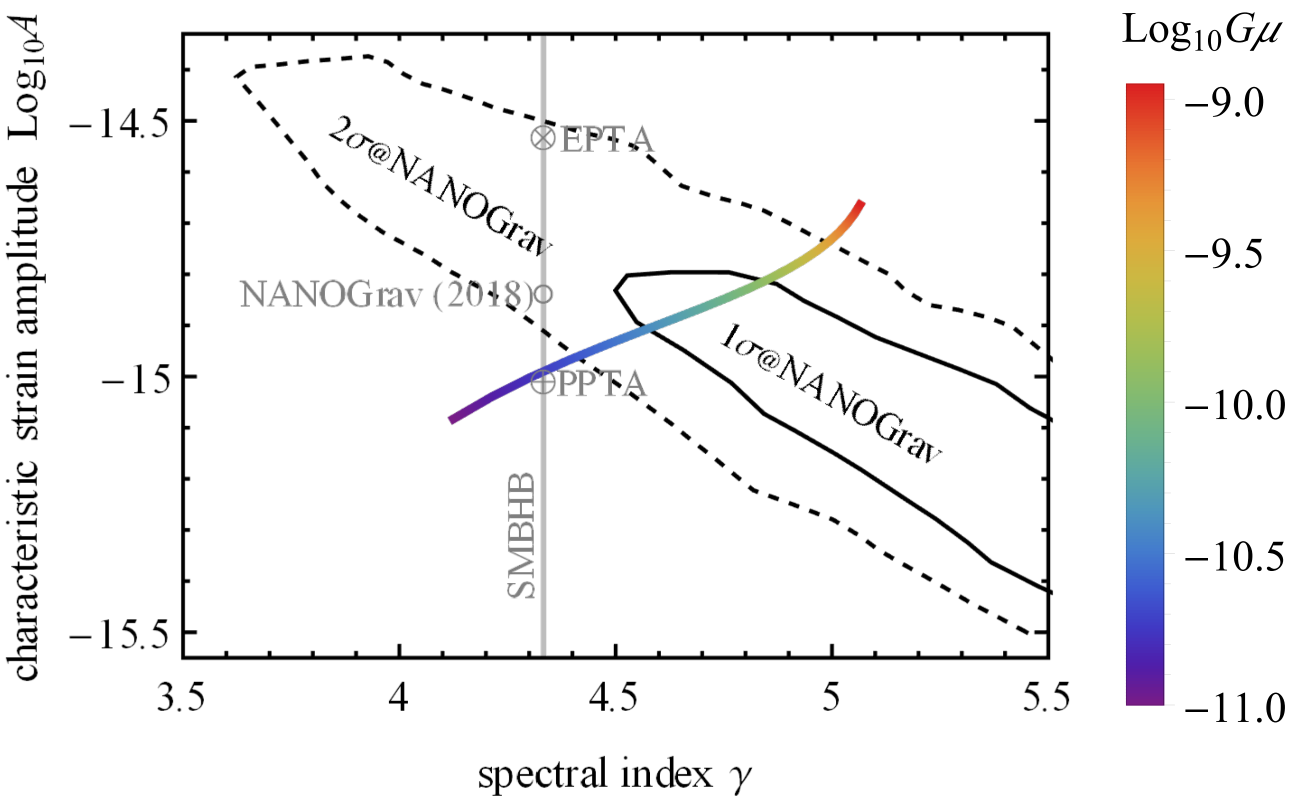


figure from [2009.06555]

GW contribute to the energy density in the universe, and hence enter into the Friedmann equation

$$H^2(T) = \frac{8\pi G}{3}\Sigma_i \rho_i(T)$$

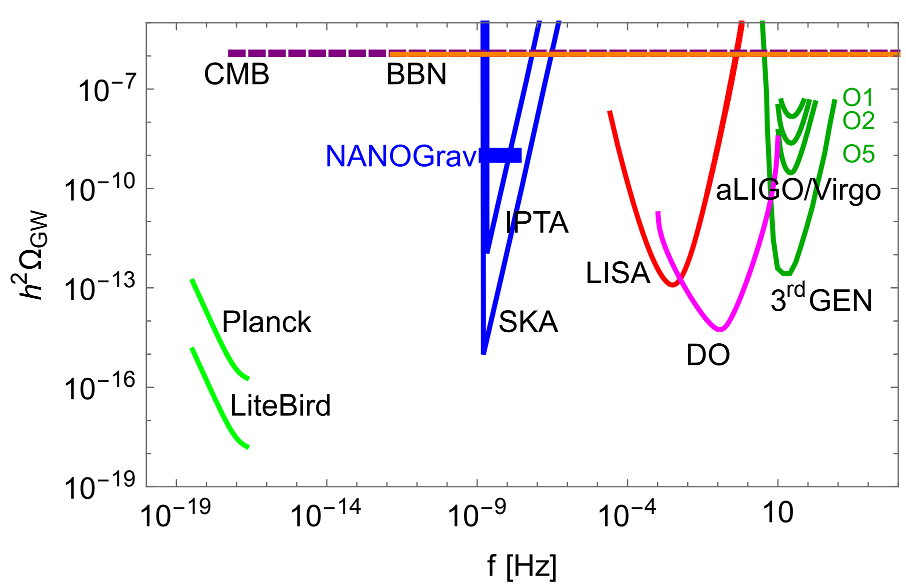
• If have large energy density in GWs at big bang nucleosynthesis $(T \sim \mathrm{Mev},\ t \sim 1\mathrm{sec})$ then larger *H which*

– Feeds into a larger freeze out temperature, and higher ratio of neutrons to protons,
– finally into over production of Helium 4

Hence bound on integrated GWs energy density generated before BBN

• Similarly, cosmic microwave background monopole and anisotropy spectrum depend on the Hubble scale at decoupling T ~ 0.3 eV

Hence bound on integrated GWs energy density generated before photon decoupling



Specifying the source: SGWB from a *causal source (radiation era)*

- Tensor anisotropic stress sourcing the GWs can be decomposed in two polarisation states $\Pi_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \Pi_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$

- Describe the source stochastically, assuming it is also statistically homogeneous+isotropic, non-polarised, gaussian.

$$\boxed{\langle \Pi_r(\mathbf{k},\tau)\, \Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\, \frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\, \delta_{rp}\, \Pi(k,\tau,\zeta)}$$

Unequal time correlator

- Back to our perturbation equation $h_r''(\mathbf{k},\eta) + 2\mathcal{H}\, h_r'(\mathbf{k},\eta) + k^2 h_r(\mathbf{k},\eta) = 16\pi G a^2 \Pi_r(\mathbf{k},\eta)$

which, again is simplified in terms of $H_r(\mathbf{k},\eta) = a(\eta) h_r(\mathbf{k},\eta)$

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$

Aim: solve this equation and find $\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0)$

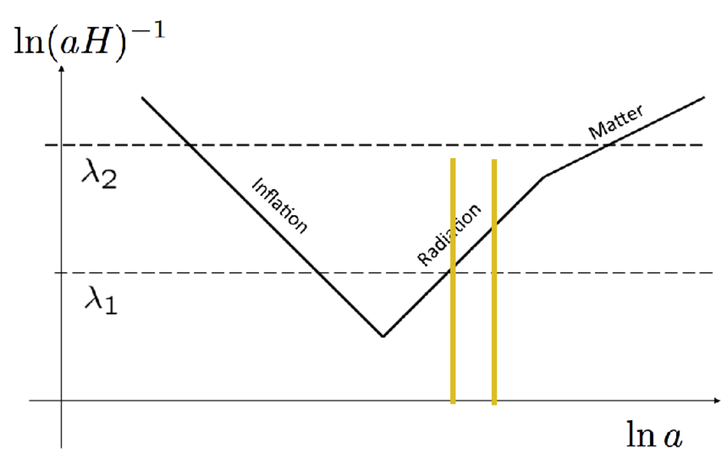
- Suppose the source acts for a finite time $\eta_{\rm in} \to \eta_{\rm fin}$ in the radiation dominated era. Can solve using Green function:

$$H_r^{\rm rad}(\mathbf{k},\eta < \eta_{\rm fin}) = \frac{16\pi G}{k} \int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\, \sin[k(\eta-\tau)]\, \Pi_r(\mathbf{k},\tau)$$

- Match onto the free solution $H_r^{\rm rad}(\mathbf{k},\eta > \eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k}) \cos(k\eta) + B_r^{\rm rad}(\mathbf{k}) \sin(k\eta)$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k} \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\, \sin(-k\tau)\, \Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k} \int_{x_{\rm in}}^{x_{\rm fin}} d\tau\, a(\tau)^3\, \cos(k\tau)\, \Pi_r(\mathbf{k},\tau)$$



GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\, \sin(-k\tau)\, \Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{x_{\rm in}}^{x_{\rm fin}} d\tau\, a(\tau)^3\, \cos(k\tau)\, \Pi_r(\mathbf{k},\tau)$$

$$\langle \Pi_r(\mathbf{k},\tau)\, \Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\, \frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\, \delta_{rp}\, \Pi(k,\tau,\zeta)$$

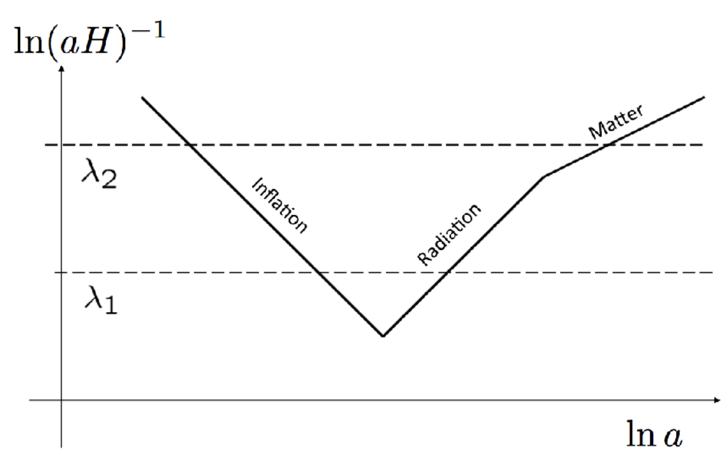
$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$ for a source in the radiation era

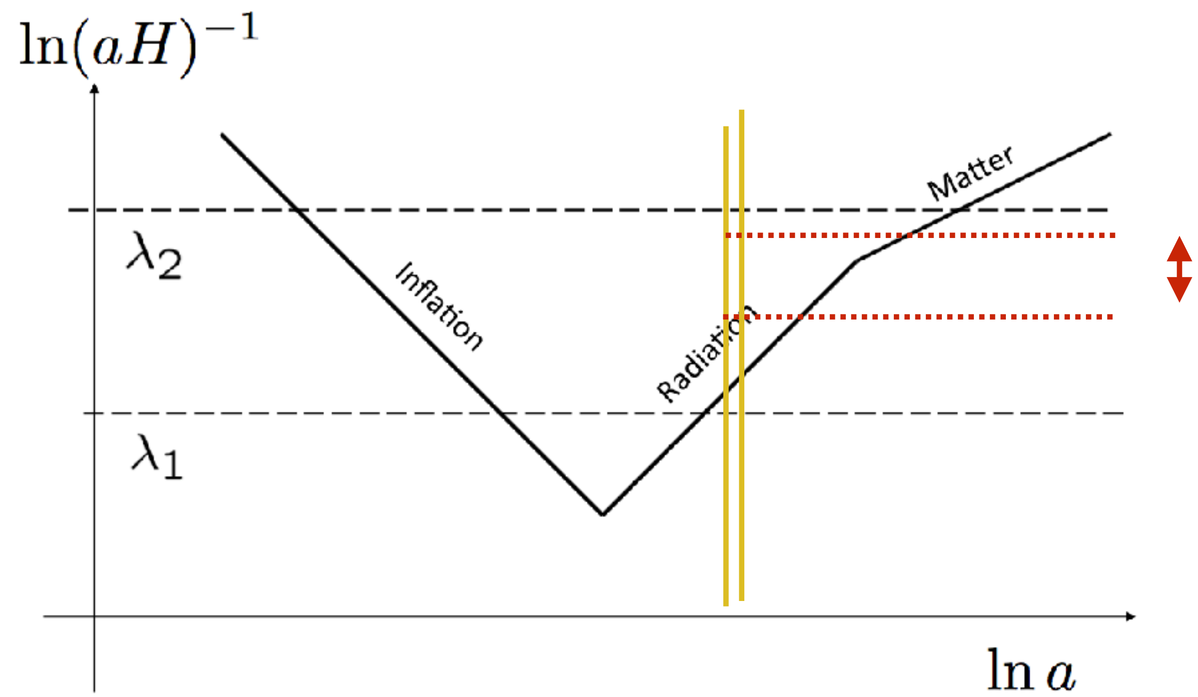
$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

$$\frac{d\rho_{GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau a^3(\tau)\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\xi a^3(\xi)\cos[k(\tau-\xi)]\Pi(k,\tau,\xi)$$

Unequal time correlation function determines the GW spectrum



A simple example



$$\frac{d\rho_{GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau a^3(\tau)\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\xi a^3(\xi)\cos[k(\tau-\xi)]\Pi(k,\tau,\xi)$$

$\Delta\eta = \eta_{\rm fin} - \eta_* \ll \mathcal{H}_*^{-1}$

$a(\tau) \to a_*$

$k\eta_{\rm in} \ll 1$

$\Pi(k,\tau,\xi) \sim$ constant over $\Delta\eta$

$\Pi(k,\tau,\xi) \to \Pi(k)$

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$\frac{d\rho_{GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau a^3(\tau)\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\xi a^3(\xi)\cos[k(\tau-\xi)]\Pi(k,\tau,\xi)$$

$a(\tau) \to a_*$

$\Pi(k,\tau,\xi) \to \Pi(k)$

with

$$\Pi(k) = \rho_\Pi \tilde{P}_{\rm GW}(k)$$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2\tilde{P}_{\rm GW}(k)$$

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$\underbrace{h^2\Omega_{GW}(k,\eta_0)}_{\mathcal{O}(10^{-11})} = \underbrace{\frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}}_{\mathcal{O}(10^{-6})} \underbrace{(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2}_{\mathcal{O}(10^{-5})} \tilde{P}_{GW}(k)$$

Value that would guarantee a detection in a not so far future

Factor depending slightly on the generation epoch through the number of relativistic d.o.f.

Only slow, very anisotropic processes have the chance to generate detectable SGWB signals!

C.Caprini

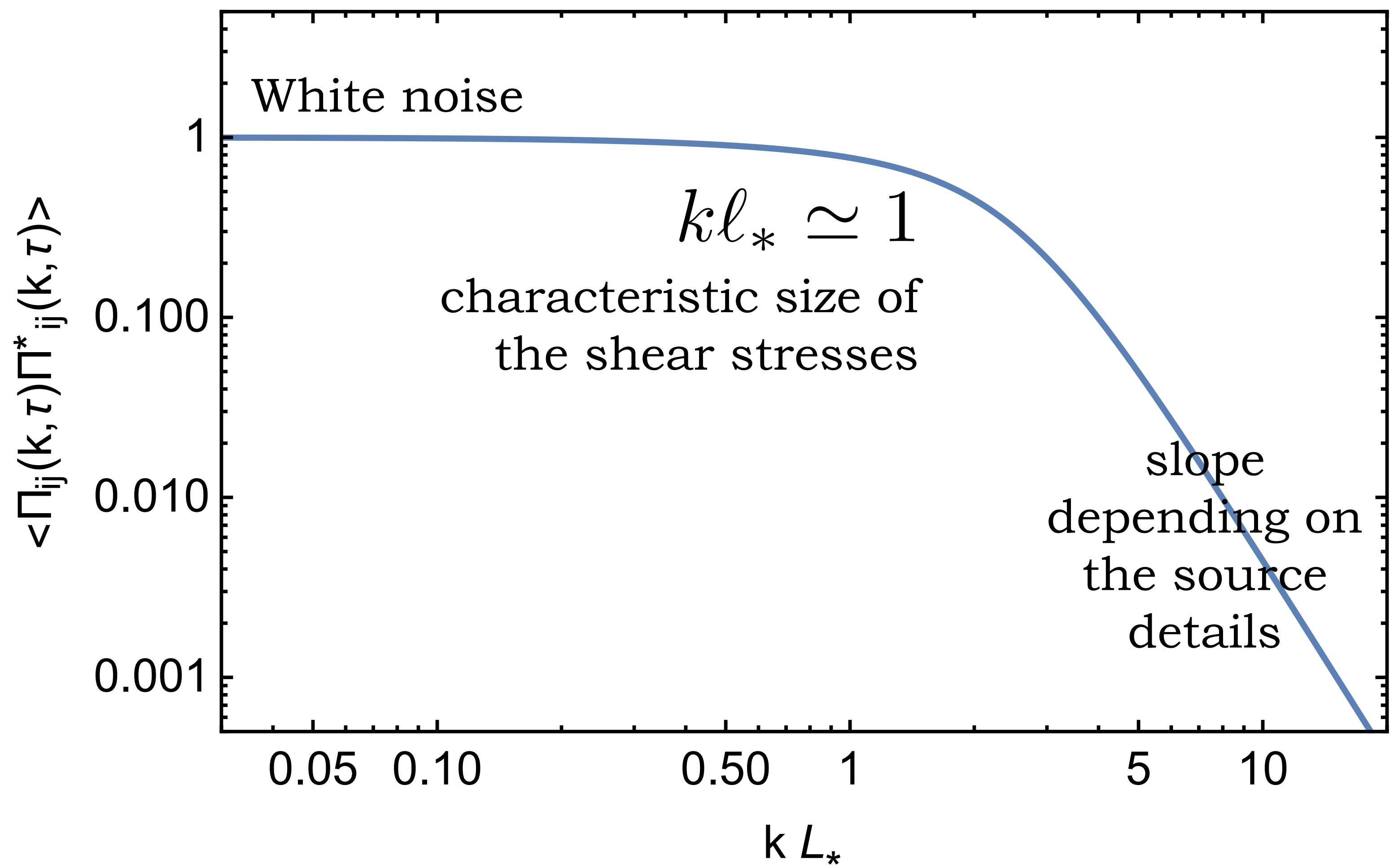
GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \tilde{P}_{\rm GW}(k)$$

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Independent on k for large enough scales (uncorrelated)

$$\boxed{\ell_* \leq H_*^{-1}}$$



C.Caprini

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \tilde{P}_{\rm GW}(k)$$

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

$$1/\eta_0 \ll \underbrace{k \ll \mathcal{H}_*}_{\text{Range of validity of the solution}} \ll \underbrace{\;}_{\text{Causality of the sourcing process}} 1/(a_*\ell_*)$$

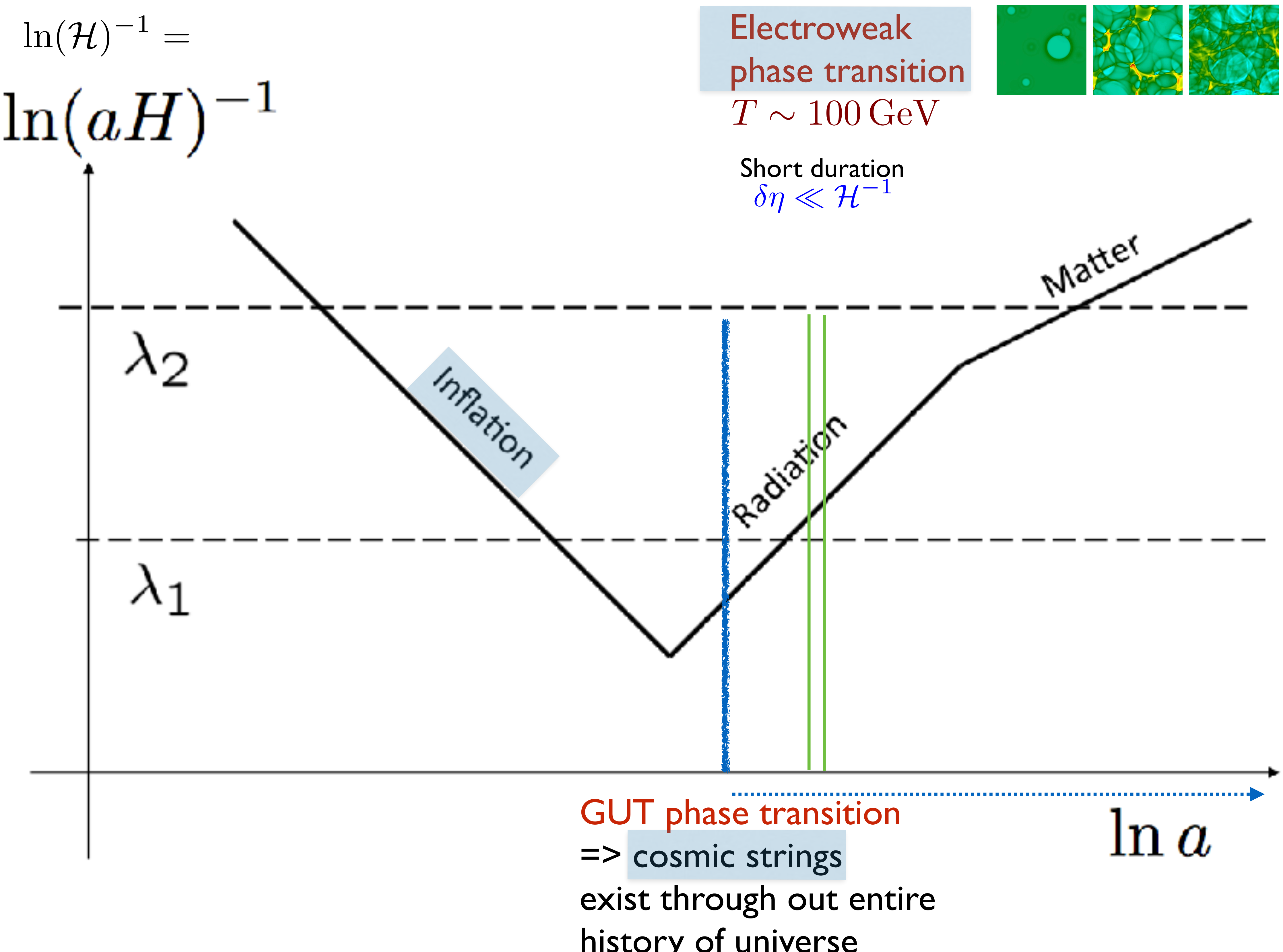
Range of validity of the solution

Causality of the sourcing process

$$\Omega_{\rm GW}(k) \propto \tilde{P}_{\rm GW}(k) \propto k^3$$

C.Caprini

Possible GW sources in the early universe

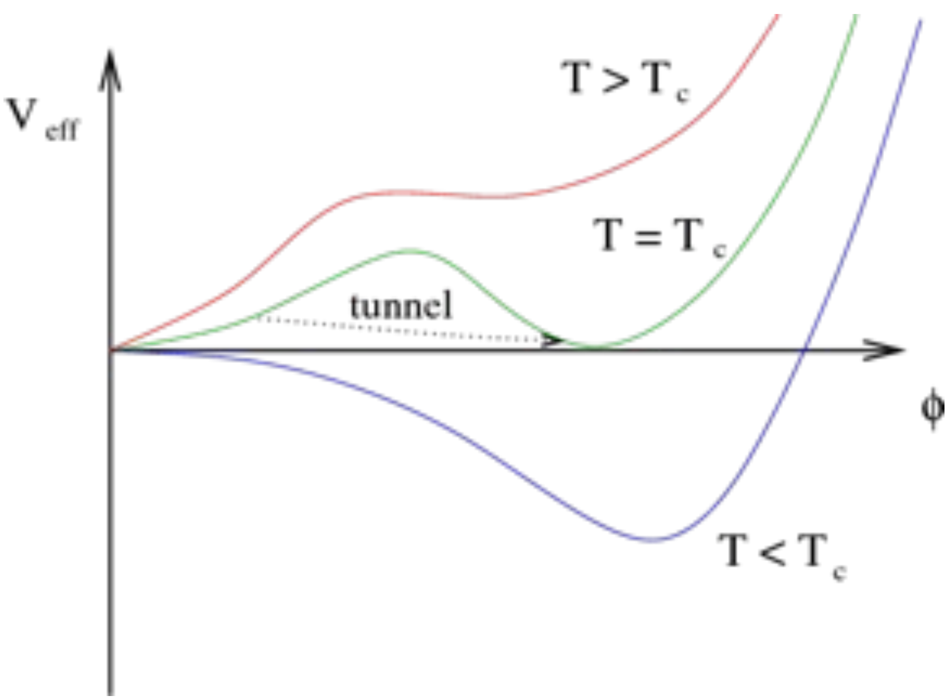


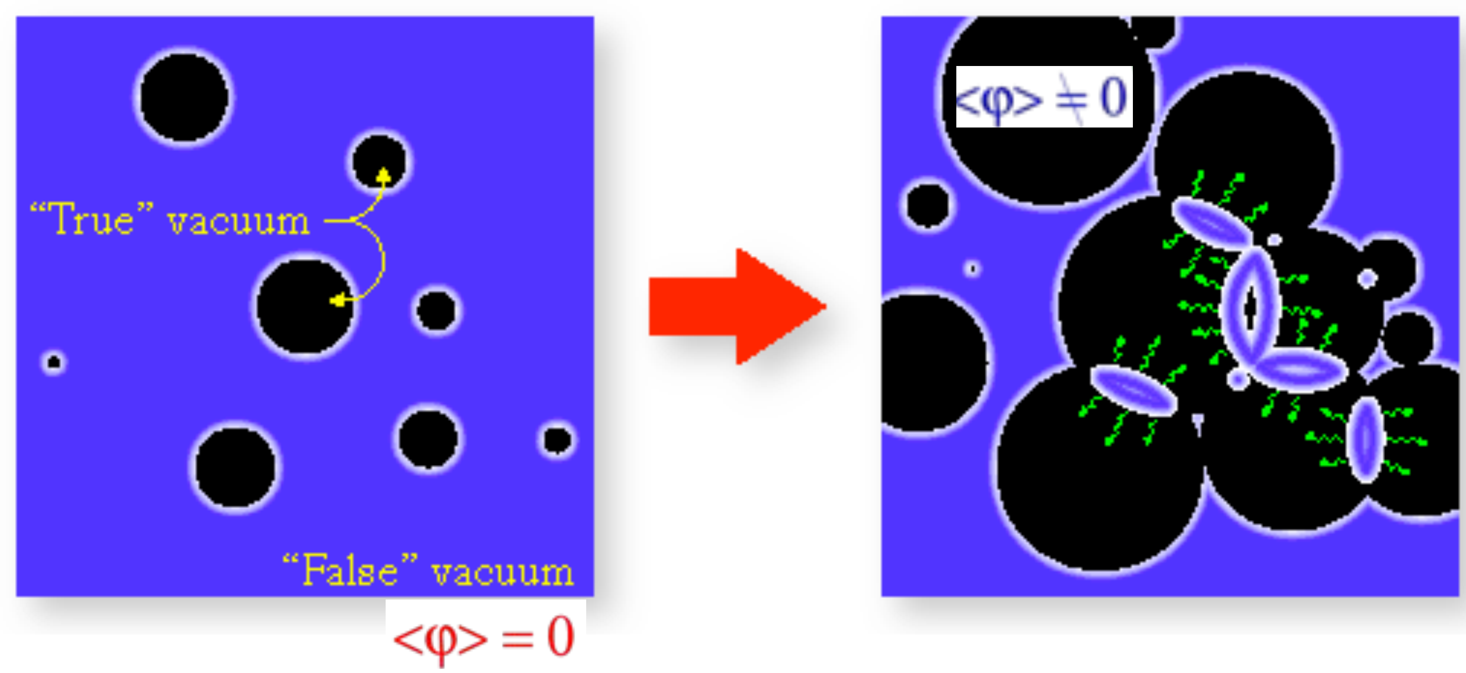
First order phase transitions

• universe expands and temperature decreases : Phase transitions; if 1st lead to GW

• potential barrier separates true and false vacua

quantum tunneling across the barrier : nucleation of bubbles of true vacuum





- collisions of bubble walls
- sound waves and turbulence in the fluid
- primordial magnetic fields (MHD turbulence)

source: Π_{ij} tensor anisotropic stress

[1512.06239]
[1910.13125]

First order phase transitions

- universe expands and temperature decreases : Phase transitions; if 1st lead to GW

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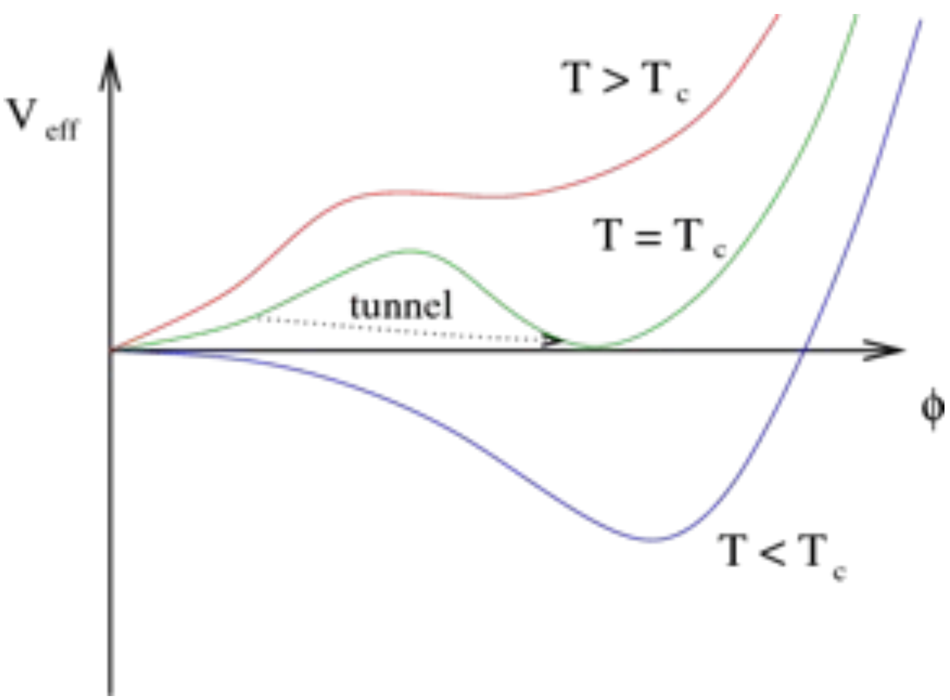
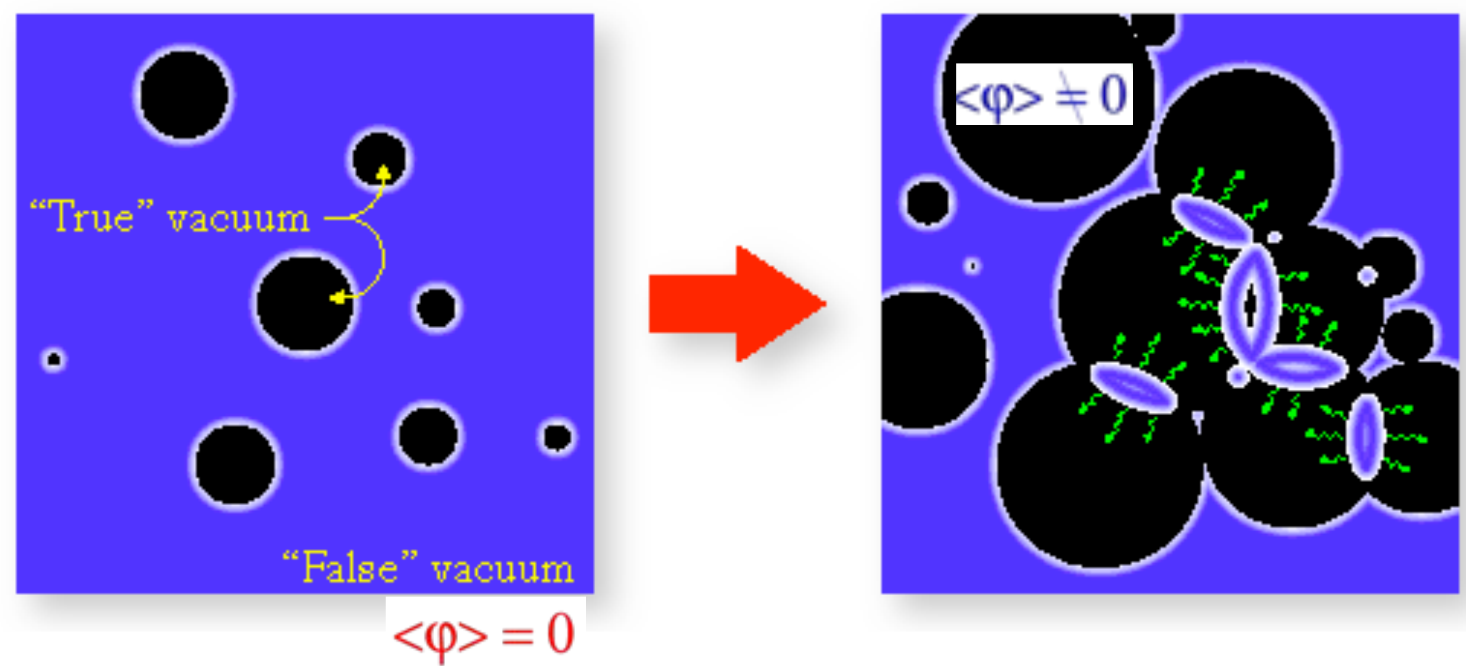
quantum tunneling across the barrier : nucleation of bubbles of true vacuum





$$\Pi_{ij} \sim \partial_i \phi \, \partial_j \phi$$

source: Π_{ij} tensor
anisotropic stress

$$\Pi_{ij} \sim \gamma^2 (\rho + p) \, v_i v_j$$

$$\Pi_{ij} \sim (E^2 + B^2) \frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$$

$$h^2 \Omega_{\rm GW} \approx h^2 \Omega_\phi + h^2 \Omega_{\rm sw} + h^2 \Omega_{\rm turb}$$

Relevant parameters:

$$T_*$$

temperature
of the PT

$$\frac{\beta}{H_*}$$

inverse duration
of the PT with respect to
Hubble time

$$(\epsilon_*)$$

$$\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}}$$

strength
of the PT

$$v_w$$

bubble
wall speed

Putting it all together

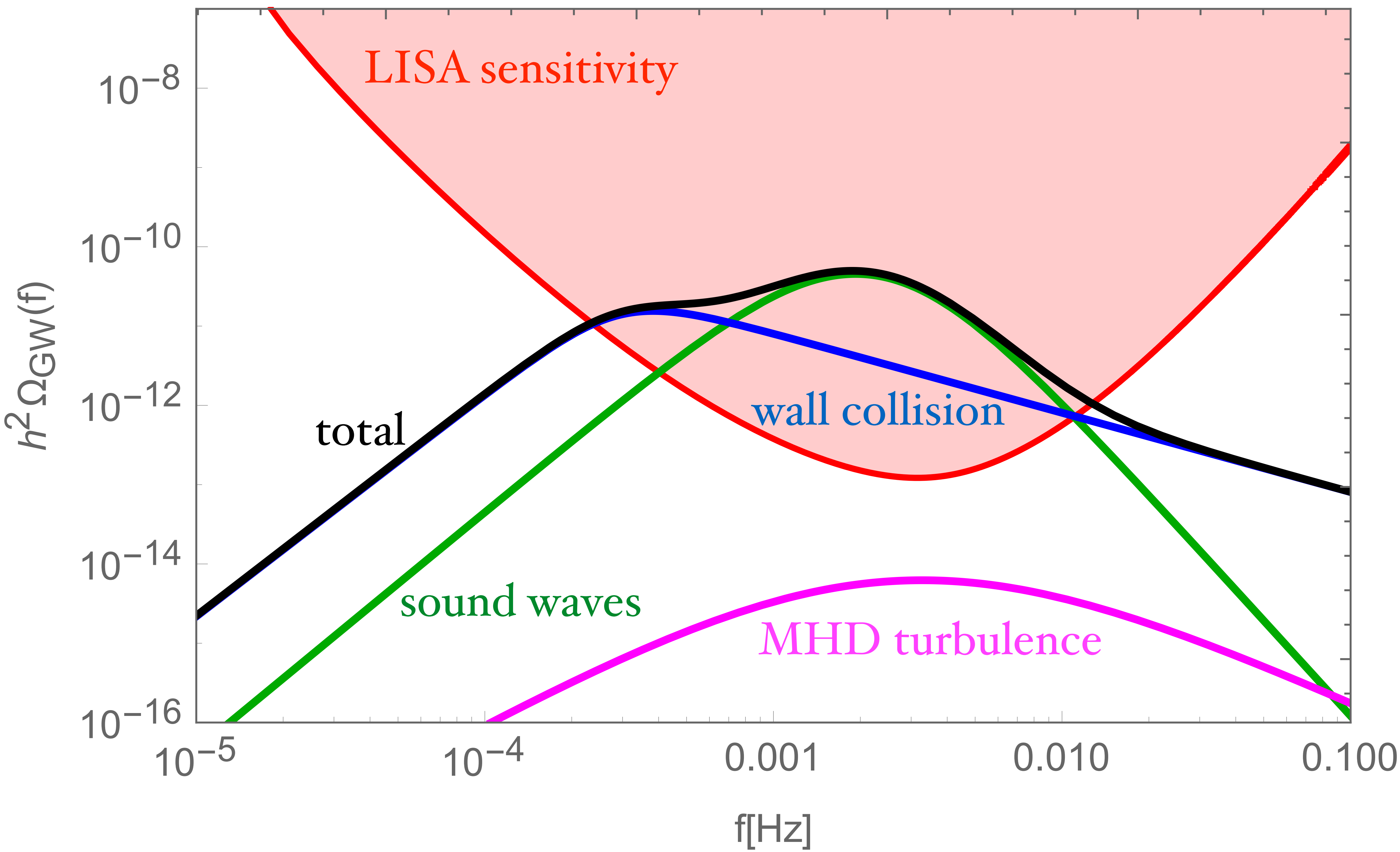
[1512.06239]

- three sources $\approx h^2\Omega_\phi,\, h^2\Omega_{\rm sw},\, h^2\Omega_{\rm turb}$
- know their dependence on the 4 parameters *[Espinosa, Konstantin,No, Servant, Caprini…]*
- predict the signal.

$$(\text{example, } T_* = 100\text{GeV},\, \alpha_{T_*} = 0.5,\, v_{\rm w} = 0.95,\, \beta/H_* = 10)$$

Example of signal

(example, $T_* = 100\text{GeV}$, $\alpha_{T_*} = 0.5$, $v_\mathrm{w} = 0.95$, $\beta/H_* = 10$)

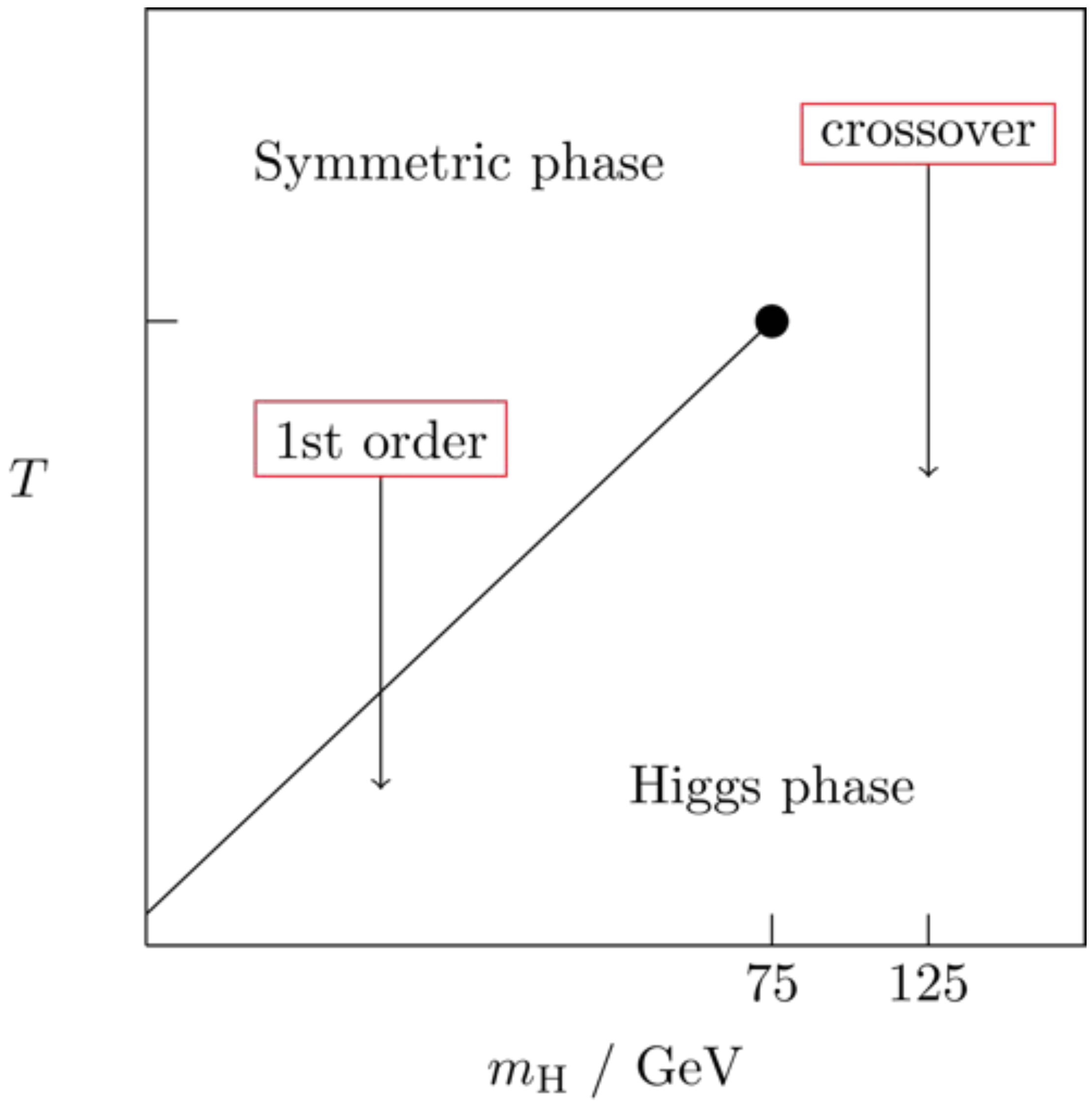


[Caprini et al, arXiv:1512.06239]

EW phase transition?

• Phase diagram for the Standard Model was found in the 1990's

[Kajantie et al, Gurtler et al, Csikor et al]



• With a Higgs mass at 125GeV, the EW PT is a crossover, and NOT first order!

• No significant departure from thermal equilibrium =>no significant GW production or baryogenesis.

• So why bother?????!

• EW PT can be first order in many extensions of the SM.

- singlet extensions of MSSM (Huber et al 2015)
- direct coupling of Higgs sector with scalars (Kozackuz et al 2013)
- SM plus dimension six operator (Grojean et al 2004)

GW background from cosmic strings

Cosmic strings: some basics

[Kibble '76]

– line-like *topological defects*, formed in a symmetry breaking phase transition $G \rightarrow H$
provided the vacuum manifold contains non-contractible loops $\Pi_1(G/H) = \Pi_1(\mathcal{M}) \neq 1$

– A lot of input/interplay with other branches of physics:
- difficult to see cosmic strings in the sky
- "easier" to see strings in the lab (vortex loops in He4, He3, superconductors, strings in NLC...)

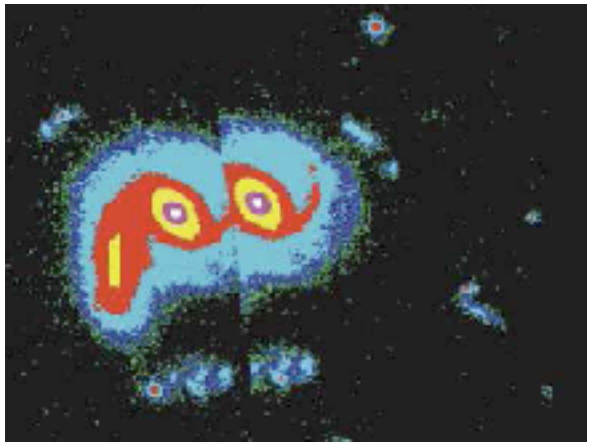
– Generically formed at the end of hybrid-like inflation *[Jeannerot et al 03]*
or in brane inflation *[Jones et al, Sarangi and Tye]*
(cosmic super-strings)

– if formed, should still exist today, they cannot disappear!

– Numerous potentially observable signatures:
Gravitational wave emission;
CMB anisotropies & B-modes;
lensing,....
particle emission
electromagnetic radiation

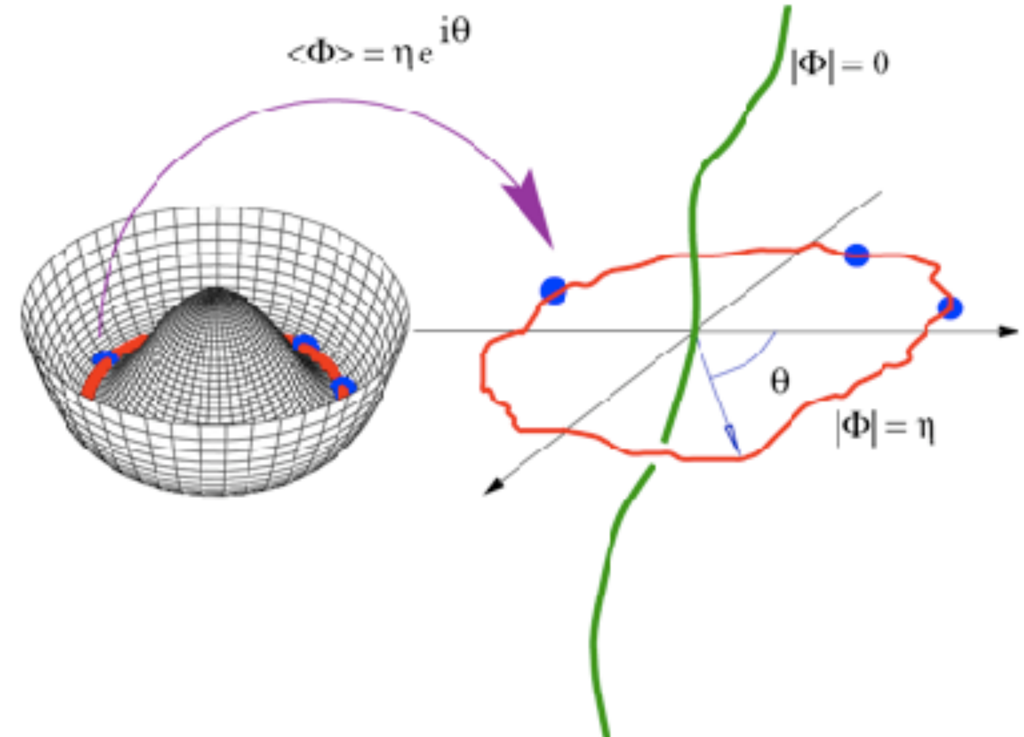
$$\boxed{G\mu \leq \text{few} \times 10^{-7}}$$

[Planck paper XXV]



– Typical example: strings in the Abelian Higgs model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - \frac{\lambda}{4}(|\phi|^2 - \eta^2)$$



infinite, or
closed loops

- Degenerate vacuum/ground state with $\langle\phi\rangle = \eta e^{i\alpha}$
- U(1) invariance $\phi \to \phi e^{i\alpha}$ broken by choice of phase
- String/vortex is a linear defect around which α changes by $2n\pi$

(n = winding number)

$G = U(1) \quad \mathcal{M} = S^1 \quad \Pi_1(\mathcal{M}) = Z$

- Energy/unit length of string: μ_n

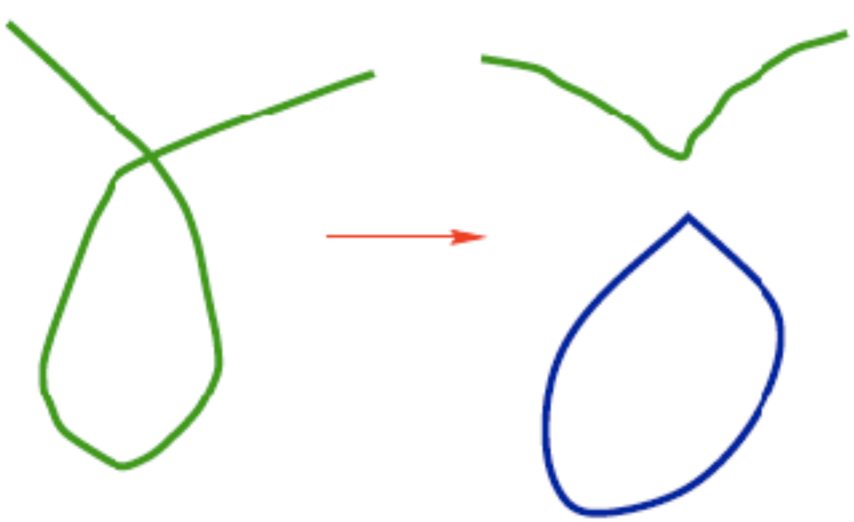
- Scales: $r \sim M^{-1}$ $G\mu \sim G\eta^2 \sim GM^2$

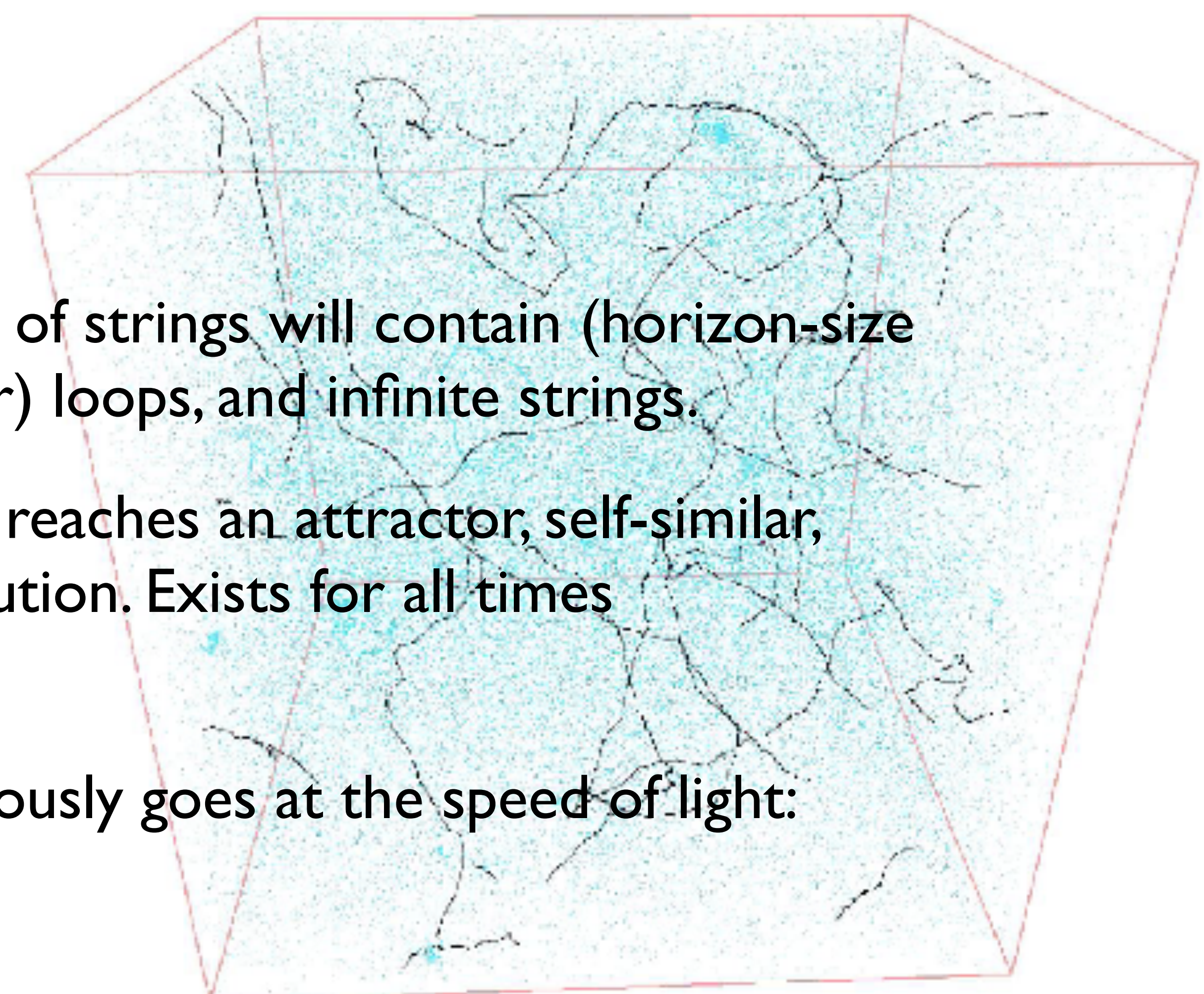
  GUT: $\sim 10^{-32}\mathrm{cm}$ $\sim 10^{-7}$

- Prototypical model of infinitely thin strings: Nambu-Goto strings $S = -\mu \int d^2\sigma \sqrt{-\det(\gamma_{ab})}$
  – Approx. dynamics of relativistic string: action = area of world-sheet

- only one free parameter $G\mu$

- intercommutation:





- Network of strings will contain (horizon-size and smaller) loops, and infinite strings.

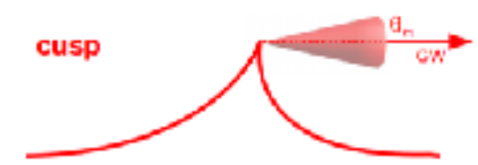
- network reaches an attractor, self-similar, scaling solution. Exists for all times

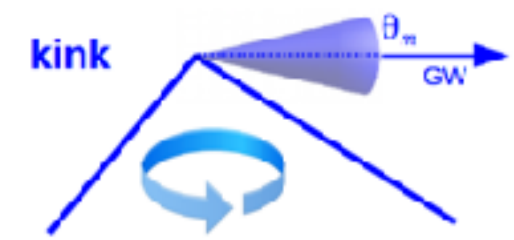
- cusps: points at which the string itself instantaneously goes at the speed of light:
- **kinks** (discontinuity in tangent vector of string)

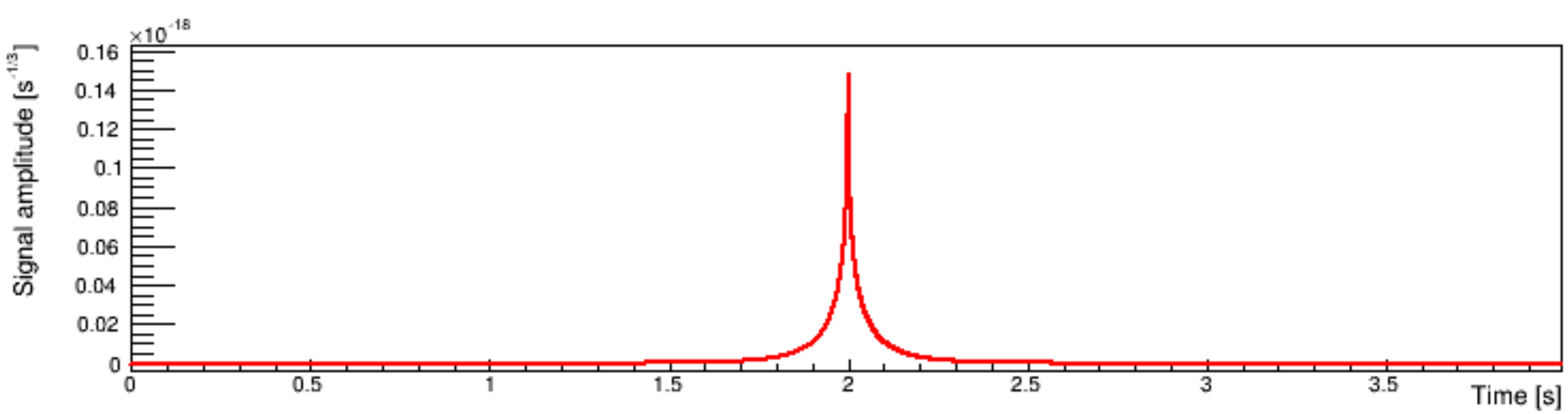
- Cosmic string loops produce 2 types of GW signals

1) sharp, non-gaussian **bursts** of gravitational waves from kinks and cusps. Their characteristic form is directly searched for by LIGO and Virgo.
   [use match-filtering techniques]

[Damour+Vilenkin 2001]







Predicted gravitational waveform produced by a cosmic string cusp.

- Cosmic string loops produce 2 types of GW signals

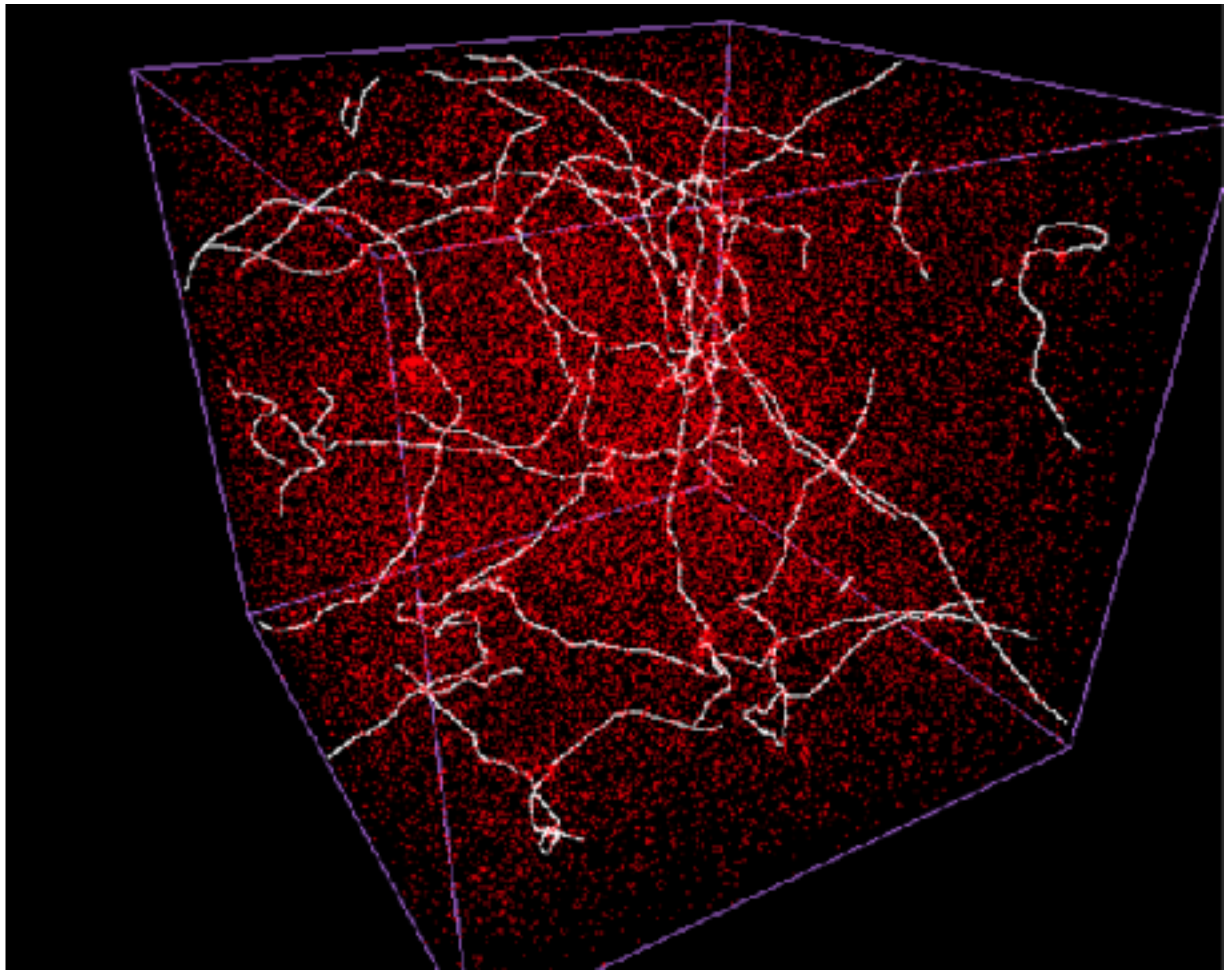
1) sharp, non-gaussian **bursts** of gravitational waves from kinks and cusps. Their characteristic form is directly searched for by LIGO and Virgo.
[use match-filtering techniques] *[Damour+Vilenkin 2001]*

2) A **stochastic GW background** ranging over many decades in frequency

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df},$$

which can be probed by e.g. pulsar timing at nHz frequencies, LIGO/Virgo…,
Sum of all the GWs emitted by oscillating loops

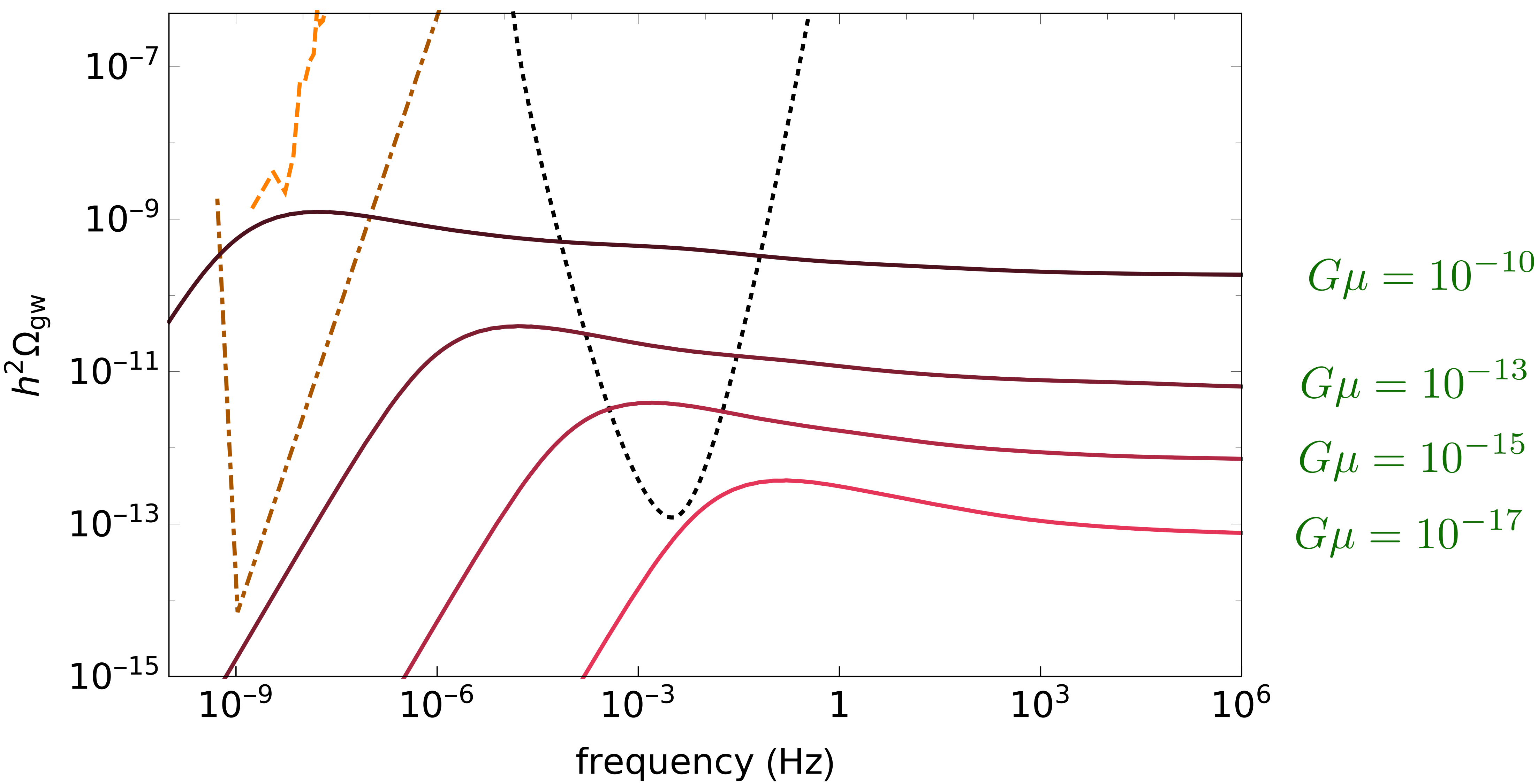
[C.Ringeval]



- Crucial quantity: $n(\ell,t)d\ell$

number density of loops with length between
$\ell \rightarrow \ell + d\ell$ at time t

Model "A"

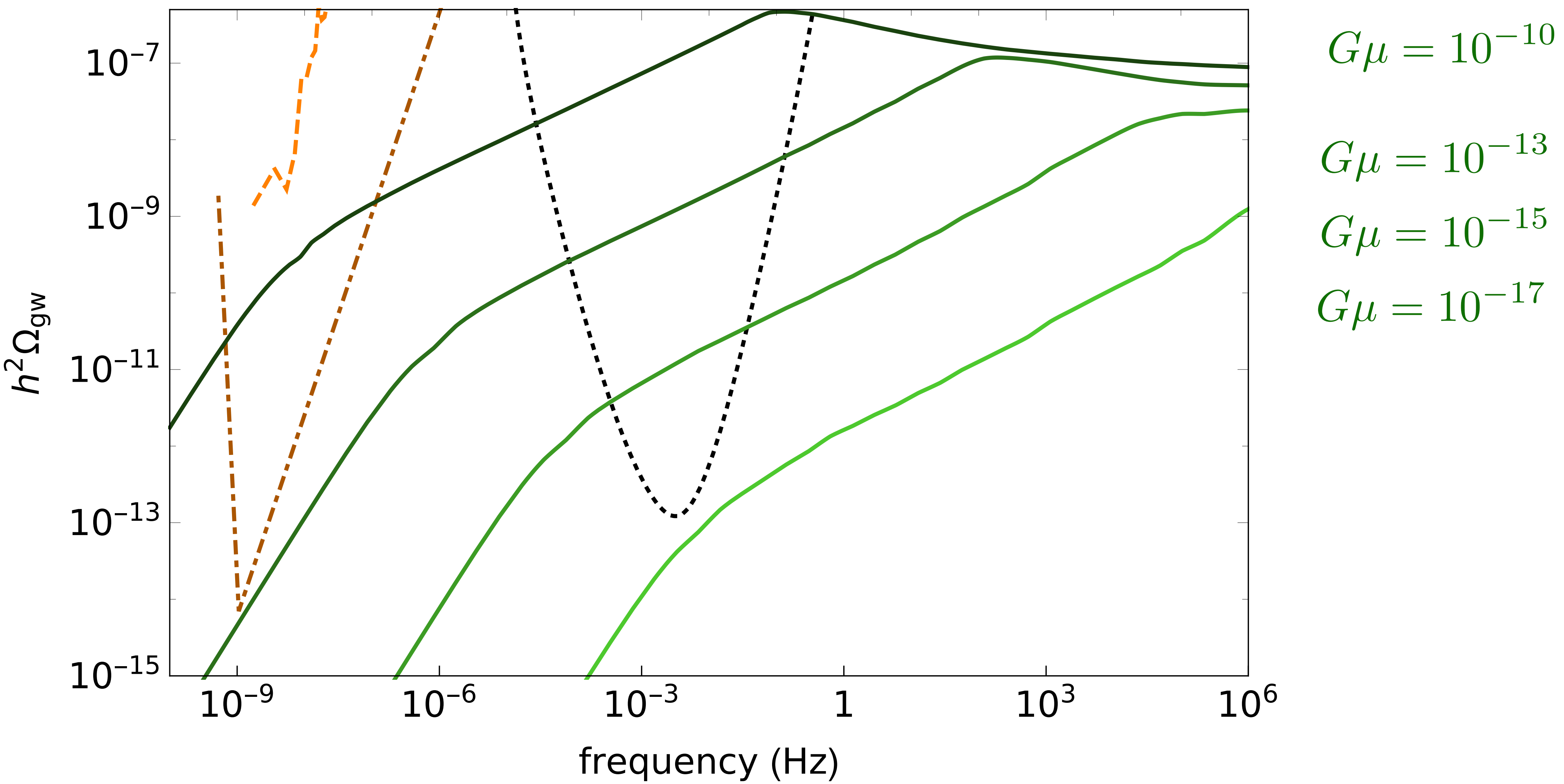


Constraints in the LISA band.

[1909.00819]

broad spectrum spanning many decades in frequency.

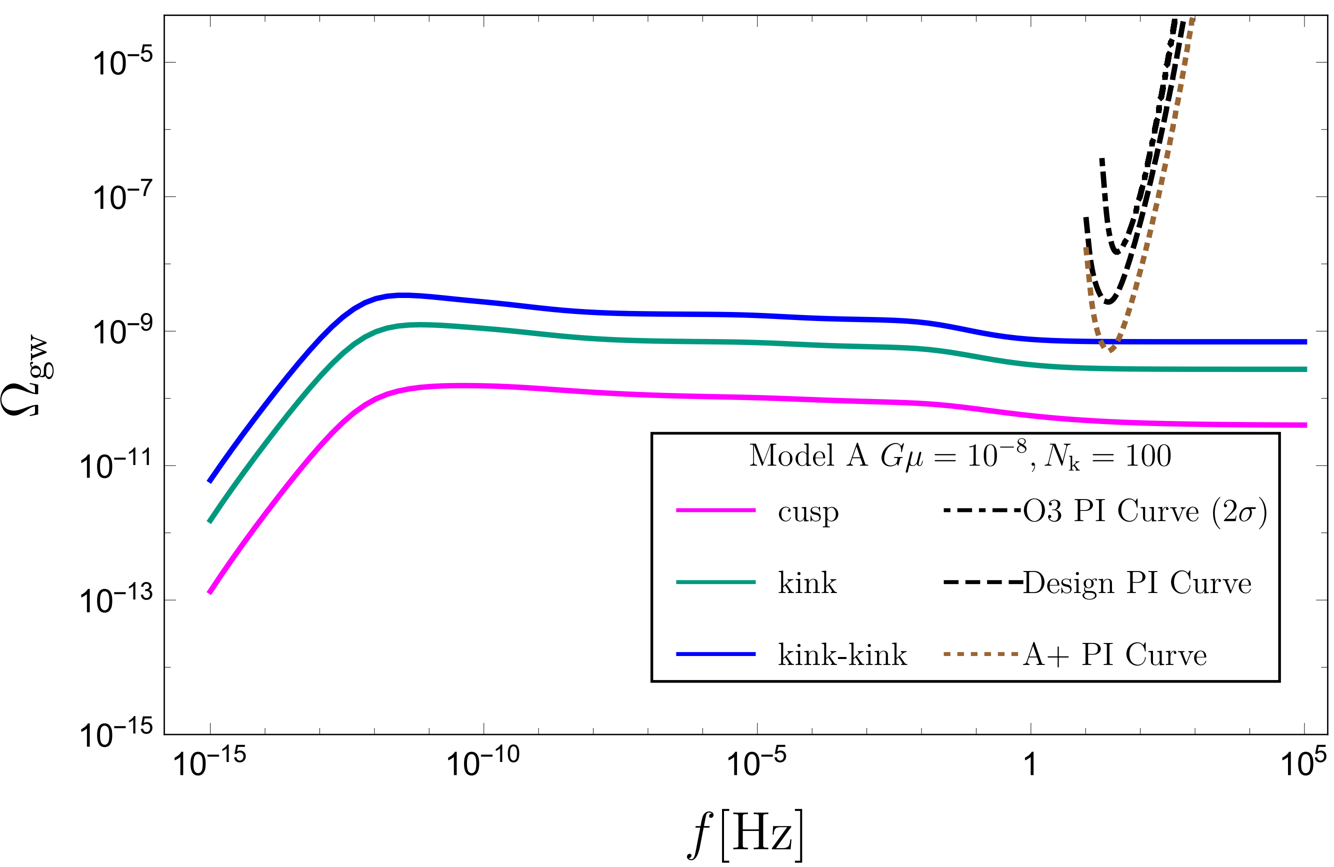
Model “B”



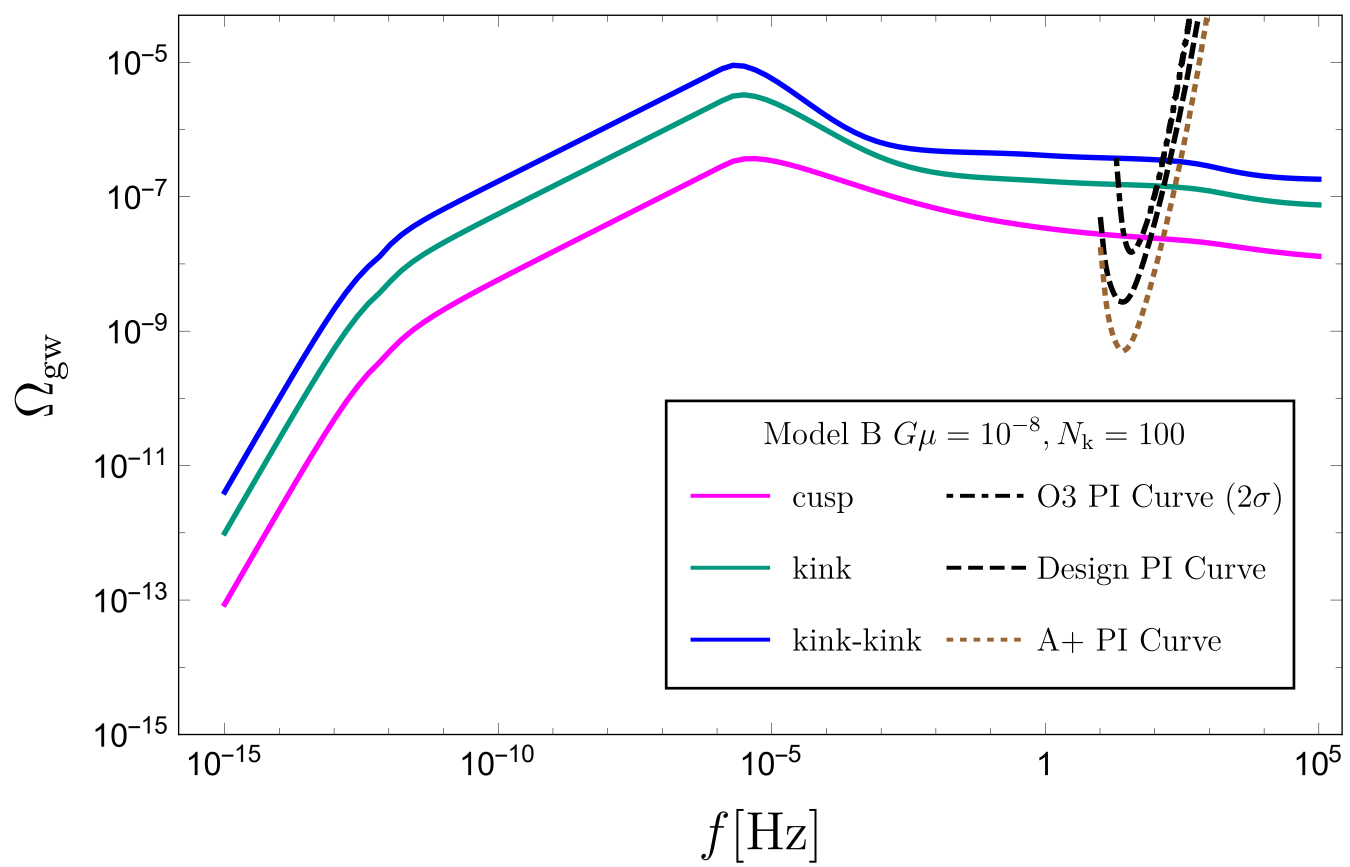
Constraints in the LISA band

[1909.00819]

Model “A”

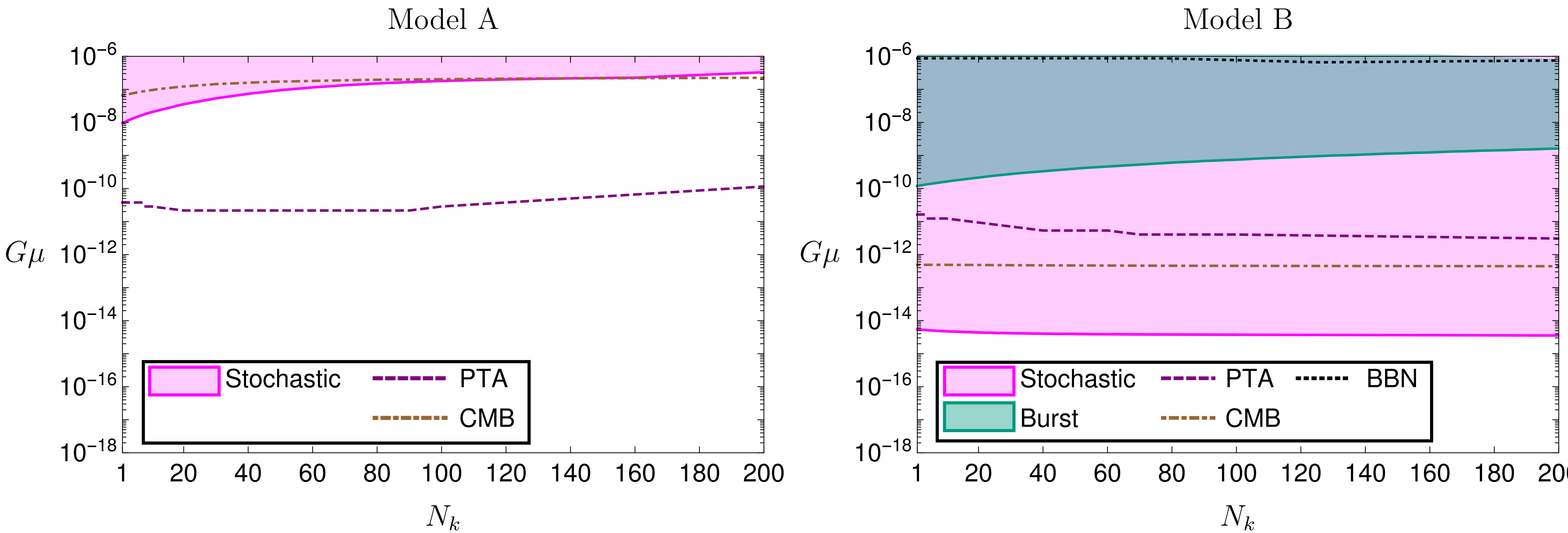


Model “B”



Constraints in the LIGO band

[2101.12248]



Constraints in the LIGO band

[2101.12248]

Conclusion

- SGWB provides a powerful means to probe the early universe, predictions are somewhat speculative, but their potential to probe fundamental physics is exceptional! Amazing discoveries may be around the corner!

- EW phase transition is at the limit of tested particle physics, but any GW signal can be accessed by LISA. This requires models beyond the standard model of particle physics —> tests of models, complementary to particle colliders

- Cosmic strings are theoretically well motivated ("just" require correct topology at GUT phase transition). GW signal spans many decades in frequency, can be accessed/constrained at PTA, LISA, LIGO/Virgo. Still work to do on models…

- Inflation: very much motivated in the standard cosmological scenario, but the standard slow-roll models lead to a SGWB which is tiny (invisible by any detector, PTA, LISA, LIGO/Virgo). Would need a seriously blue-tilted spectrum to be observable at these frequencies.

Gravitational waves and cosmology

late-time universe

Individual sources
and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star- black-hole binary (NS-BH)…

– Expansion rate $H(z)$
– H_0, Hubble constant
– Ω_m
– beyond $\Lambda\mathrm{CDM}$
dark energy $w(z)$ and dark matter
– modified gravity (modified GW propagation)
– astrophysics; eg BH populations, PISN mass gap?

Very early universe
$t \gtrsim t_{Pl}$

Stochastic background
of GWs of cosmological origin

Some motivation for modified gravity

– Understand GR better.
– GR not been tested independently on galactic and cosmological scales: a bit of an extrapolation of our limited knowledge of GR to assume its true on cosmological scales:
– Late time acceleration/dark energy => interest in revising the theory of gravity. Could making gravity weaker on cosmological scales explain late time acceleration?

Some constraints on modified gravity

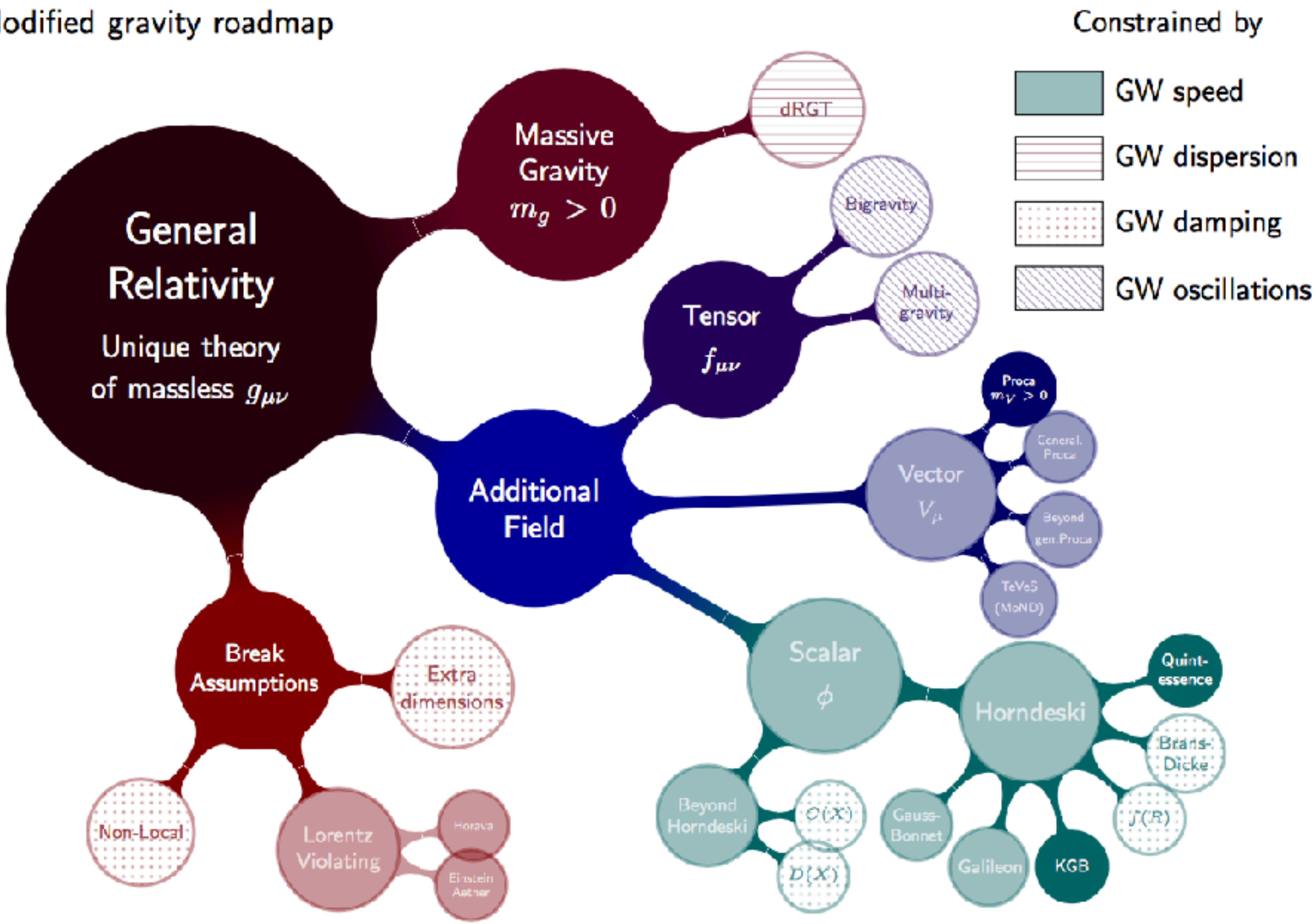
– Any MG theory must reduce to GR on the scales on which this has been tested, i.e. on solar system scales (deflection angle of light by sun, Cassini etc). And *at the same time* lead to significant differences from GR on large scales. "Screening"
– And it must be mathematically consistent and stable.

Some conditions to have modified gravity

Lovelock's theorem: Einstein's equations are the only *2nd order local* e of m for a *metric* derivable from an action in *4D*. As a result, if modify GR, then will necessarily have one or more of:

• Extra d of f.
• Higher derivatives
• Higher dimensional space-time
• Non-locality.

Modified gravity roadmap

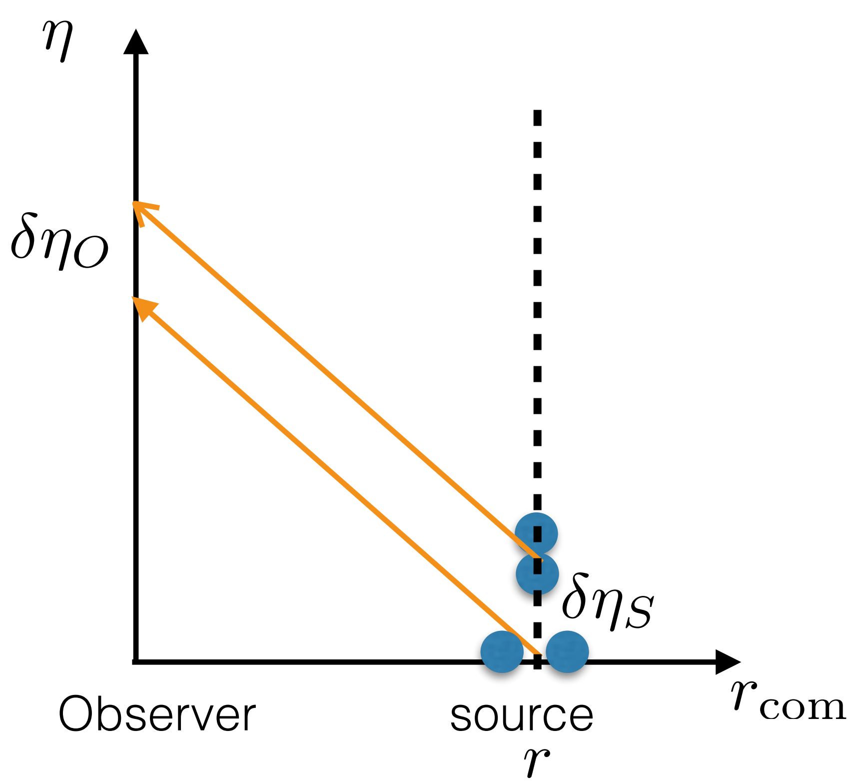


[Ezquiaga and Zumalacarregui 1807.09241

Modified gravity: effect on GWs from binaries?

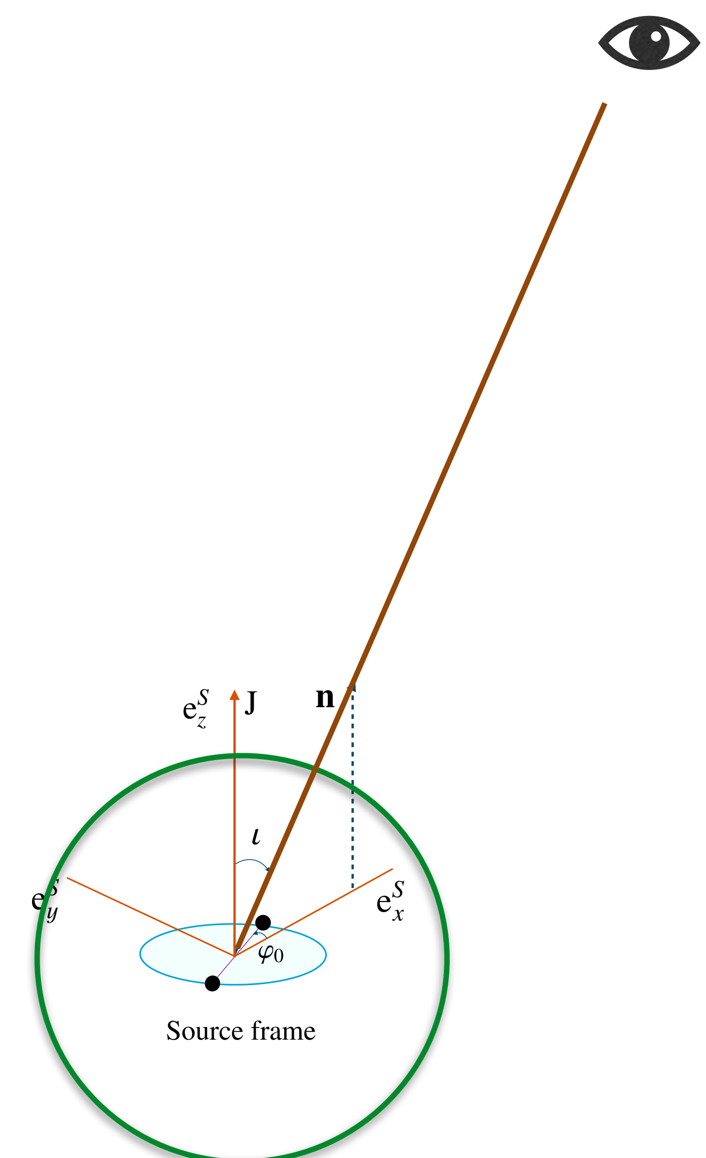
Turn on expansion in FRW universe.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$$



Idea: in local wave-zone of the source (scales, large relative to source, small relative to Hubble), know solution

Then propagate it in FRW space-time to observer



$$r_{\rm phys} = a(t_{\rm emis})r$$

$$a(t) \sim \text{constant}$$

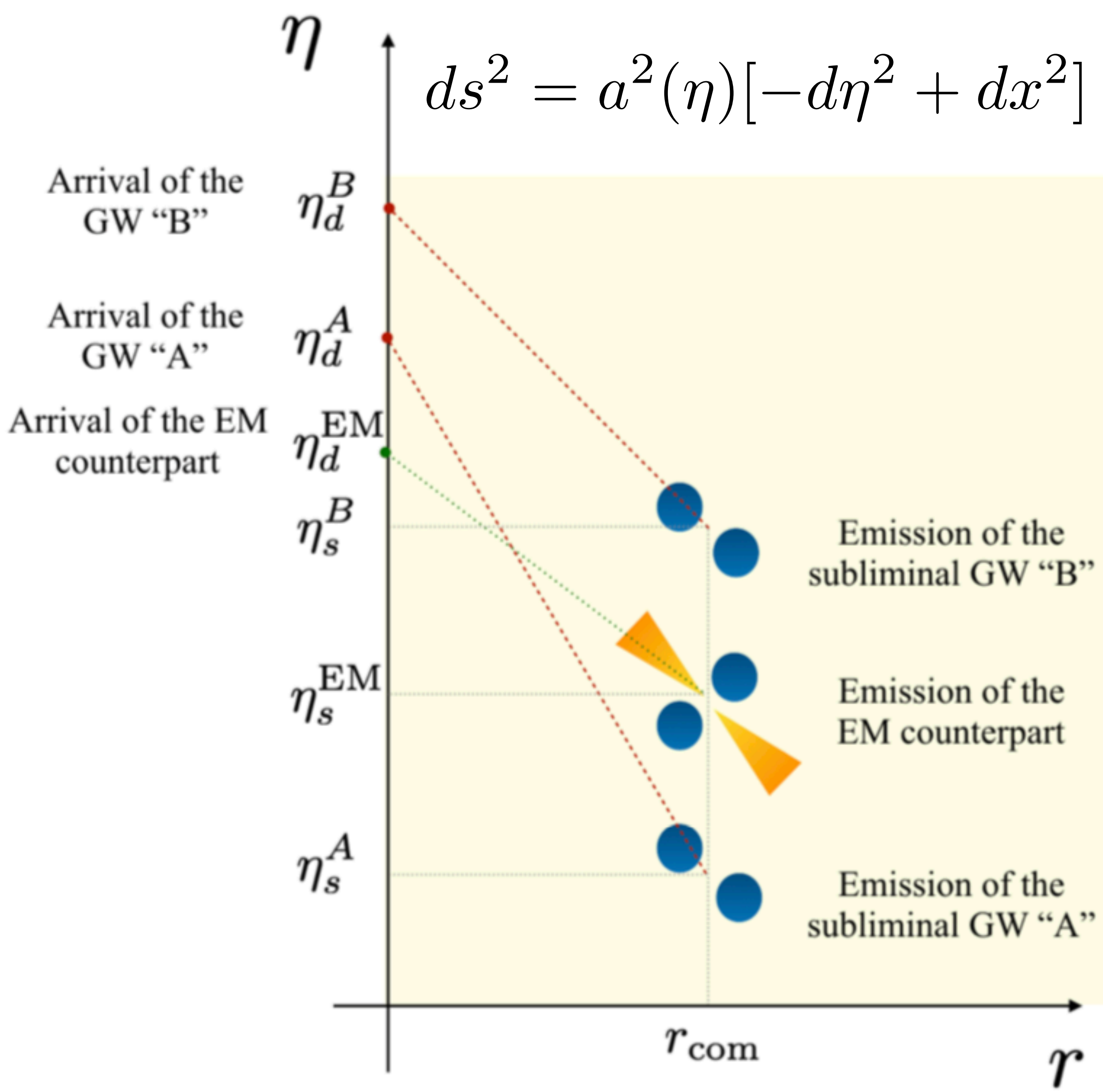
Modified propagation equation!

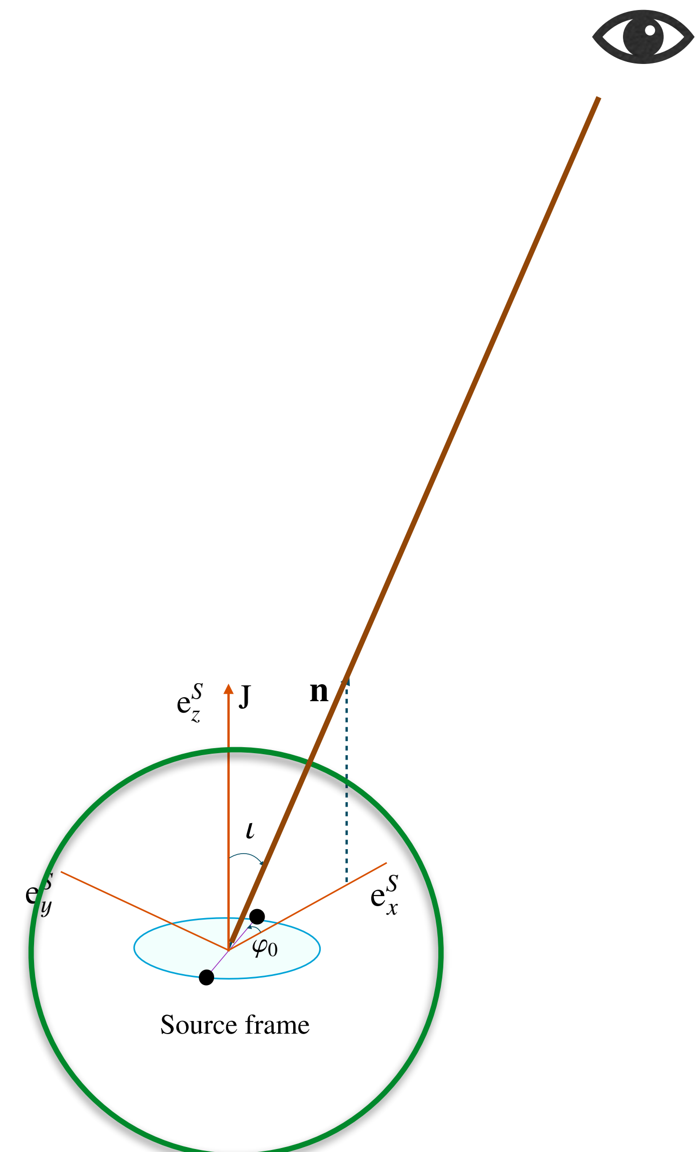
(photons unaffected)

– Assume

- background is exactly as in Lambda CDM: modified gravity alters only the dynamics of perturbations (scalar and tensor).
- Modified gravity affects the *propagation* of GWs; do **not** consider effect on their *generation* at the source (assume GR waveform).
- Consider only standard 2 TT polarisations of GWs (no extra polarisations)

Modified gravity: effect on GWs from binaries?





$$r_{\rm phys} = a(t_{\rm emis})r$$

$$a(t) \sim \text{constant}$$

$\boxed{\text{Modified propagation equation!}}$

(photons unaffected)

– Modified GW *propagation* effects:

1) A **modified dispersion relation**,

$$E^2 = p^2c^2 + m_g^2c^4 + A_\alpha p^\alpha c^\alpha$$

=> GWs have a frequency dependent speed, different from speed of light

[GW train travels at slightly different velocities, as a function of the frequency of the wave, leads to GW dephasing which accumulates over cosmological distances travelled from source to detector. If not detected -> constraints]

2) **Modified friction term**, namely relative to GR, GW energy dissipates differently with the expansion of the universe.

[Therefore effects amplitude of wave, => will enter into the luminosity distance which will be modified. Expect a degeneracy with H0.]

$$\boxed{h'' + 2[1 + \alpha_M(\eta)]\frac{a'}{a}h' + k^2 c_T^2(\eta, k/a)h = 0,}$$

friction term

phase velocity from dispersion relation

$$h'' + 2[1 + \alpha_M(\eta)]\frac{a'}{a}h' + k^2 c_T^2(\eta, k/a)h = 0,$$

friction term

phase velocity from dispersion relation

Group velocity: from modified dispersion relation

$$\boxed{v_g \simeq c\left[1 - \frac{\hat{\alpha}_j}{2}\left(\frac{f_d}{a}\right)^j\right]}$$

<=>

$$j = \alpha - 2.$$

$$E^2 = p^2c^2 + m_g^2c^4 + A_\alpha p^\alpha c^\alpha$$

Massive gravity, j=-2.

Observable quantities

$$\boxed{h'' + 2[1+\alpha_M(\eta)]\frac{a'}{a}h' + k^2 c_T^2(\eta, k/a)h = 0,}$$

- **Modified luminosity distance:** $h \propto \frac{1}{d^{\rm GW}}$

$$d^{\rm GW}(z) = d_{\rm EM}(z)\exp\left[\int_0^z \frac{\alpha_M(z)}{1+z}dz\right]$$

M. Lagos et al Phys. Rev. D 99, 083504 (2019),

- **Time delay between EM waves and GWs:**

$$\Delta t_d^{\rm GW-EM} = (1+z_s)\tau + \frac{f_{R,d}^j}{2}\mathcal{T}_j$$

$$\mathcal{T}_j \equiv \int_0^{z_s} dz' \hat{\alpha}_j \frac{(1+z')^j}{H_0\sqrt{\Omega_{m,0}(1+z')^3+\Omega_\Lambda}},$$

[$f_{R,d}$ = GW reference frequency used to compute the time-delay, eg merger frequency, tau = GRB-GW emission time-delay at the source]

- from frequency dependence of GW velocity, modified **phase evolution** of GWs relative to GR.

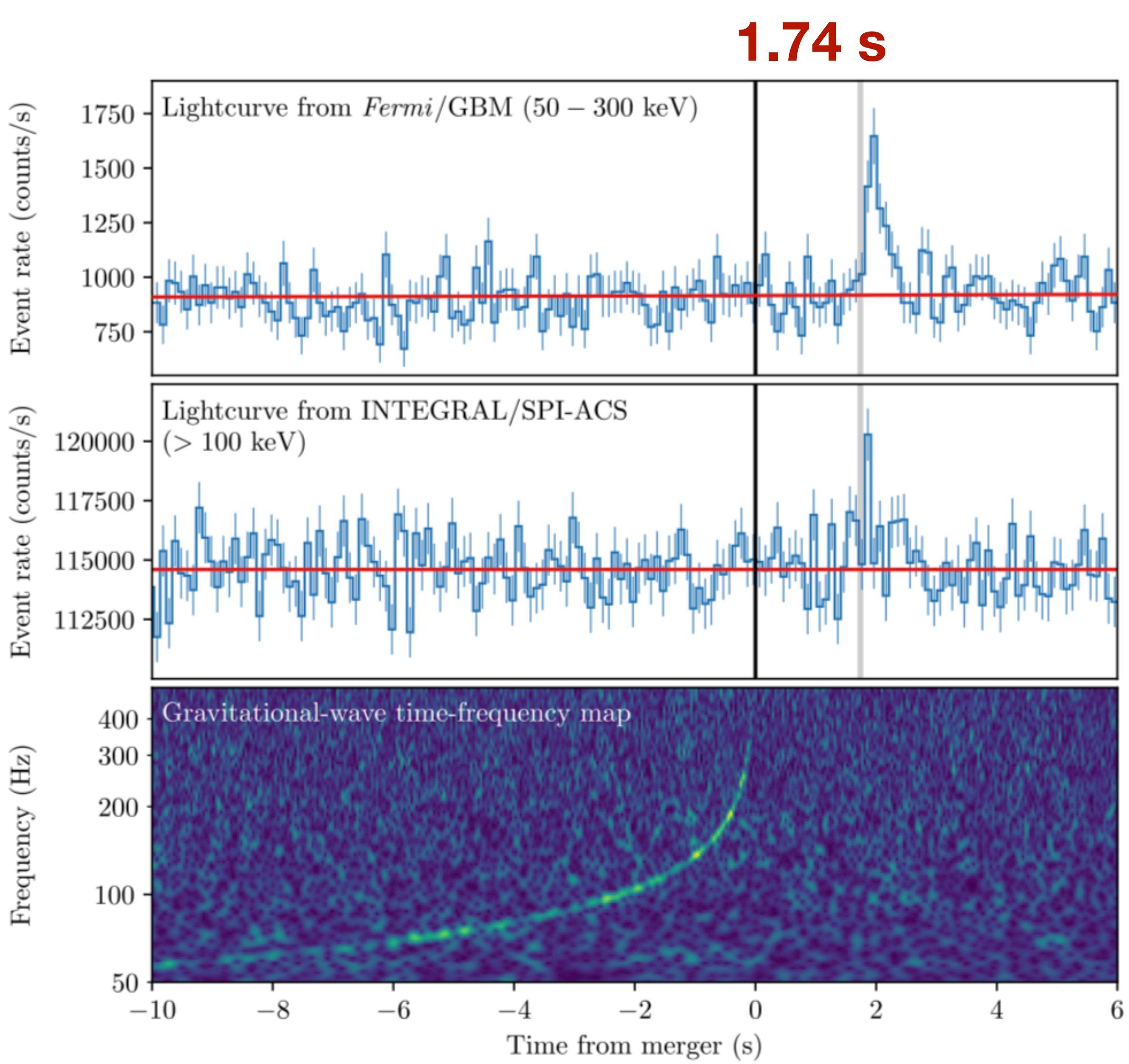
$$\psi(f_d) = \psi_{\rm GR}(f_d) + \pi\mathcal{T}_j\frac{f_d^{j+1}}{j+1}, \quad \text{when } j \neq -1$$

$$\psi(f_d) = \psi_{\rm GR}(f_d) + \pi\mathcal{T}_j \ln f_d, \quad \text{when } j = -1,$$

Mirshekari, S., Phys. Rev. D 85, 024041 (2012)

All these estimators depend on the redshift of the source and H0

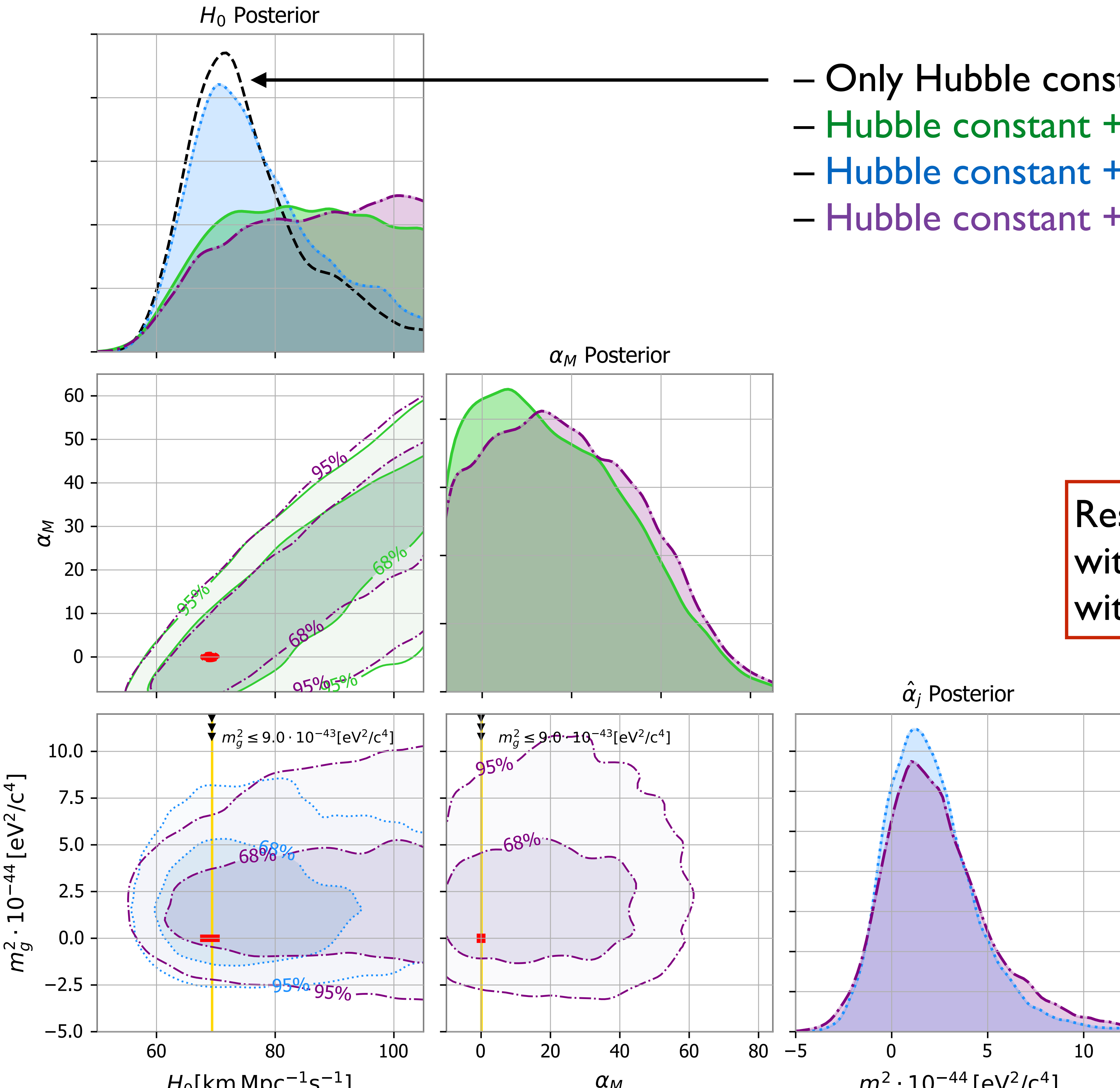
Data from GW170817



B. P. Abbott +, APJL, 848:L13 (2017)

$$-3 \times 10^{-15} \leq \frac{c_{GW} - c}{c} \leq +7 \times 10^{-16}$$

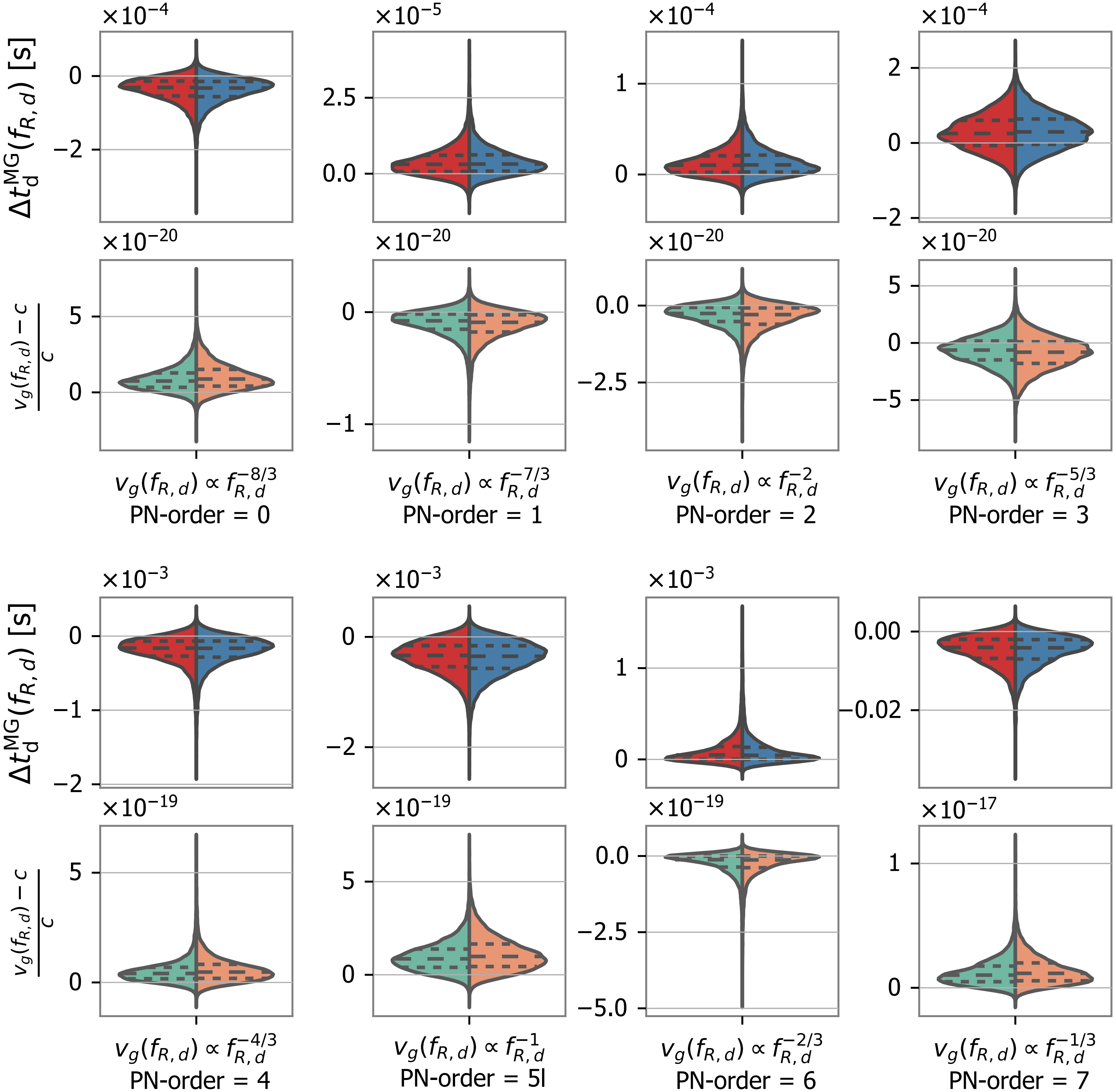
Results with GW170817 (massive gravity)



– Only Hubble constant [check of method]
– Hubble constant + GW friction
– Hubble constant + GW dispersion
– Hubble constant + GW friction + GW dispersion

Results in **good agreement** with GR (m_g = 0 and friction=0) within 1 sigma in all cases.

j=-8/3, -7/3,….-1/3



time delay due to modified gravity negligible (relative to current uncertainty on observed time delay, ~0.05 for all dispersion relations studied => tight constraint on alpha_j and hence on modified speed

speed GWs at the merger Constrained to a $\lesssim 10^{-17}$ precision

red/green: without friction term
orange/blue: with friction term