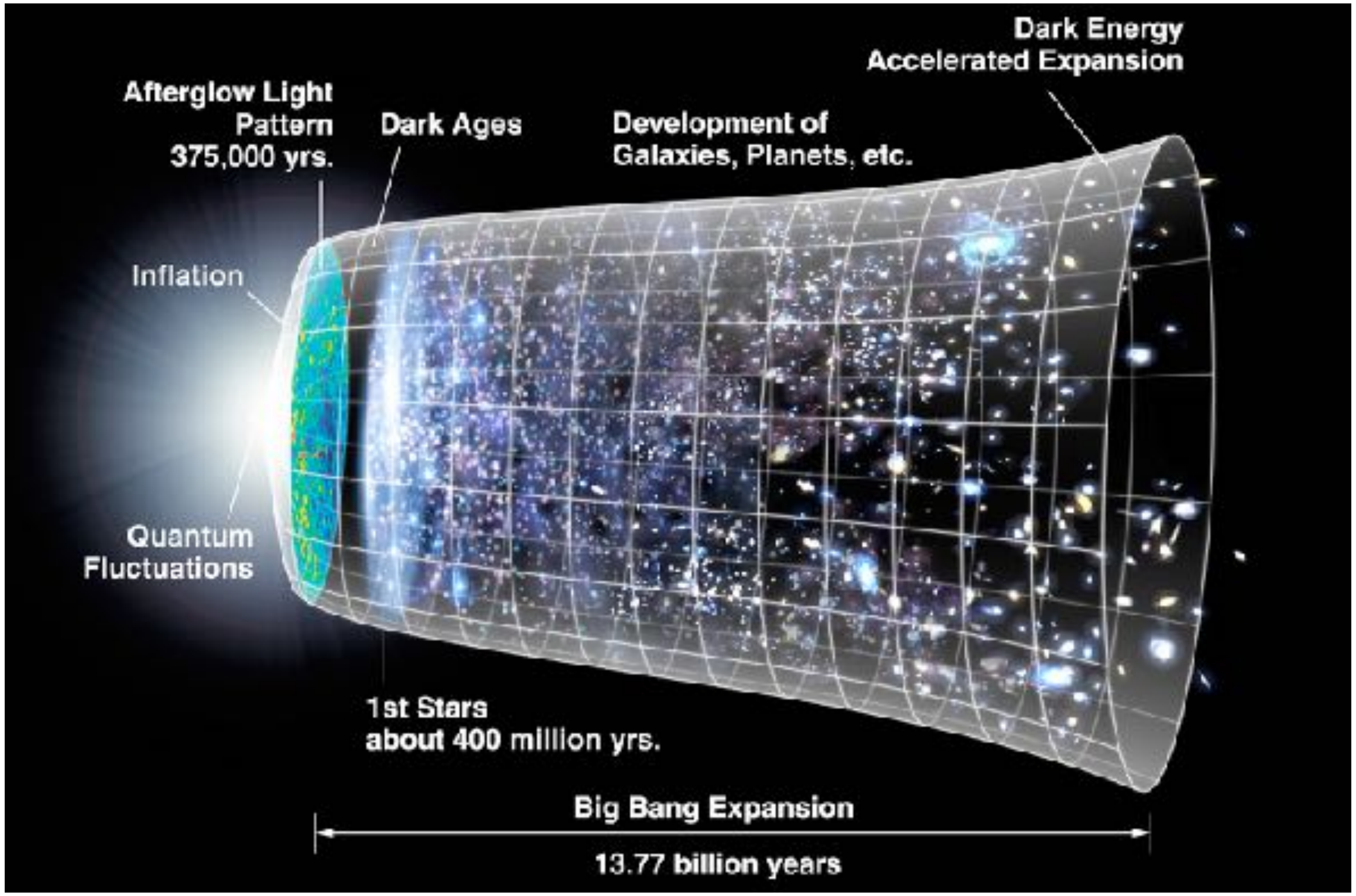
Gravitational waves and cosmology



Danièle Steer
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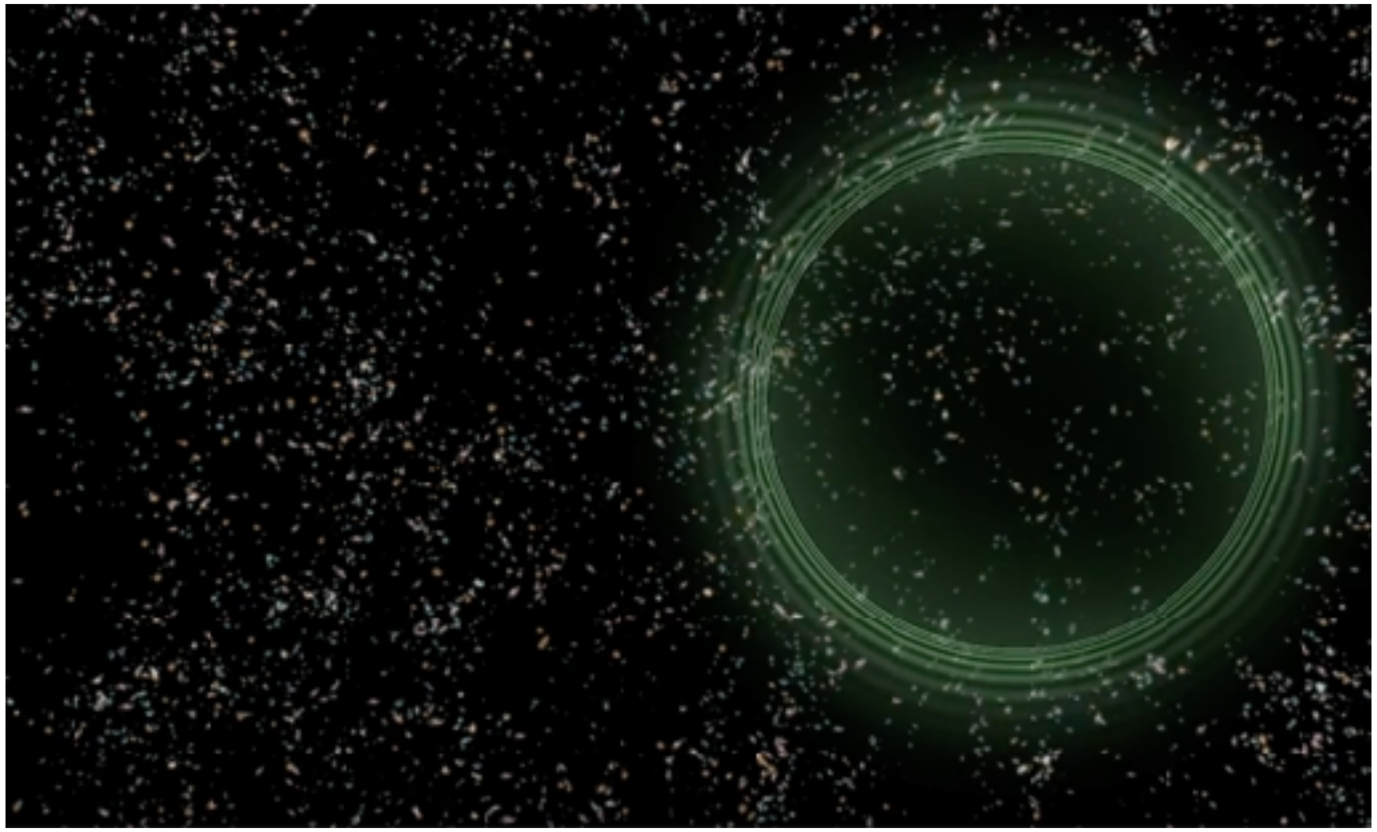
Gravitational waves and cosmology



1) properties of the late-time universe

- *from observed GW signal from **individual** sources (binaries) at cosmological distances*

e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star- black-hole binary (NS-BH)…

[LIGO-Virgo: BNS event GW170817 with its EM counterpart;
BBH and NS-BH events in GWTC-2 [*arXiv:2010.14527]*.
Most distant source at distance ~ 5Gpc, i.e z ~0.75

Gravitational waves and cosmology

1) properties of the late-time universe

– FLRW universe:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

– Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$

– redshift: $1 + z = \frac{a(t_0)}{a(t)}$

– Hubble parameter

$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_\Lambda (1+z)^{3(1+w(z))}\right]^{1/2}$$

dark energy equation of state $\frac{p_{\rm DE}(z)}{\rho_{\rm DE}(z)}$

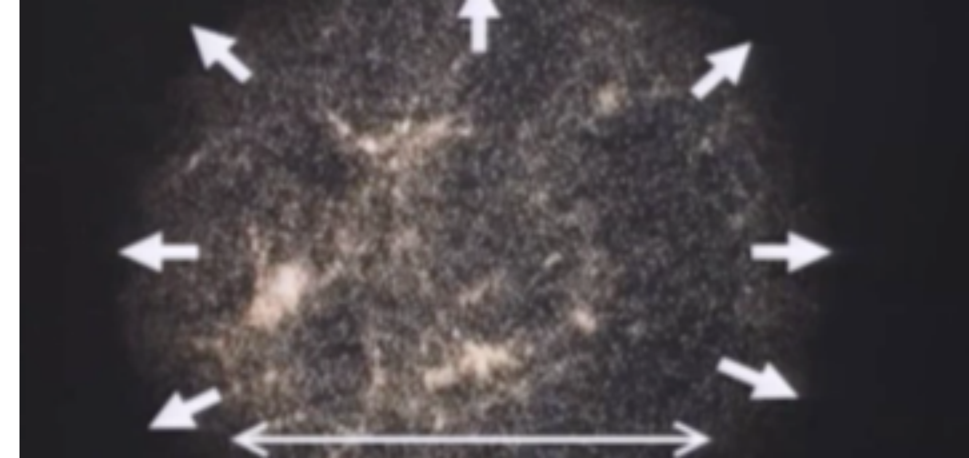
Hubble constant

Characterizes local time-scale in universe

$c/H_0 \sim \mathrm{Gpc}$

energy fraction in matter today

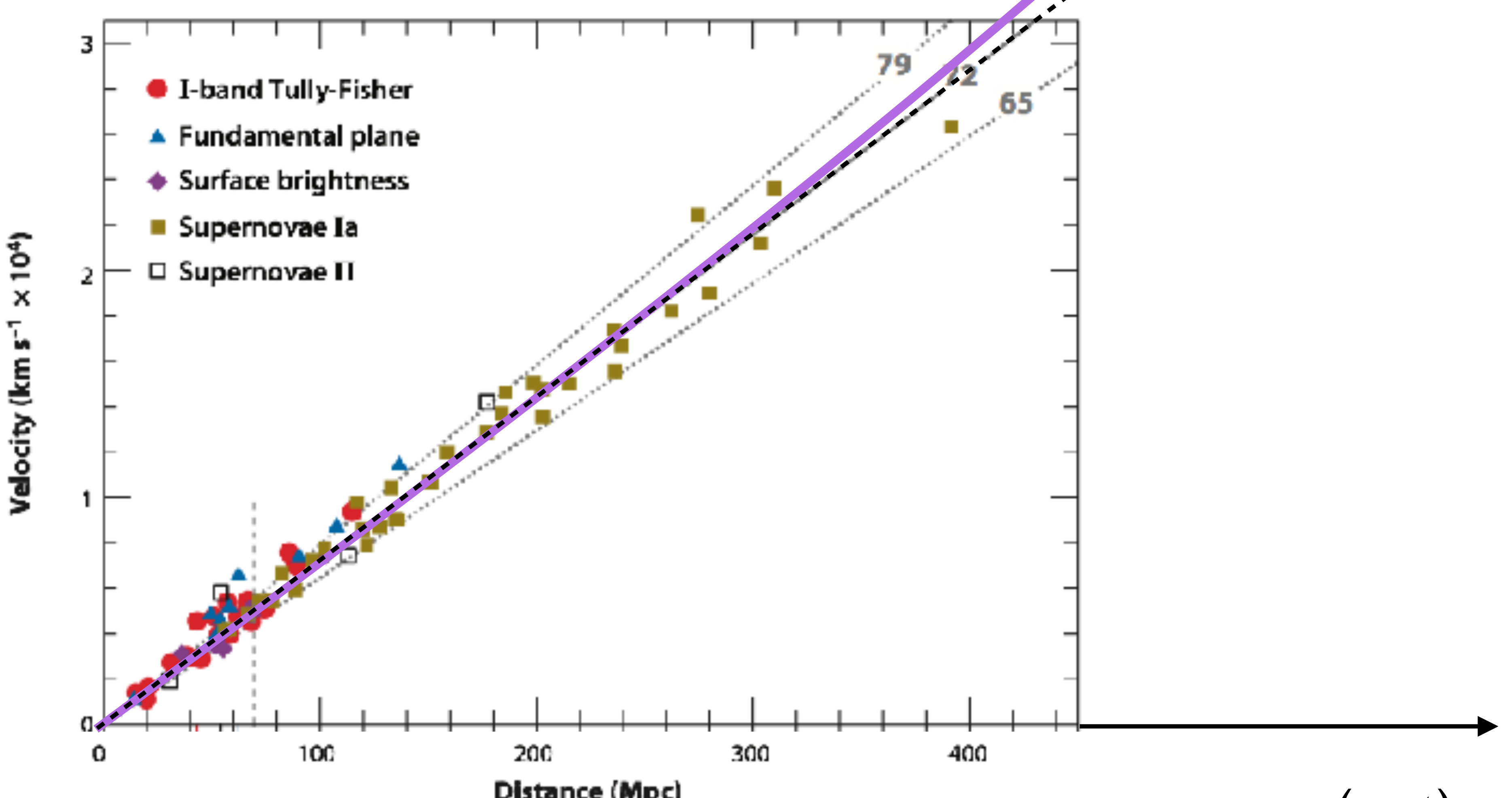
Dark energy



Gravity is an attractive force…
=> So one would expect the expansion of the universe
to decrease with time.
=> deceleration: in the past, objects were faster

• In the nearby universe (Hubble law)

$$v_H = H_0 \times d$$



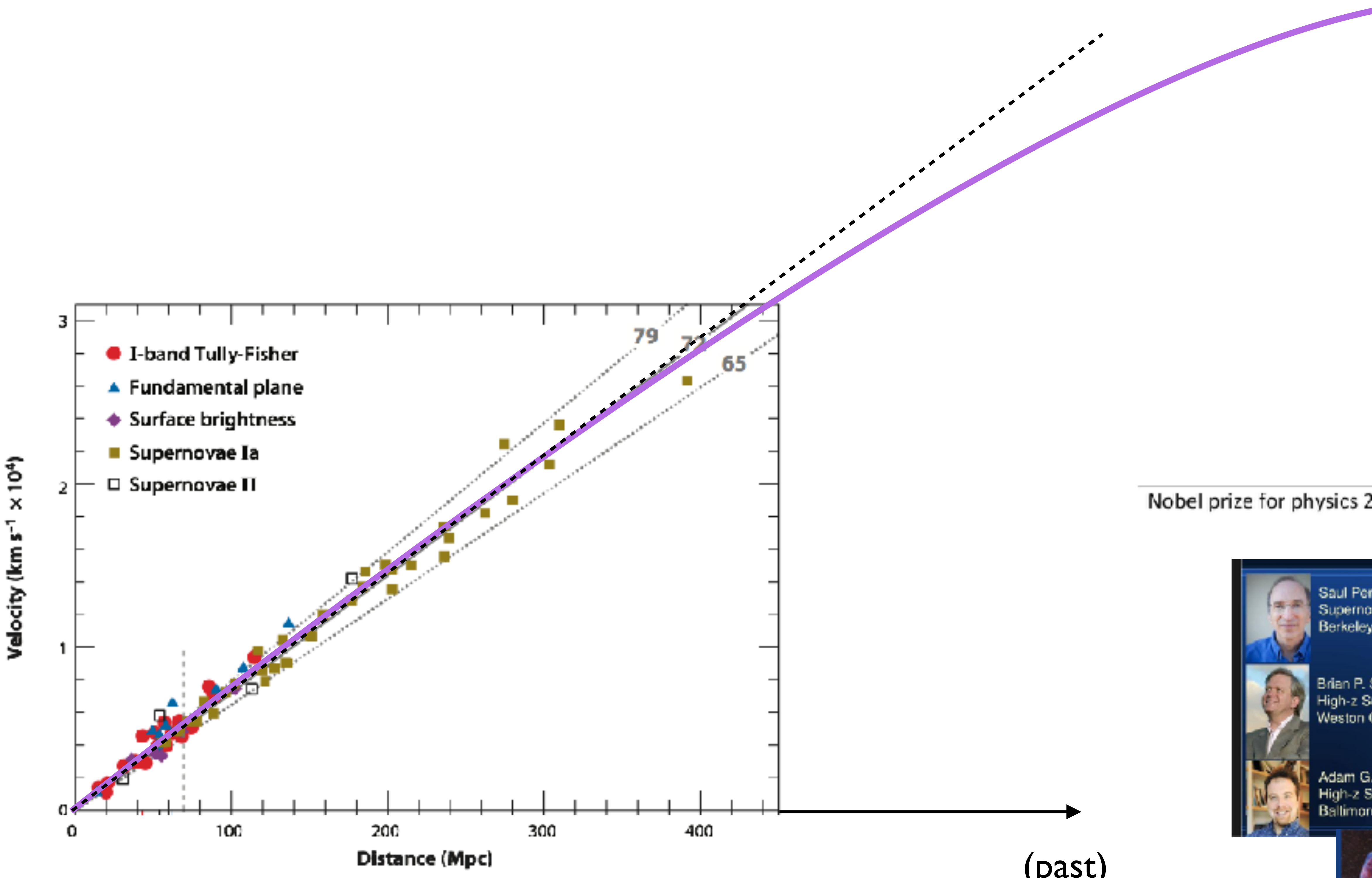
(past)

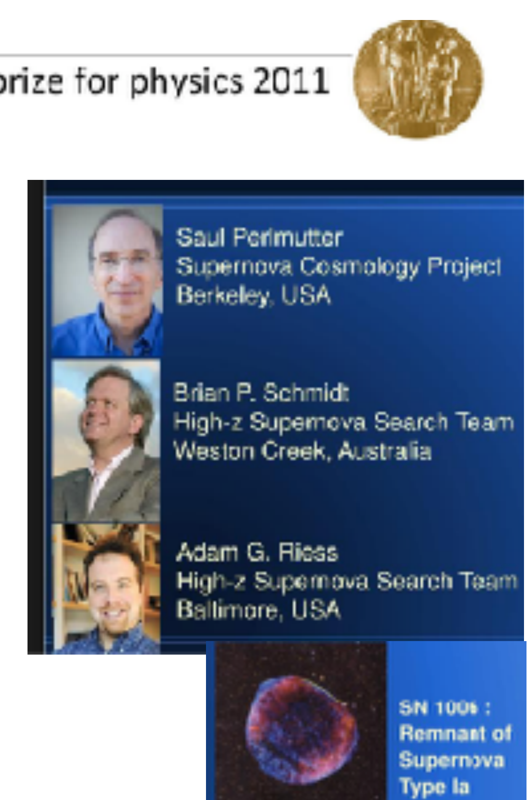
[2001, Hubble Space Telescope Key Project]

[C. de Rham]

Observations indicate exactly the contrary!

– objects were slower in the past
– the expansion of the universe is accelerated.



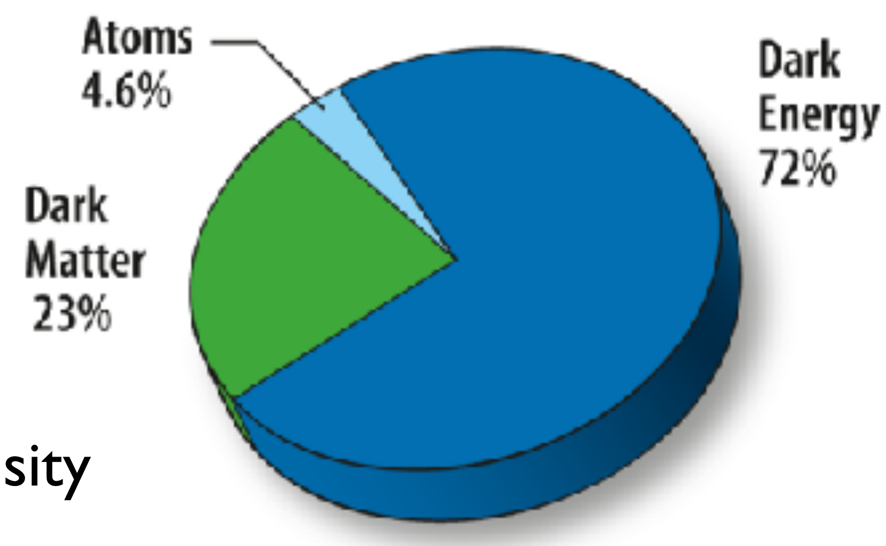


[C. de Rham]

Source of this acceleration?

– ordinary matter doesn't give rise to acceleration.
– nor dark matter in which galaxies are evolving

One idea: introduce a new substance — *dark energy*



• doesn't interact directly with us

• is diluted everywhere in the universe with ~constant density

$$\rho \sim 10^{-28} \mathrm{kg/m}^3$$

= approx 1 mosquito / (10000 km)^3!


One thing which dark energy is not… contribution of the Higgs boson to the vacuum energy:

$$\rho \gtrsim 10^{+29} \mathrm{kg/m}^3$$

Gravitational waves and cosmology

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dark energy equation of state $\frac{p_{\rm DE}(z)}{\rho_{\rm DE}(z)}$

Hubble constant

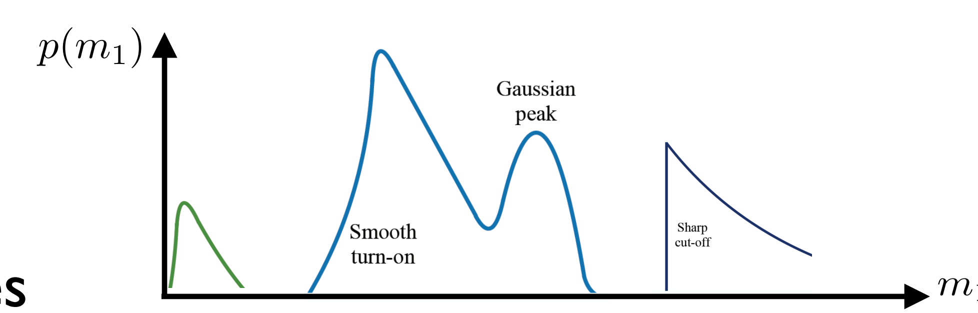
Characterizes local time-scale in universe

$c/H_0 \sim \mathrm{Gpc}$

energy fraction in matter today

+ possibly modified gravity (modified GW propagation)

+ astrophysics, e.g. origin of black holes?
e,g. distribution in mass of populations of binaries



Gravitational waves and cosmology

2) properties of the very early universe $\quad t \gtrsim t_{Pl}$

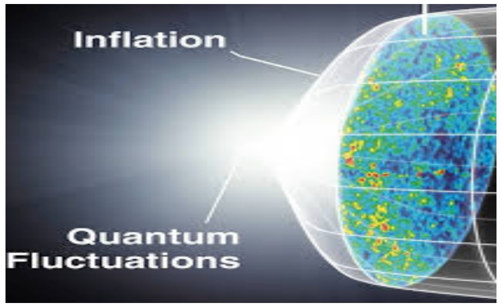
- *not individual sources, but observation of a stochastic GW background (SGWB) of cosmological origin*

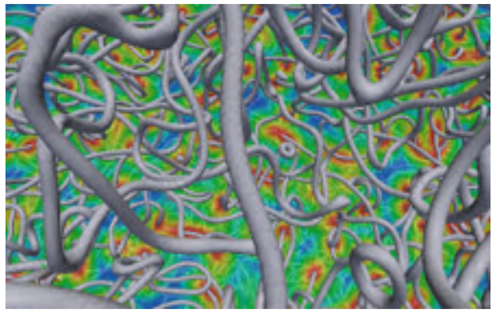
- SGWB: superposition of GWs arriving at random times and from random directions, overlapping so much that individual waves not detectable

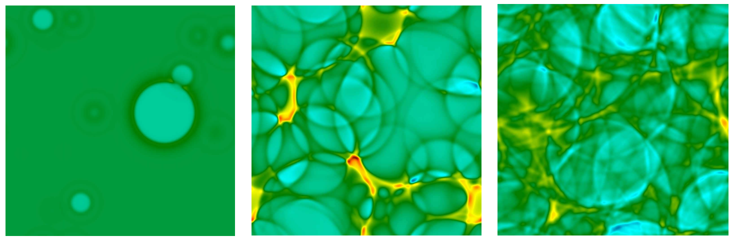
- Analogue of the CMB of photons, but crucial difference due to the weakness of GW interactions

$$T_{dec} \sim 3000\text{K}, \;\; z_{dec} \sim 1100$$

Sources?
– quantum processes during inflation
– primordial black holes
– Phase transitions in Early universe
– topological defects, eg cosmic strings
– …..







– reminder: particles which decouple from primordial plasma at $t \sim t_{dec}$ or $T \sim T_{dec}$ give snapshot of state of universe at that time.

$t < t_{dec}$
$T > T_{dec}$
they are coupled and interactions obliterate all information.

- In thermal equilibrium when

$$\Gamma \sim n\sigma|v| > H$$

rate of process maintaining thermal equilibrium (Γ)

Number density of particles (n)

X-section for interaction (σ)

typical velocity ($|v|$)

For light/massless particles at temperature T

$n \sim T^3$

$v \sim 1$

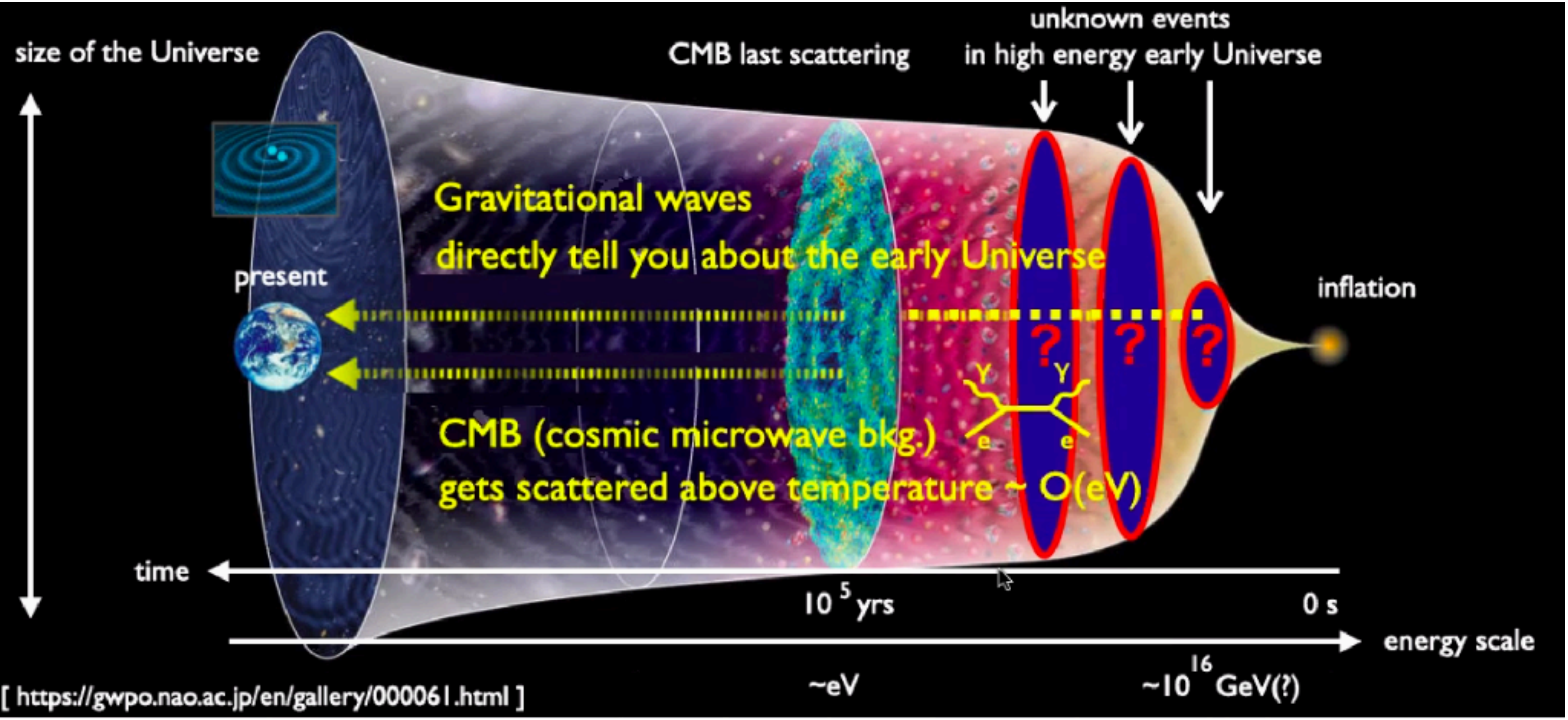
$H^2 \sim T^4 M_{\rm Pl}^{-2}$

and drop out when $\Gamma \sim H$

- Neutrinos: $\sigma \sim G_F^2 T^2$
$$\left(\frac{\Gamma}{H}\right)_{\rm neutrino} \sim \left(\frac{T}{1{\rm MeV}}\right)^3$$

- Gravitons $\sigma \sim G_N^2 T^2 \sim \dfrac{T^2}{M_{\rm Pl}^4}$
$$\left(\frac{\Gamma}{H}\right)_{\rm graviton} \sim \left(\frac{T}{M_{\rm Pl}}\right)^3$$

– gravitons decoupled below Planck scale!
– do not loose memory of conditions when produced
– retain spectrum/shape/typical frequency & intensity of physics at corresponding high energy scales.



– if SGWB can be detected, information on universe at early times.

– Huge potential to bring information on high energy phenomena in Early universe, at times even earlier than the CMB.

Gravitational waves and cosmology

late-time universe

Individual sources and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star- black-hole binary (NS-BH)…

- Expansion rate $H(z)$
- H_0, Hubble constant
- Ω_m
- beyond $\Lambda\mathrm{CDM}$
  dark energy $w(z)$ and dark matter
- modified gravity (modified GW propagation)
- astrophysics; eg BH populations, PISN mass gap?

Very early universe
$t \gtrsim t_{Pl}$

Stochastic background
of GWs of cosmological origin

- quantum processes during inflation
- primordial black holes
- Phase transitions in Early universe
- topological defects, eg cosmic strings
- …..

Plan

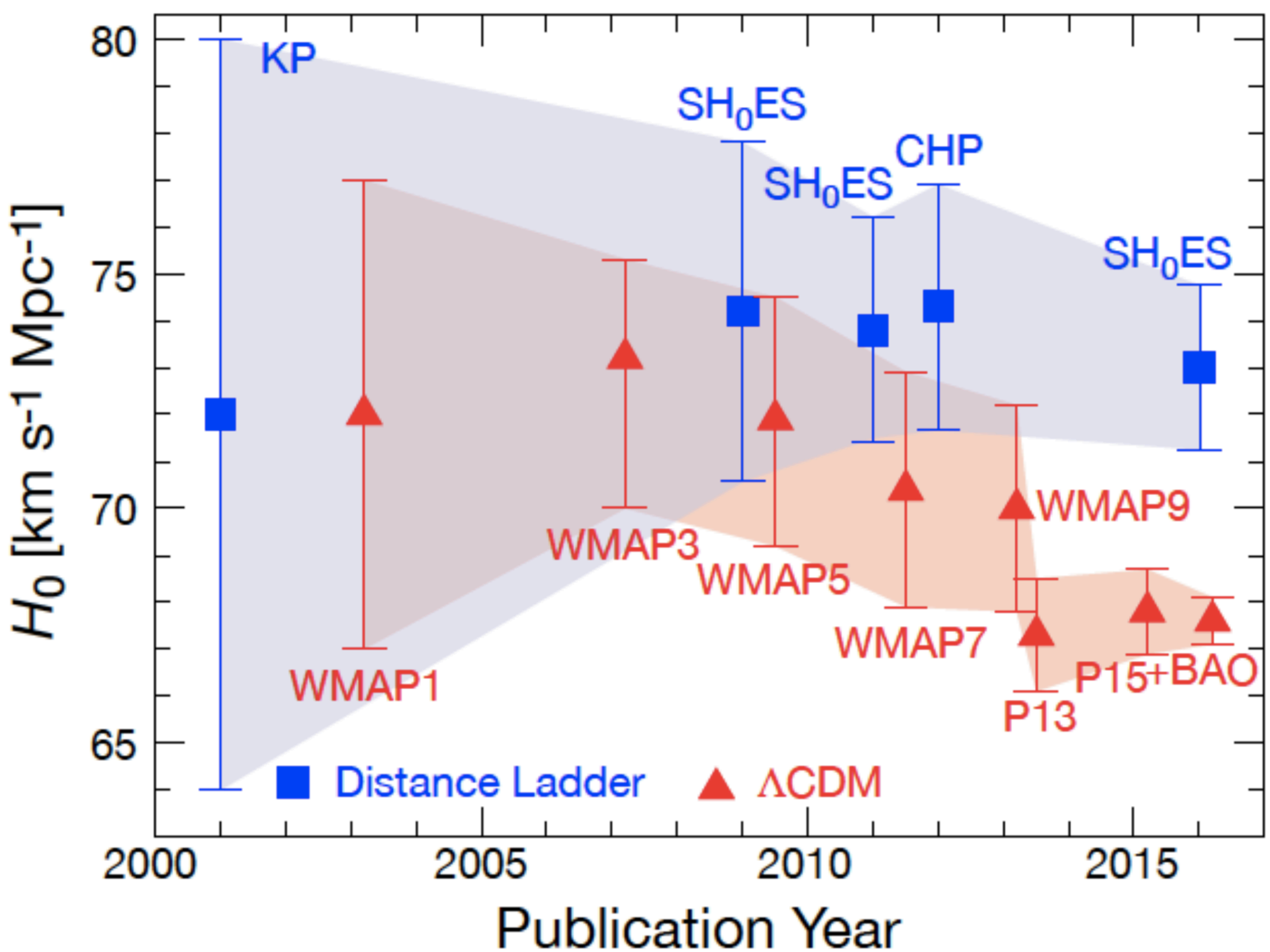
Part 1) Late time cosmology with GWs

1) Status on cosmological parameters.
2) Binaries: Characteristic scales and detectors.
3) gravitational waveform in an expanding universe
4) distance measurements with GWs; accuracy?
5) Determining the redshift, 4 methods.
6) tests of modified gravity with GWs

Part 2) probing the very early universe with GWs

I) Status on cosmological parameters.
Value of H_0

[W.Freedman, 1706.02739]



- Blue: determined from nearby universe with a calibration based on the Cepheid distance scale : distance ladder.

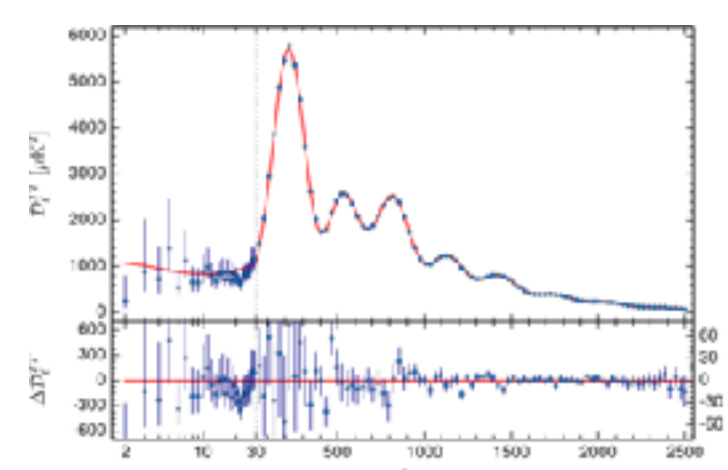
$$H_0 = 74.0 \pm 1.4 \ \ \mathrm{km/s/Mpc}$$

[SH0ES collaboration, Reiss et al]

- Red: from early universe CMB physics, assumes LCDM

$$H_0 = 67.74 \pm 0.46 \ \mathrm{km/s/Mpc}$$

[Planck collaboration]



[Hubble, 1929] $H_0 \sim 500 \mathrm{km/s/Mpc}$

- In the nearby universe

(On *larger* distance scales, more general relation, depending on matter content of universe, curvature.)

$$v_H = H_0 \times d$$

measuring distances in cosmology particularly difficult

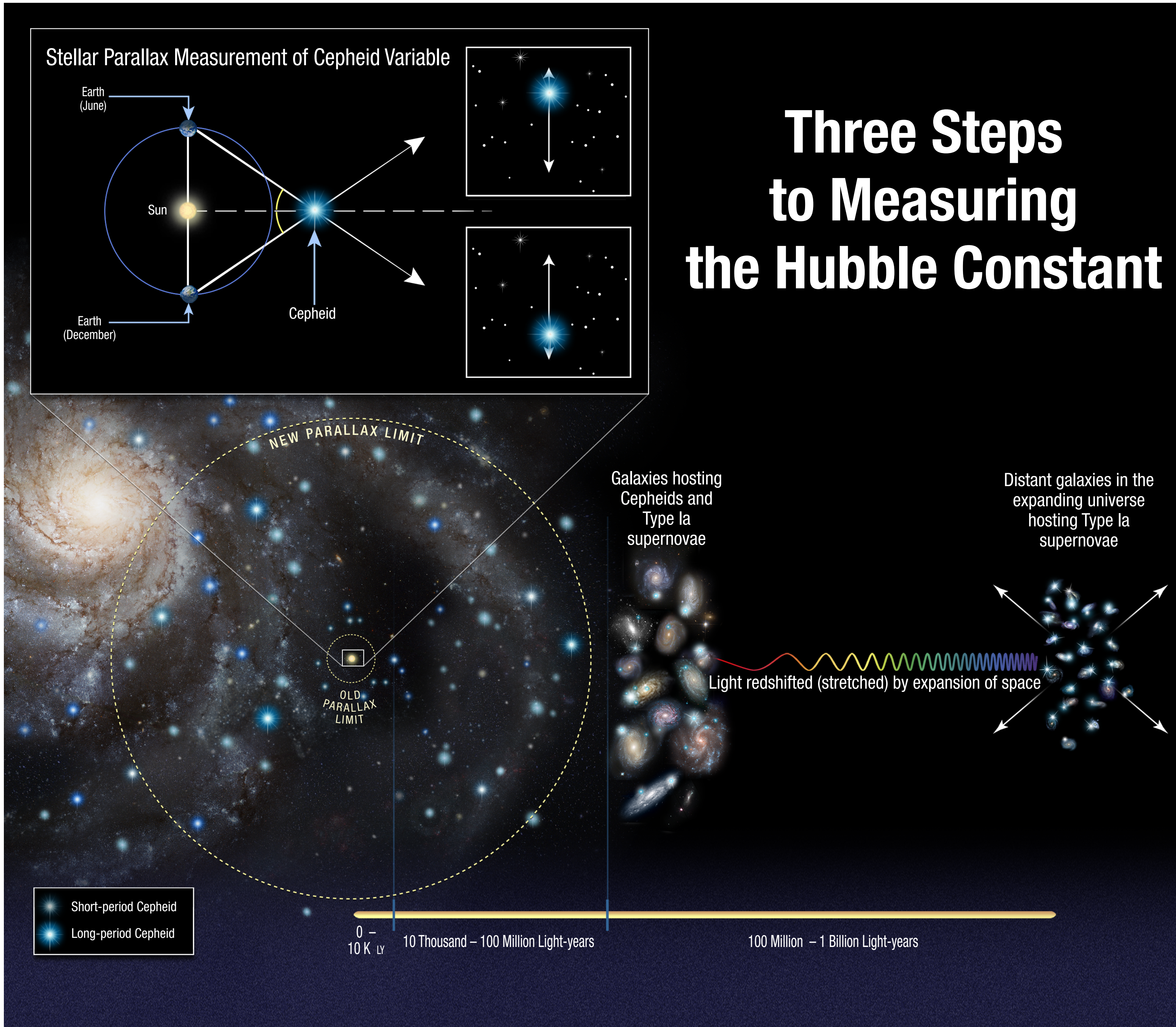
$$v_H = c\,z$$

radial Hubble flow velocity

z from doppler effect: photons, whilst they propagate to us, loose energy

$$f_O = \frac{f_S}{(1+z)}$$

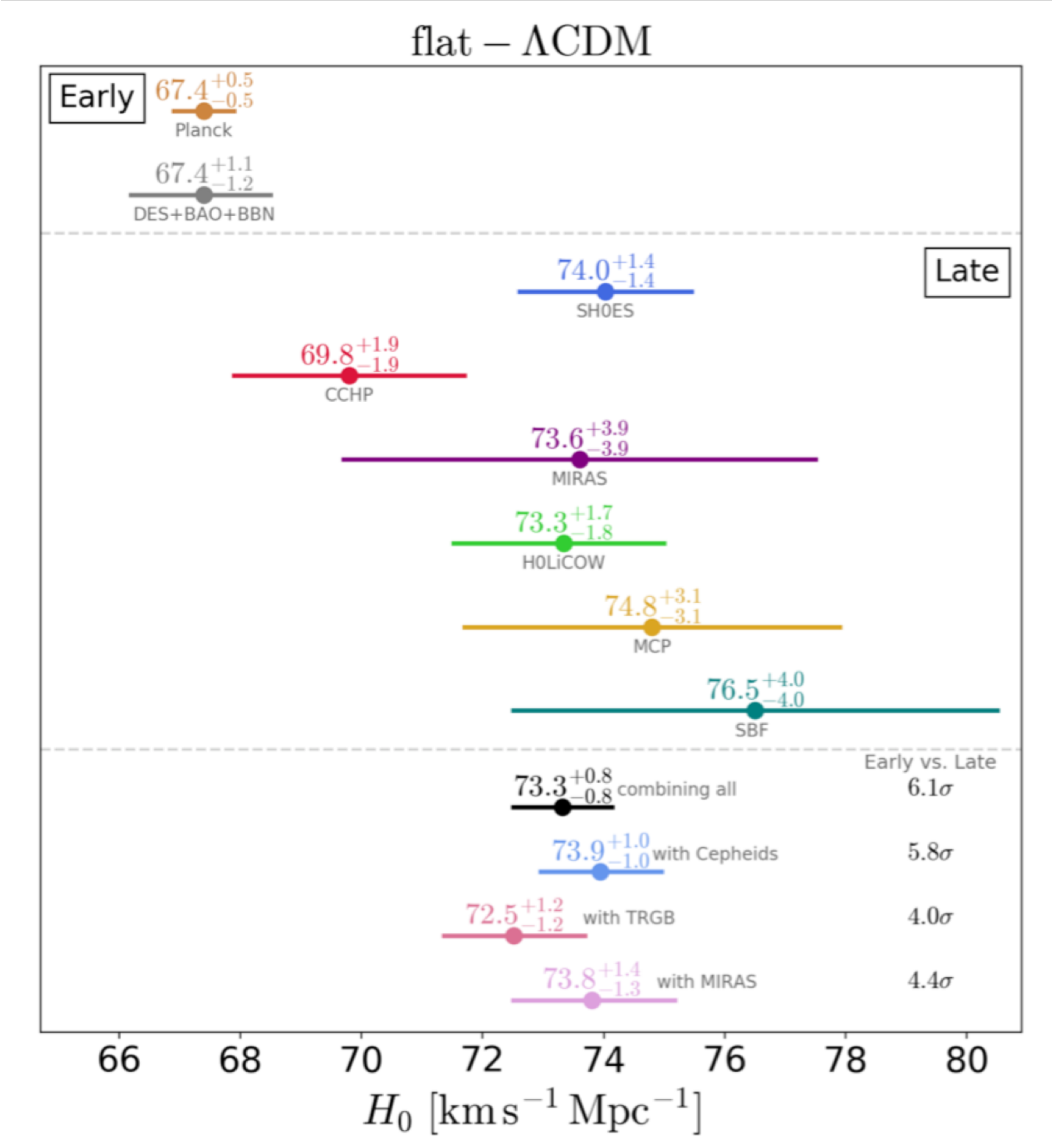
standard candles, of known luminosity and spectrum: Cepheid variables and type 1a supernovae.



Hubble constant tension

[Verde, Treu and Reiss 1907.10625]

Carnegie-Chicago Hubble Program 1907.05922



- Tension between measurements that calculate the sound horizon at decoupling (+assumption of Lambda CDM) and those that do not.

Real discrepancy or unknown systematic errors?

- Physics beyond the current standard model of cosmology e.g. exotic early dark energy? *modified gravity*? Magnetic fields?

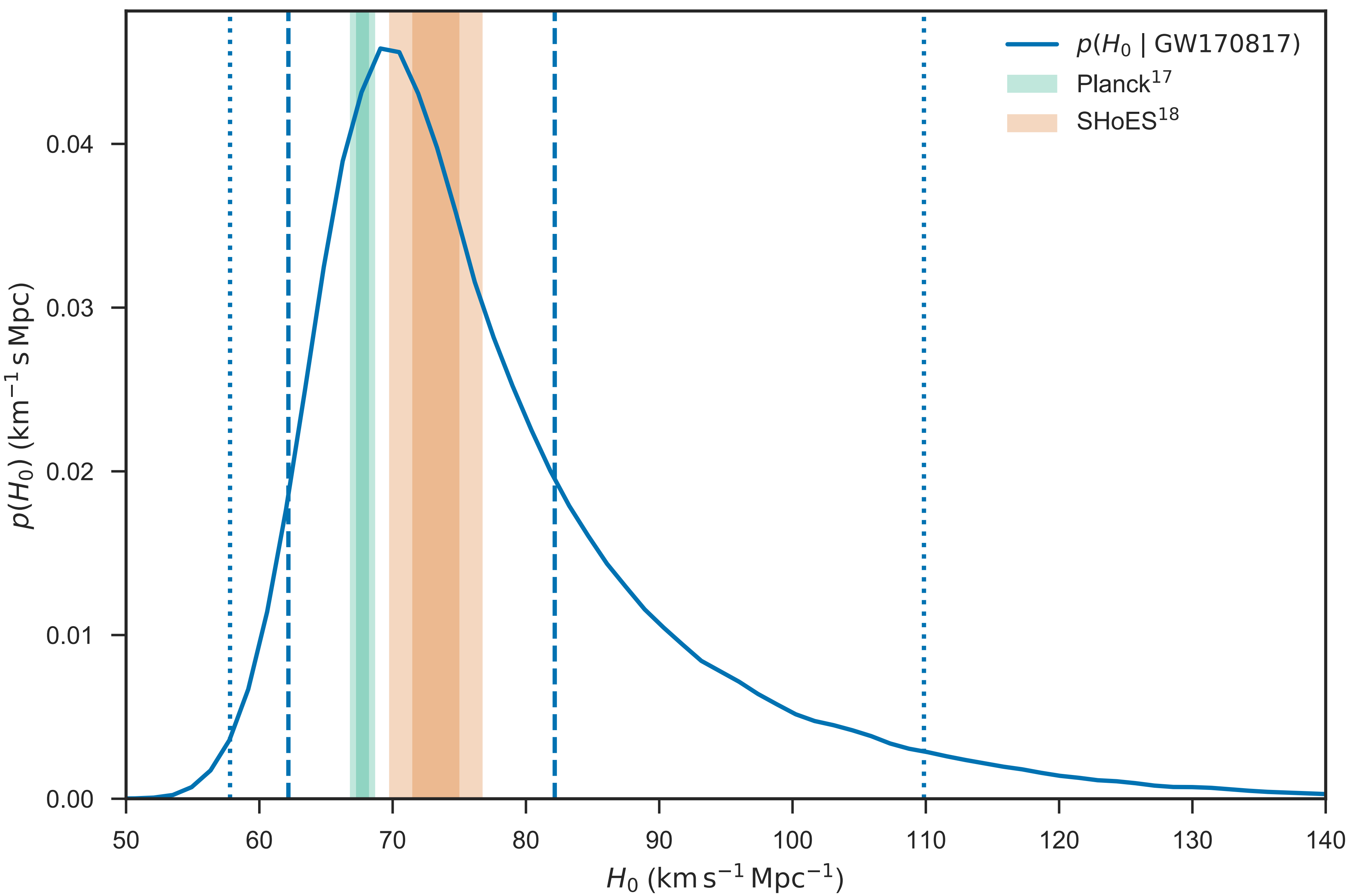
[1908.03663]

Hubble constant with GWs from binaries

$v_H = H_0 \times d$

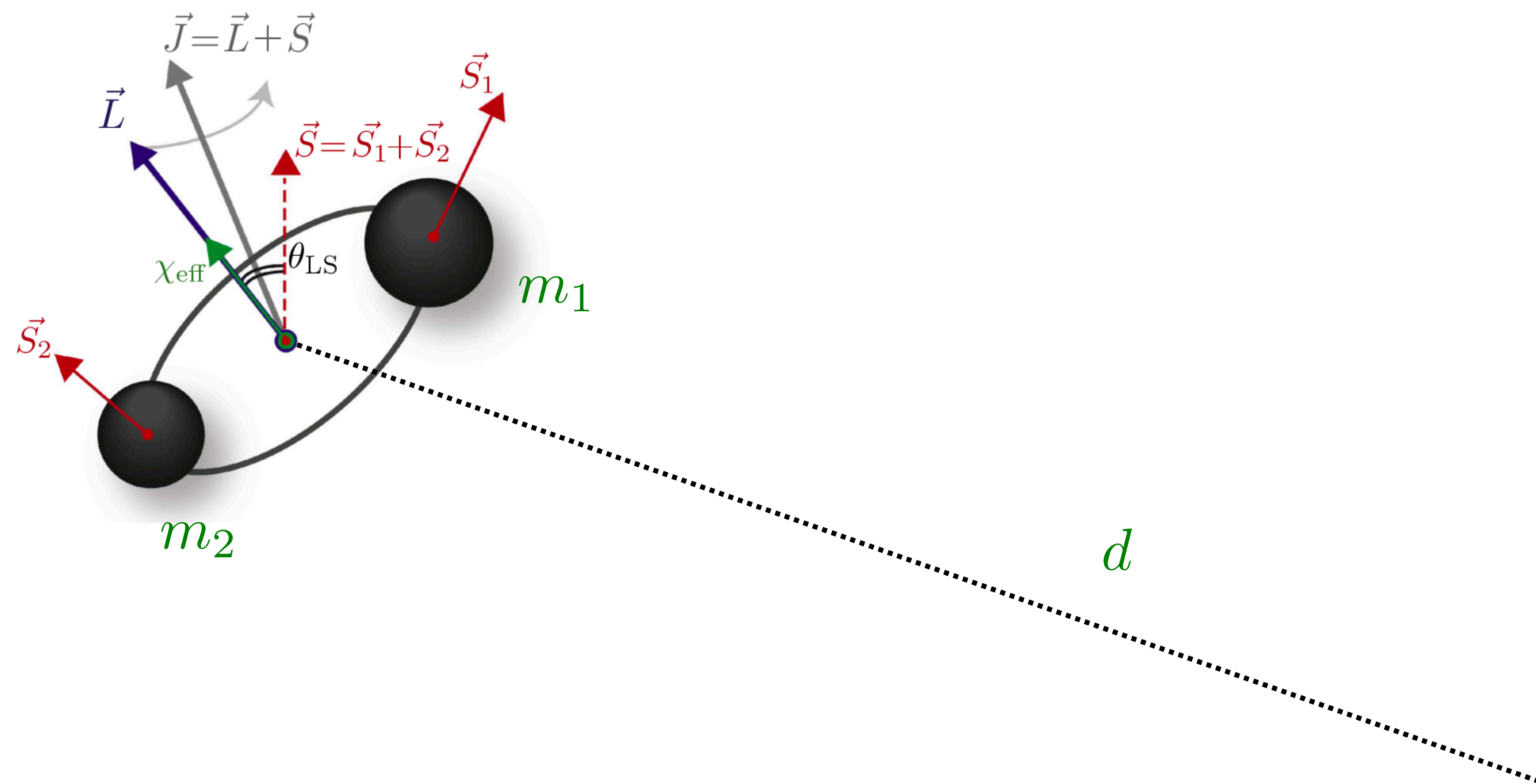
First measurement of H0 using GW170817

[1710.05835]

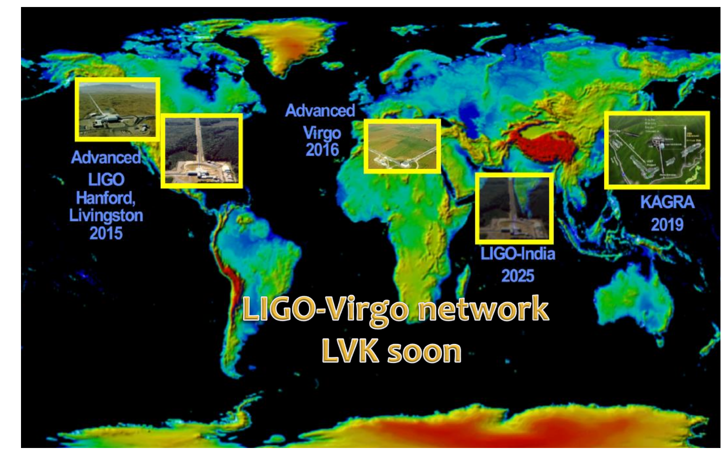


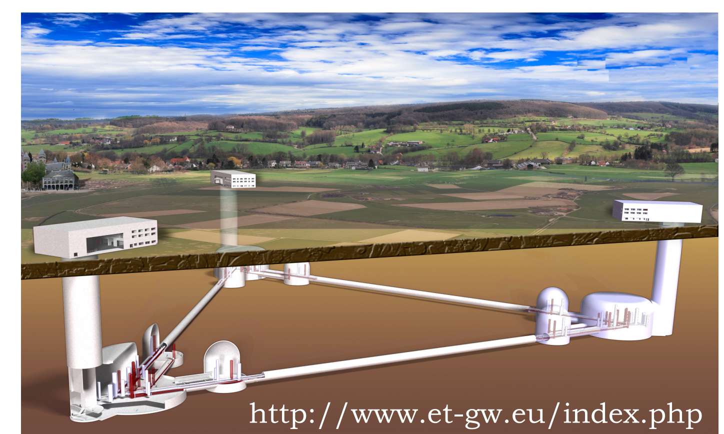
$$H_0 = 69^{+17}_{-8}\ \mathrm{km/Mpc/s}$$

2) GWs from binaries: Characteristic scales?

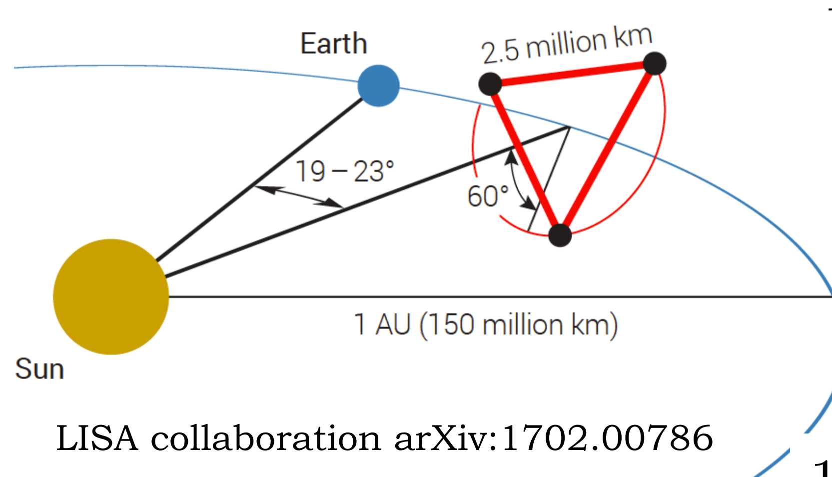


$10\,\mathrm{Hz} < f < 5\,\mathrm{kHz}$





$1\,\mathrm{Hz} < f < 10^4\,\mathrm{Hz}$



LISA collaboration arXiv:1702.00786

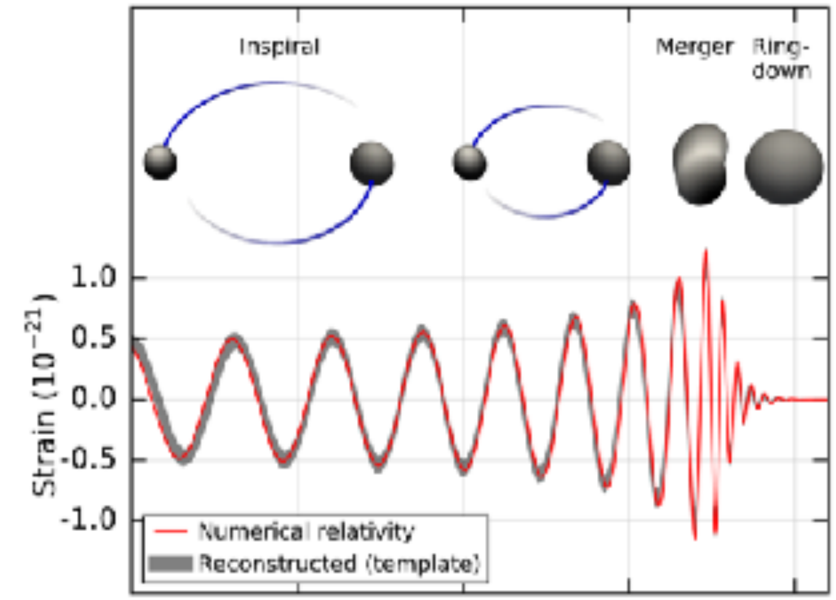
$10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$

– Focus on the inspiral phase.
– Initially neglect expansion (z<<1)

$$f_{\rm GW} = \frac{1}{\pi}(GM_c)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

$$\tau = t - t_c$$

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$



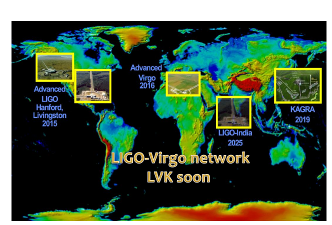
dominant quadrupolar mode calculated in previous lectures; to lowest order in the Newtonian expansion, for *point particles of mass m1 and m2*; no tidal effects, no spins,…, assuming circular orbit; and using quadrupole formula

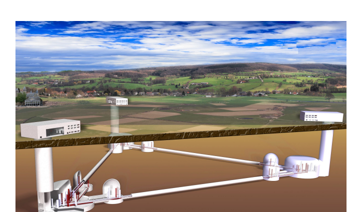
• *<u>Merger frequency</u>:* Assuming merger at innermost stable circular orbit (ISCO)

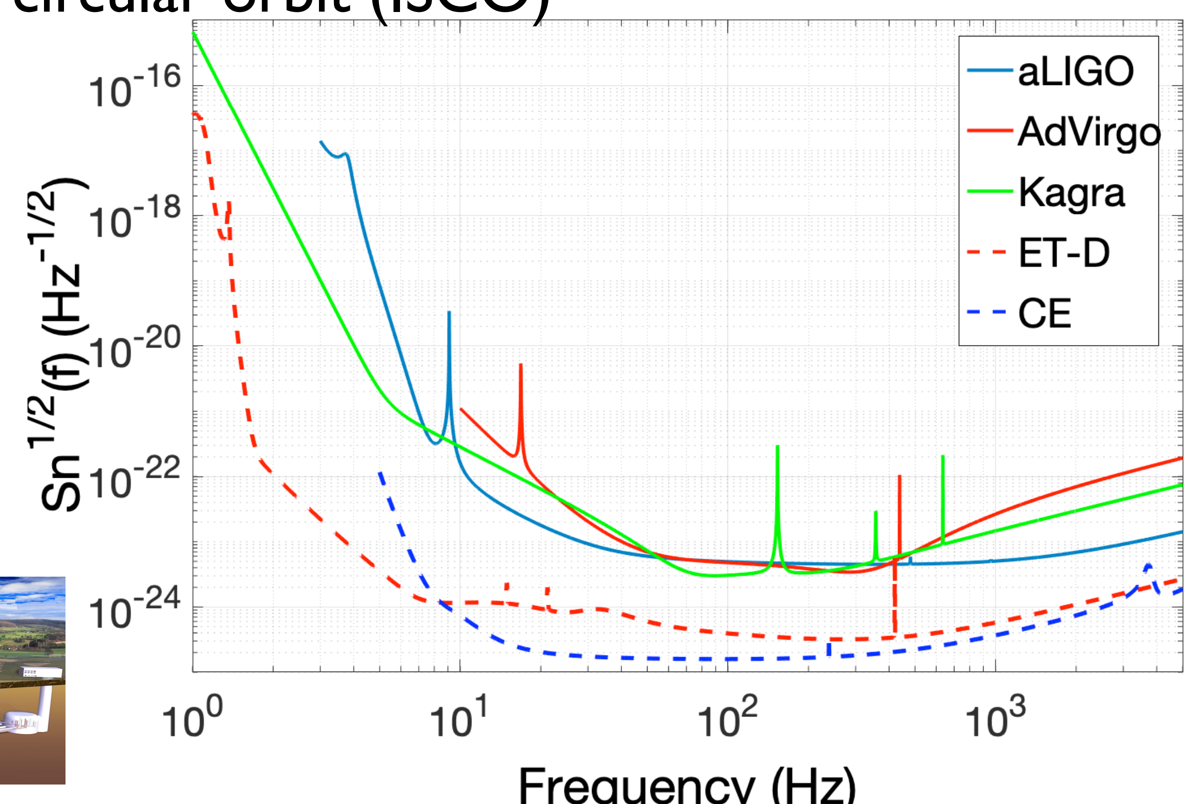
$$\boxed{f_{\rm merger} = \frac{1}{6^{3/2}\pi}\left(\frac{c^3}{GM}\right)}$$

M=total mass
Follows from Keplers laws

• BNS, $m_{1,2} \sim 1.4 M_\odot$ $\quad f_{\rm merger} \sim 1.5\,{\rm kHz}$
• stellar mass BHs, $m_{1,2} \sim 35 M_\odot$ $\quad f_{\rm merger} \sim 60\,{\rm Hz}$







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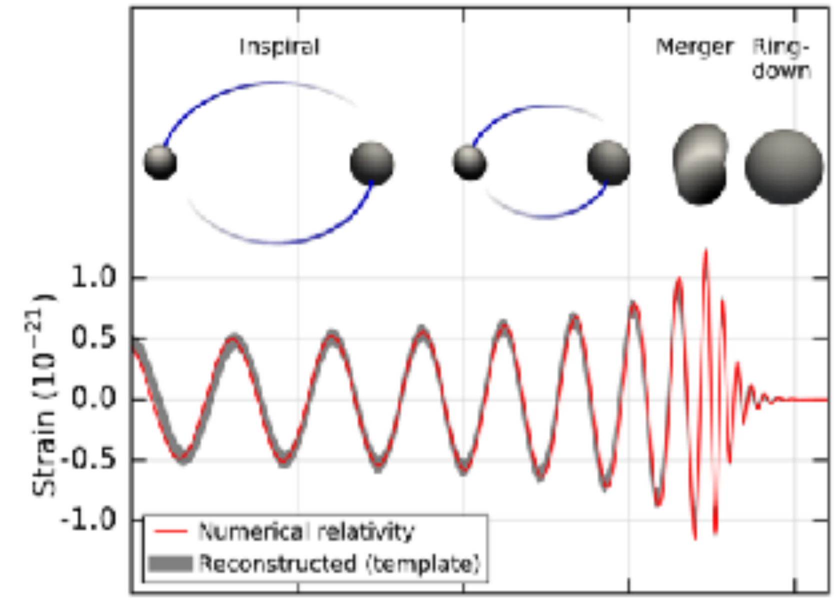

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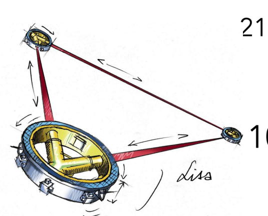
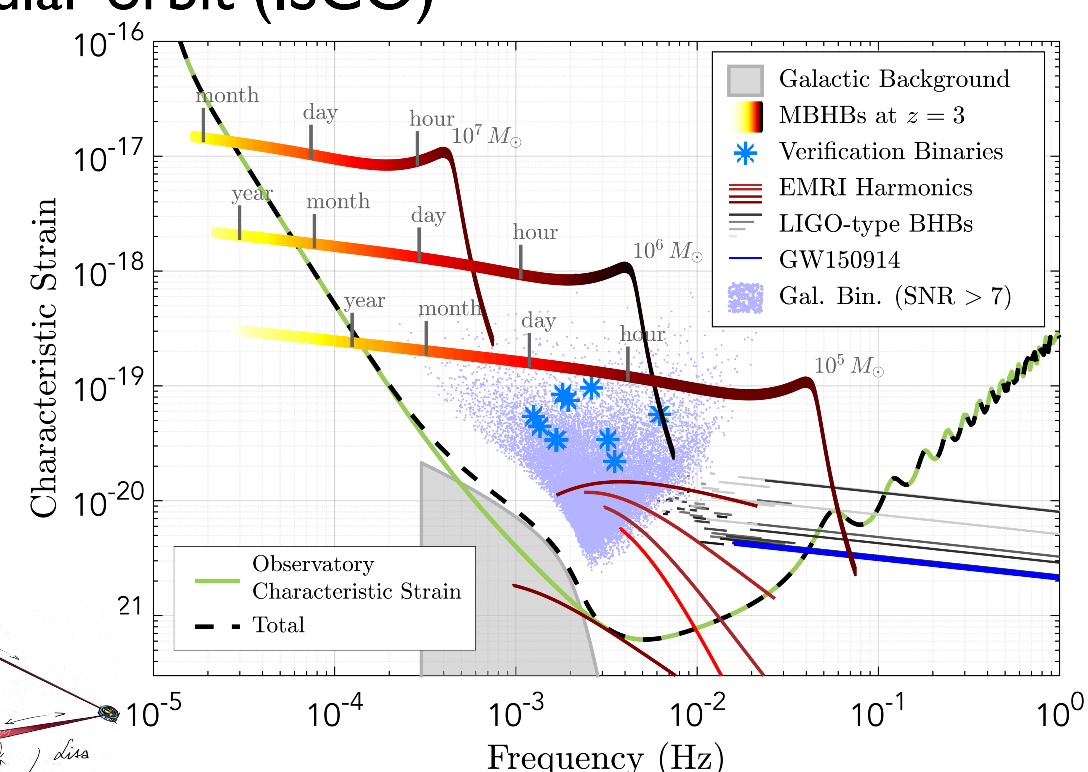
• stellar mass BHs, $m_{1,2} \sim 35 M_\odot$ $f_{\mathrm{merger}} \sim 60\,\mathrm{Hz}$
• Supermassive BBHs, $m_{1,2} \sim 10^6 M_\odot$ $f_{\mathrm{merger}} \sim 10^{-3}\,\mathrm{Hz}$

LISA





- *Time to merger*

$$f_{\rm GW} = \frac{1}{\pi}(GM_c)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

If GWs enter frequency band of a detector at observed frequency $f_{\rm low}$

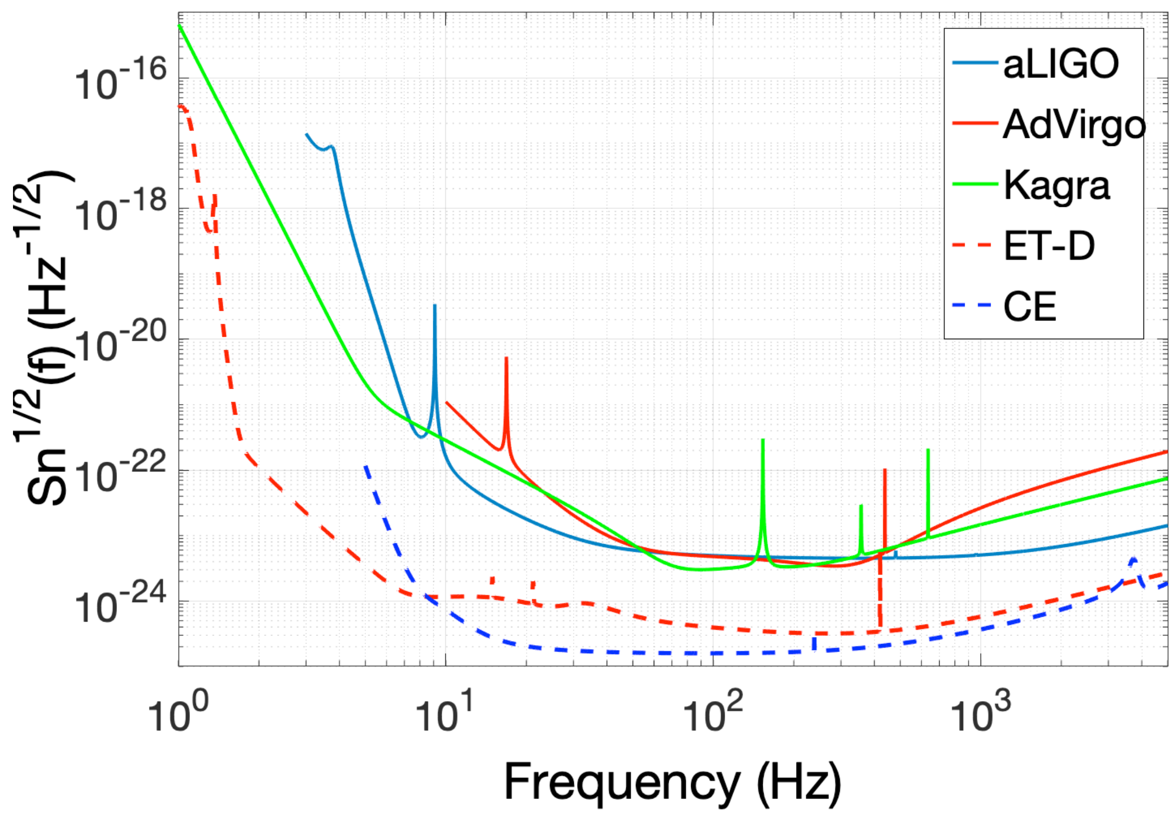
$$\boxed{T \sim 10^{-3} f_{\rm low}^{-8/3}\left(\frac{c^3}{G\mathcal{M}}\right)^{5/3}}$$

- BNS, entering LIGO-Virgo detector window at observed frequency $f_{\rm low} \sim 20\,{\rm Hz}$ $\quad T \sim 4\,{\rm min}$
  $m_{1,2} \sim 1.4 M_\odot$ $\quad f_{\rm merger} \sim 1.5\,{\rm kHz}$

- BNS, entering ET detector window at observed frequency $f_{\rm low} \sim 1\,{\rm Hz}$ $\quad T \sim 5\,{\rm days}$

=> cannot neglect the rotation of the earth

=>Given the merger rates for BNS, BBH and BH-NS, expect a typical BNS signal will be overlapped by a number of BBH signals, which may merge at similar times

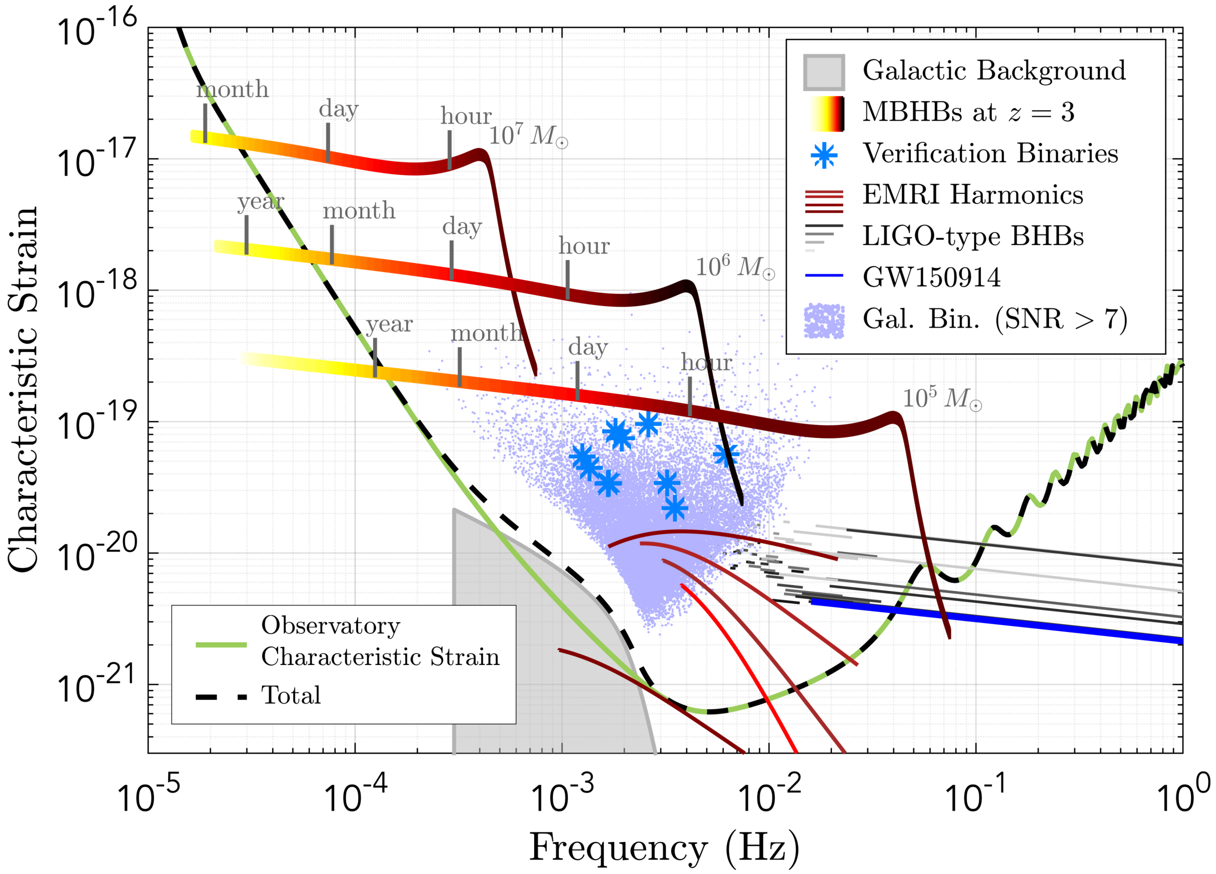


- stellar mass BHs entering LIGO-Virgo detector window

$m_{1,2} \sim 35 M_\odot$ $\quad T \sim 0.1\,{\rm s}$

$f_{\rm merger} \sim 60\,{\rm Hz}$

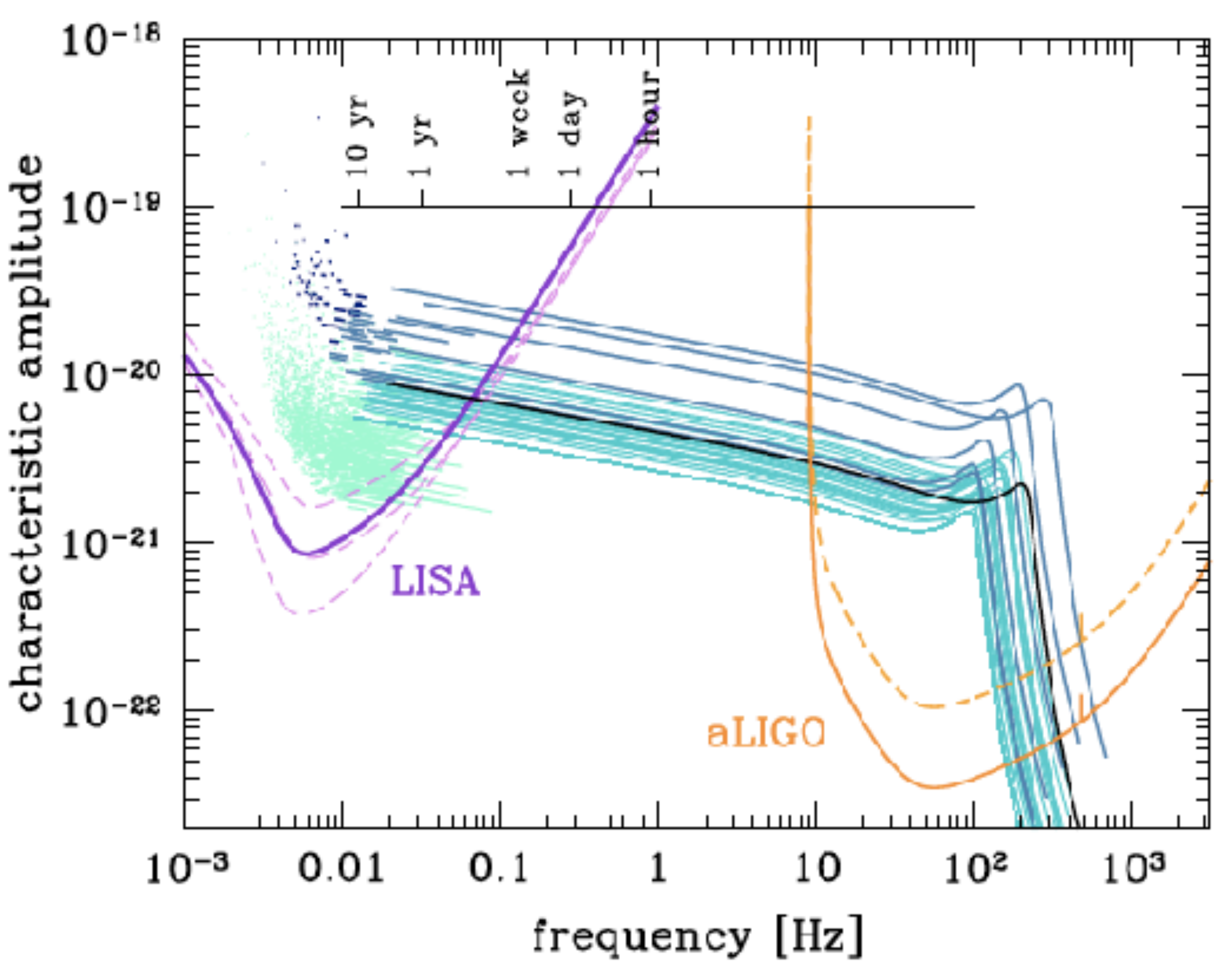
LISA



- Supermassive BBHs,

$$m_{1,2} \sim 10^6 M_\odot$$

$$T \sim 1\,\text{month}$$



- stellar mass BHs entering LISA detector window

$$f_{\rm low} \sim 10^{-2}\,\text{Hz}$$

$$T \sim 20\,\text{yrs}$$

- ***Amplitude/distance***

$$\boxed{h \sim \frac{4c}{R}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(\pi f)^{2/3}}$$

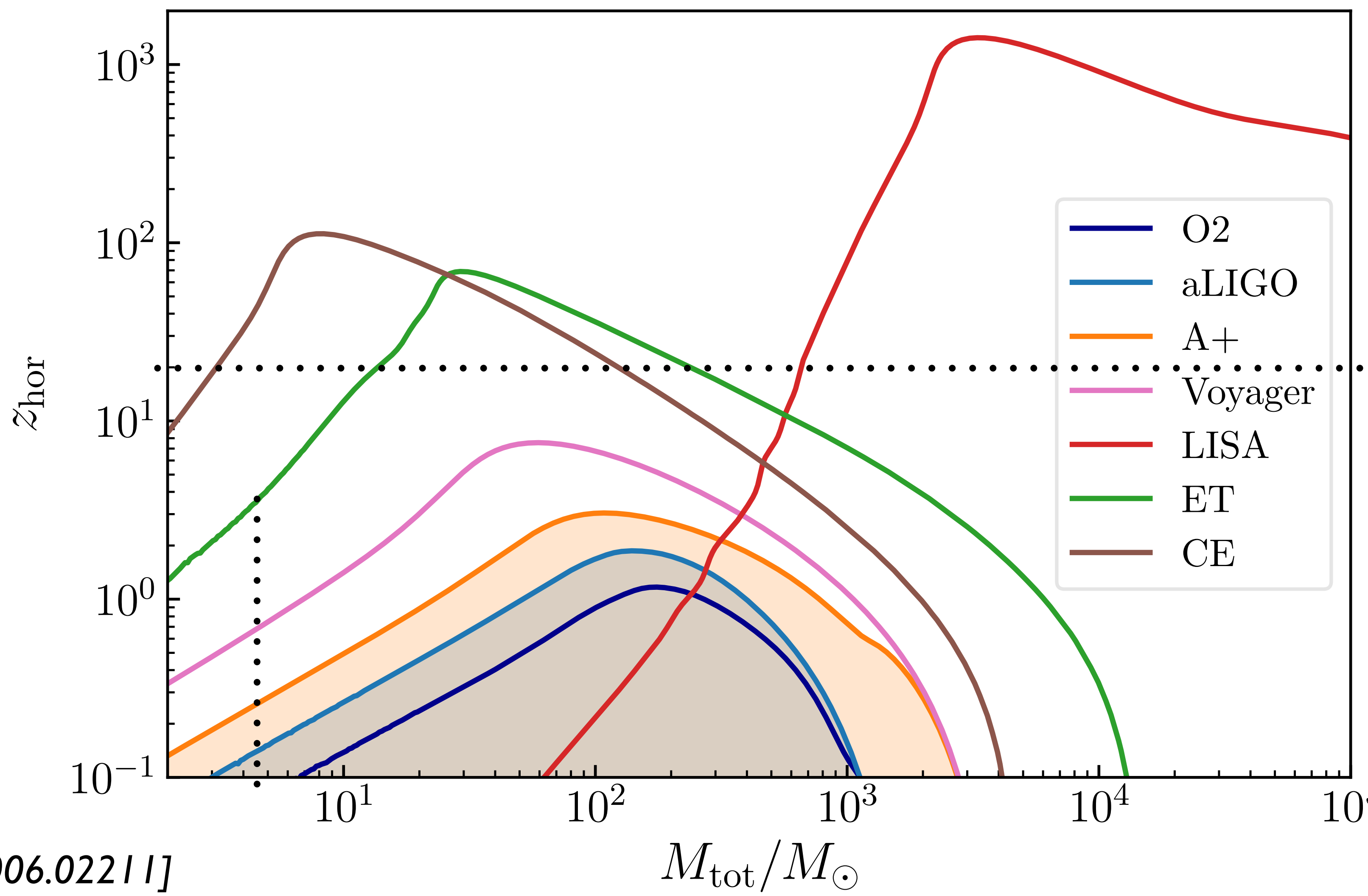
- stellar mass BHs in LIGO-Virgo

$$m_{1,2} \sim 35 M_\odot$$
$$f_{\mathrm{merger}} \sim 60\,\mathrm{Hz}$$

$$h \sim 10^{-21},\, R \sim 400\,\mathrm{Mpc}$$

converted to a redshift *assuming* the Planck values of cosmological parameters (z ~ 0.1)

Horizon redshift as a function of total source frame mass for an SNR detection threshold of rho=8. For LISA assumes 4 yrs obsv.



[2006.02211]

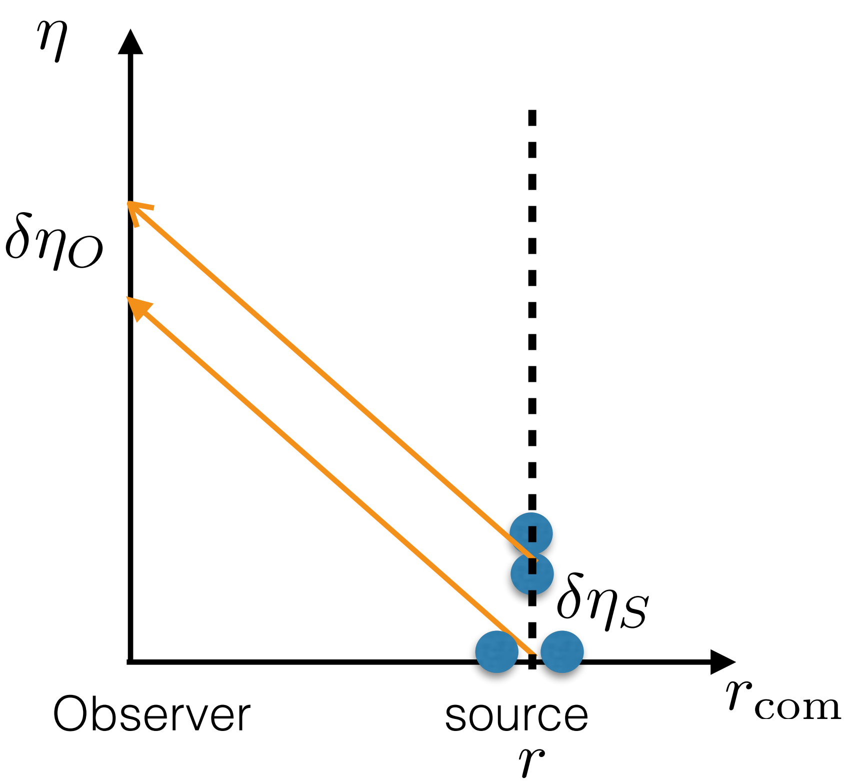
z>20; dark era preceding birth of first stars: any detected BHs must be primordial

Conclusion: 1) cannot neglect expansion of the universe
2) Want to use data to extract H0 and not assume a value of H0.

3) Inspiral of compact binaries at cosmological distances

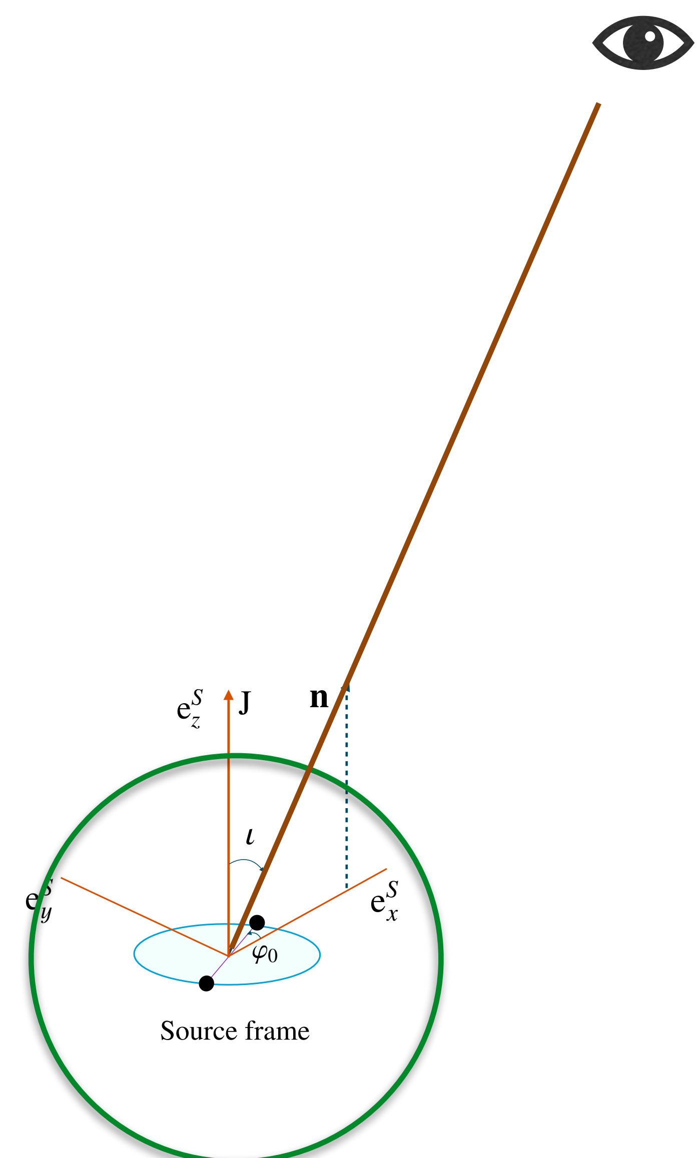
Turn on expansion in FRW universe.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$$



Idea: in local wave-zone of the source (scales large relative to source, small relative to Hubble), know solution

Then propagate it in FRW space-time to observer



$$r_{\rm phys} = a(t_{\rm emis})r$$

$$a(t) \sim \text{constant}$$

– amplitude redshifts, plus time dilatation

$$dt_O = \frac{a(t_O)}{a(t_S)}dt_S = (1+z)dt_S \qquad f_O = \frac{f_S}{(1+z)}$$

Close to source, as measured by time t_S of clock of the source,

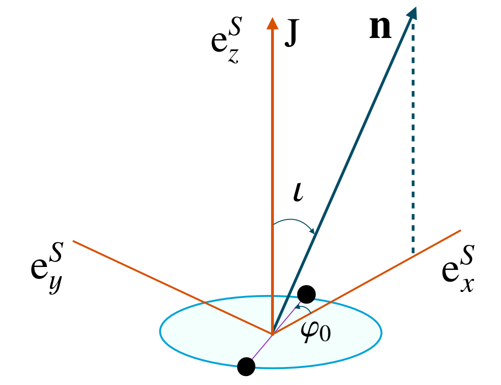
$$h_+(t_S, i) = \frac{4}{R}(G\mathcal{M})^{5/3}(\pi f_S(t_S^{\rm ret}))^{2/3}\left(\frac{1+\cos^2 i}{2}\right)\cos(2\Phi_S(t_S^{\rm ret}))$$

$$h_\times(t_S, i) = \frac{4}{R}(G\mathcal{M})^{5/3}(\pi f_S(t_S^{\rm ret}))^{2/3}\cos i\,\sin(2\Phi_S(t_S^{\rm ret}))$$

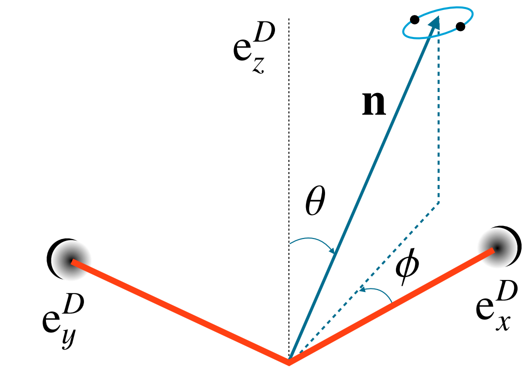
$R = r_{\rm phys} = a(t_{\rm emis})r$

frequency of GW

time to coalescence $t_S^{\rm ret} = t_S - t_c$



Source frame



Detector frame

Quadruple formula

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(G\mathcal{M})^{5/3}f_S^{11/3}$$

source frame chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$f_S(t_S^{\rm ret}) = \frac{1}{\pi}\left(\frac{5}{256\,t_S^{\rm ret}}\right)^{3/8}(G\mathcal{M})^{-5/8}$$

$$\Phi_S(t_S) \equiv \Phi_c + 2\pi\int_{t_c}^{t_S} dt'_S f_S(t'_S) = -2\left(\frac{t_S^{\rm ret}}{5G\mathcal{M}}\right)^{5/8} + \Phi_c$$

From source to observer:

- Perturbed FRWL metric (ignoring scalars and vectors):

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$|h_{ij}| \ll 1$

$h^i_i = \partial_j h^j{}_i = 0$

- from Einstein equations

$$\ddot{h}_{ij}(\vec{x},t) + 3H\dot{h}_{ij}(\vec{x},t) - \frac{\nabla^2}{a^2} h_{ij}(\vec{x},t) = 16\pi G \Pi^{TT}_{ij}(\vec{x},t)$$

source: tensor anisotropic stress

$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$

- away from the source, in conformal time and Fourier space

$$h''(\vec{k},\eta) + 2\mathcal{H}h'(\vec{k},\eta) + k^2 h(\vec{k},\eta) = 0$$

with $\mathcal{H} = \dfrac{a'}{a} = \dfrac{2}{\eta^2}$

- Change of variable $Q(\vec{k},\eta) = a(\eta)h(\vec{k},\eta)$

$$Q'' + \left(k^2 - \frac{a''}{a}\right) Q = 0$$

- Thus for sub-Hubble modes

$$\boxed{h(\vec{k},\eta) = \frac{A(\vec{k})}{a(\eta)} e^{ik\eta} + \frac{B(\vec{k})}{a(\eta)} e^{-ik\eta}}$$

<u>Plane waves with redshifting amplitude</u>

- **from source to observer**

$$h_+(t_S, i) = \frac{4}{a(t_O)R}(G\mathcal{M})^{5/3}\left(\pi f_S(t_S^{\mathrm{ret}})\right)^{2/3}\left(\frac{1+\cos^2 i}{2}\right)\cos(2\Phi_S(t_S^{\mathrm{ret}}))$$

$$h_\times(t_S, i) = \frac{4}{a(t_O)R}(G\mathcal{M})^{5/3}\left(\pi f_S(t_S^{\mathrm{ret}})\right)^{2/3}\cos i \sin(2\Phi_S(t_S^{\mathrm{ret}}))$$

still measured in source clock; want it in the observers clock.

$$dt_O = (1+z)dt_S$$

$$f_S = (1+z)f_O$$

$$\Phi_S(t_S) = 2\pi\int_{t_{c,S}}^{t_S} dt'_S f_S(t'_S) = 2\pi\int_{t_{c,O}}^{t_O} dt'_O f_O(t'_O) = \Phi_O(t_O)$$

phase is constant along null geodesics

$$h_\times(t_S, i) = \frac{4}{a(t_O)R}(G\mathcal{M})^{5/3}(\pi f_S(t_S^{\rm ret}))^{2/3}\cos i \sin(2\Phi_S(t_S^{\rm ret}))$$

$$\downarrow$$

$$f_S = (1+z) f_O$$

$$h_\times(t_O, i) = \frac{4}{a(t_O)R}(G\mathcal{M})^{5/3}(\pi f_O(t_O^{\rm ret})(1+z))^{2/3}\cos i \sin(2\Phi_O(t_O^{\rm ret}))$$

$$\downarrow$$

$$h_\times(t_O, i) = \frac{4}{a(t_O)R(1+z)}(G\mathcal{M}(1+z))^{5/3}(\pi f_O(t_O^{\rm ret}))^{2/3}\cos i \sin(2\Phi_O(t_O^{\rm ret}))$$

$$= \frac{4}{d_L}(G\mathcal{M}_z)^{5/3}(\pi f_O(t_O^{\rm ret}))^{2/3}\cos i \sin(2\Phi_O(t_O^{\rm ret}))$$

Redshift absorbed in a shift in the chirp mass

$$m_{1,2}^{\rm det}(z) = (1+z) m_{1,2}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}$$

redshifted / detector frame masses

BNS system with

$m_{1,2} \sim 1.4 M_\odot$ $\mathcal{M} \sim 1.21 M_\odot$

at z=1 has $\mathcal{M}_z \sim 2.42 M_\odot$

Luminosity distance

$$d_L(z) = a(t_O)R(1+z) = \sqrt{\frac{L}{4\pi F}}$$

$$d_L(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{\mathrm{d}z'}{\left[\Omega_m(1+z')^3 + \Omega_\Lambda(1+z')^{3(1+w(z'))}\right]^{1/2}}$$

at observer

$$h_+(t,i) = \frac{4}{d_L(z)} (G\mathcal{M}_z)^{5/3} (\pi f(t^{\rm ret}))^{2/3} \left(\frac{1+\cos^2 i}{2}\right) \cos(2\Phi(t^{\rm ret}))$$

$$h_\times(t,i) = \frac{4}{d_L(z)} (G\mathcal{M}_z)^{5/3} (\pi f(t^{\rm ret}))^{2/3} \cos i \sin(2\Phi(t^{\rm ret}))$$

where from

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(G\mathcal{M})^{5/3} f_S^{11/3} \qquad \text{with} \qquad f_S = (1+z)f$$

$$(1+z)\frac{d[f(1+z)]}{dt} = \frac{96}{5}\pi^{8/3}(G\mathcal{M})^{5/3} f^{11/3}(1+z)^{11/3}$$

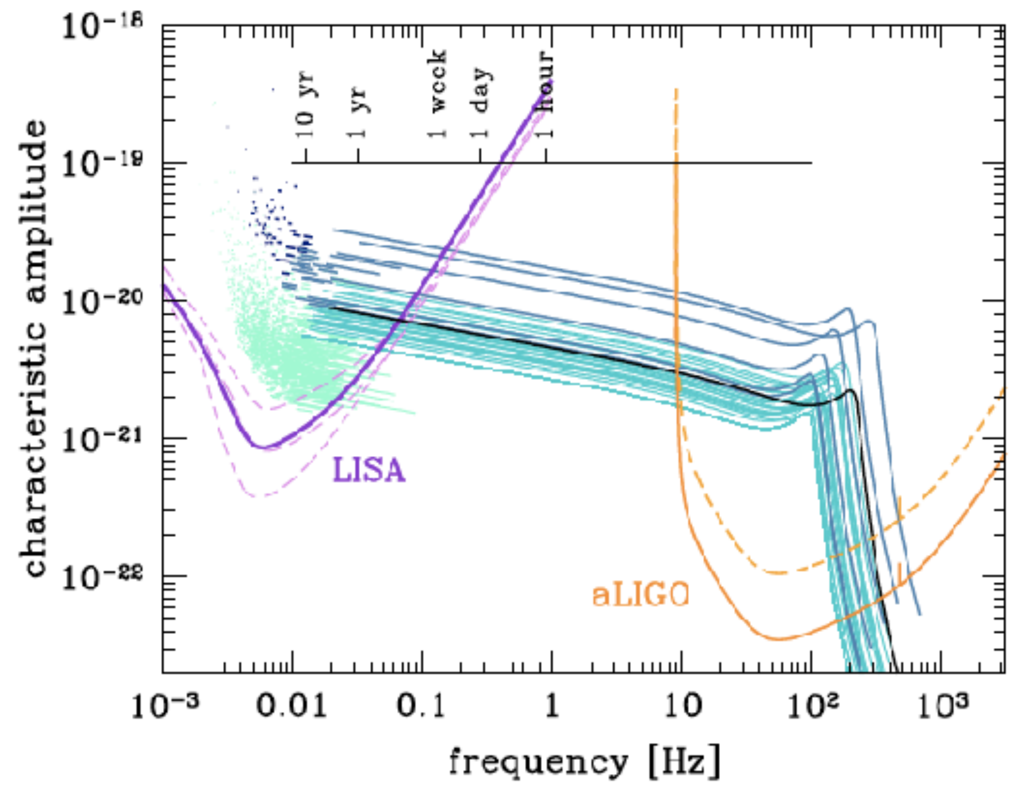
assuming z is constant during the observation time

(could lead to a bias for stellar-mass binaries entering LISA band and then coalesce in LIGO band),

$$\frac{df}{dt} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_z)^{5/3} f^{11/3}$$

$$\Phi(t^{\rm ret}) = -2\left(\frac{t^{\rm ret}}{5G\mathcal{M}_z}\right)^{5/8} + \phi_c$$

Phase depends on redshifted chirp mass



at observer

$$h_+(t,i) = \frac{4}{d_L(z)}\left(G\mathcal{M}_z\right)^{5/3}\left(\pi f(t^{\rm ret})\right)^{2/3}\left(\frac{1+\cos^2 i}{2}\right)\cos(2\Phi(t^{\rm ret}))$$

$$h_\times(t,i) = \frac{4}{d_L(z)}\left(G\mathcal{M}_z\right)^{5/3}\left(\pi f(t^{\rm ret})\right)^{2/3}\cos i\,\sin(2\Phi(t^{\rm ret}))$$

Phase depends on redshifted chirp mass

$$\mathcal{M}_z = (1+z)\mathcal{M}$$

- phase information cannot extract z (unless source frame masses known for some reason)

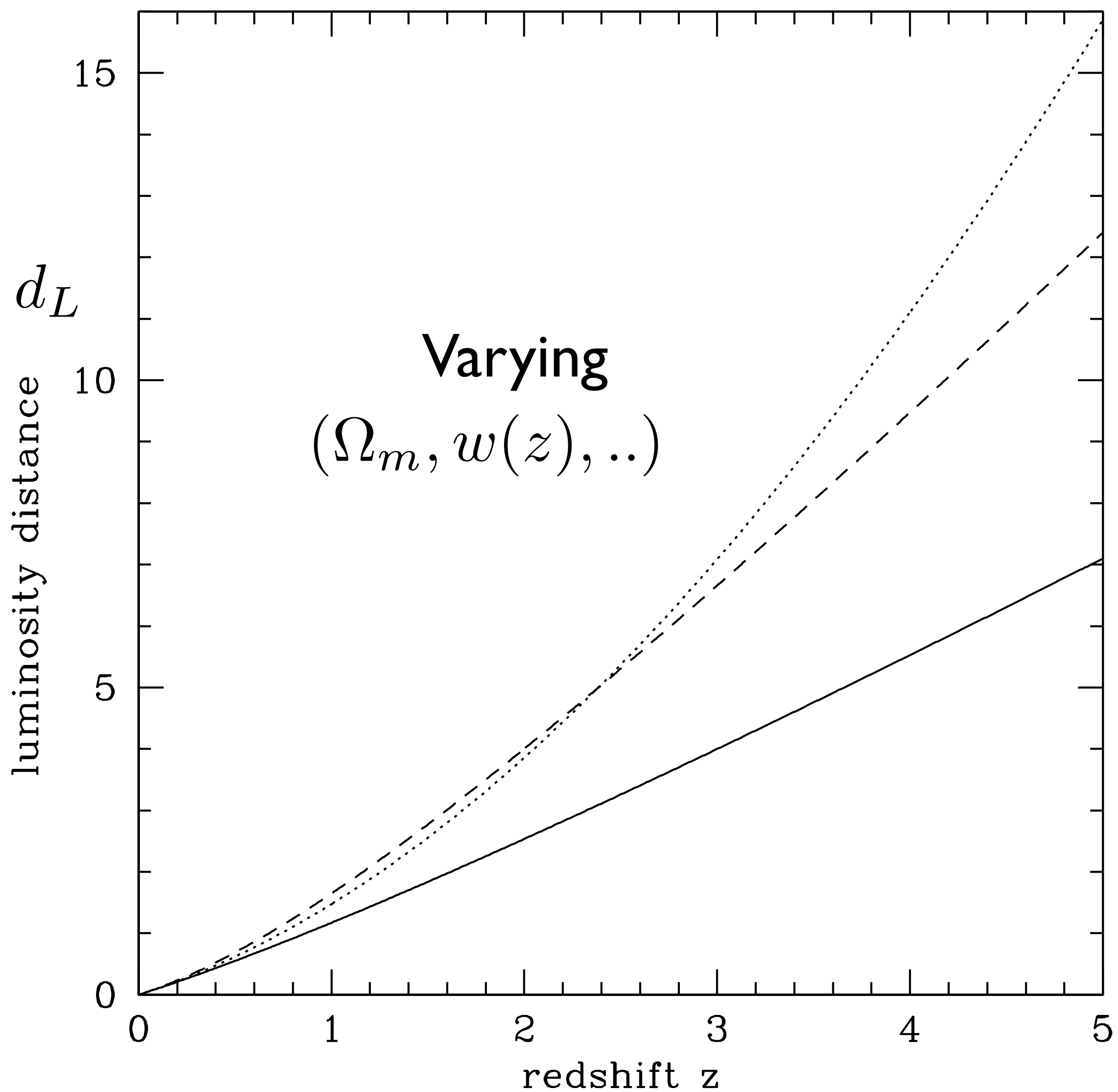
- amplitude information cannot extract z

perfect degeneracy between source masses, redshift, spins.. (gravity is scale-free)

The redshift does change the waveform, but in a way that can be exactly compensated by a shift of the masses from their source to detector values and by replacing the comoving distance with the luminosity distance.

<u>Some extra non-gravitational information is necessary to determine z.</u>

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{\mathrm{d}z'}{\left[\Omega_m (1+z')^3 + \Omega_\Lambda (1+z')^{3(1+w(z'))}\right]^{1/2}}$$



For $z \ll 1$ (LIGO-Virgo): $d_L = \dfrac{cz}{H_0}$

For larger z (ET, LISA), dL depends on other cosmological parameters. Can potentially access

$$H(z) = H_0 \left[\Omega_m (1+z')^3 + \Omega_\Lambda (1+z')^{3(1+w(z'))}\right]^{1/2}$$

Cosmological parameter determination:

1) their accuracy will depend on accuracy of dL measurement

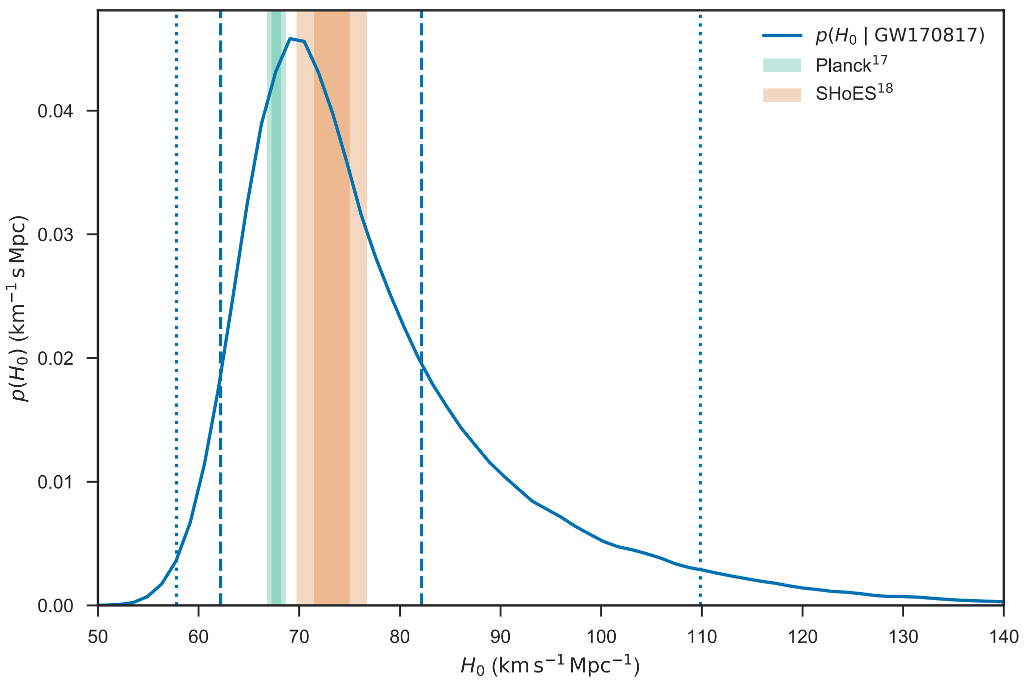
2) and require z, with some accuracy.

$$cz = H_0 \, D_L$$

$$\frac{\Delta H_0}{H_0} \sim \frac{\Delta z}{z} + \frac{\Delta D_L}{D_L}$$

4) Accuracy of distance measurement

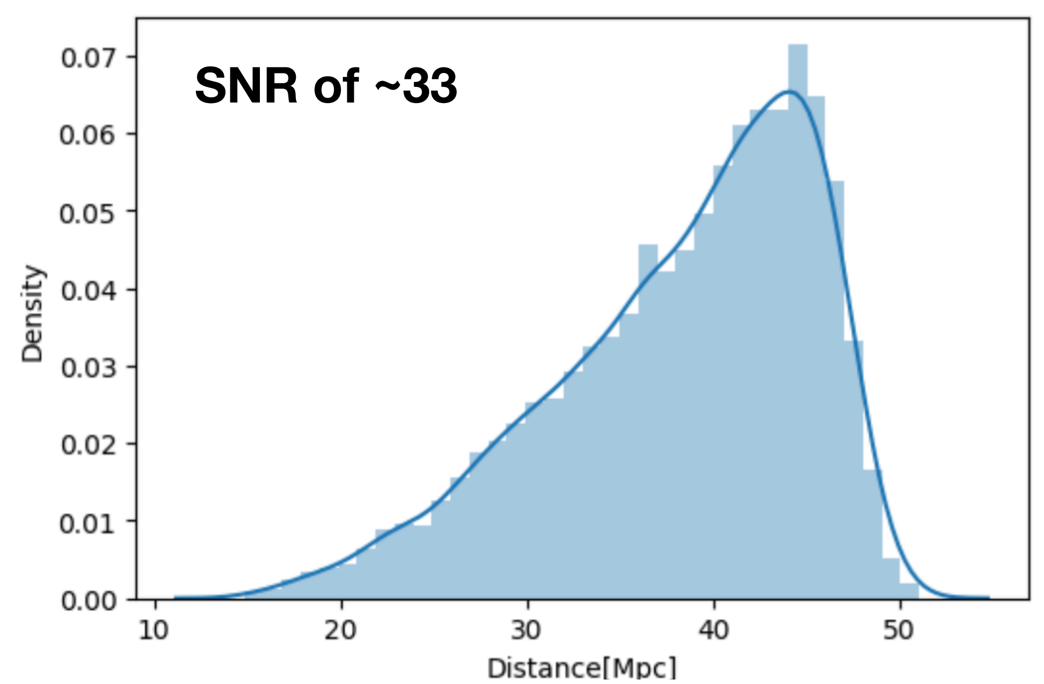
GW170817



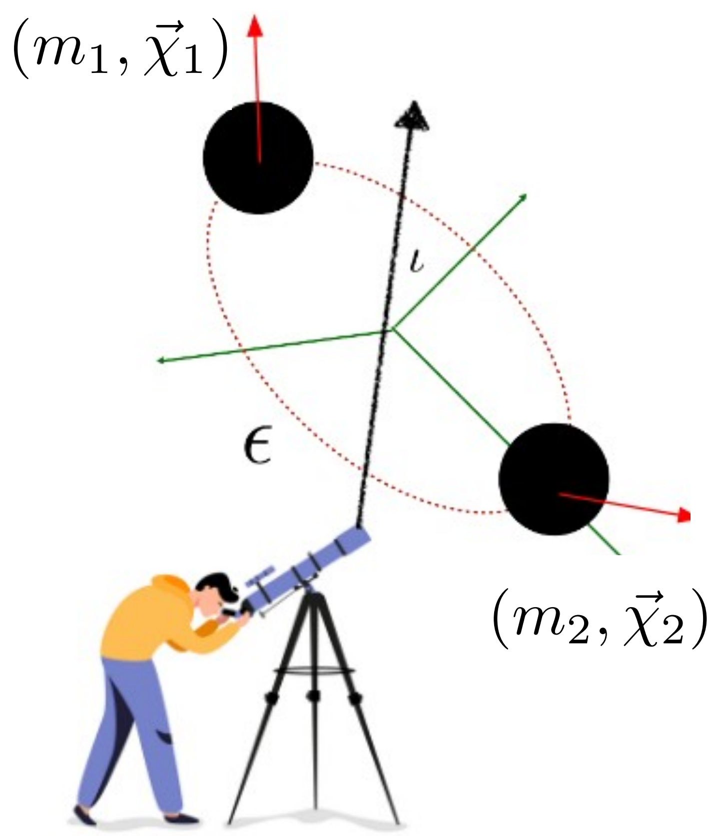
~15% error on distance measurement

posterior probability density for distance

Public GW170817 PE samples on GWOSC



Estimate of error on luminosity distance



- In an ideal world, one could measure two polarisations separately:

$$h_+(t,i) = \frac{4}{d_L(z)} (G\mathcal{M}_z)^{5/3} (\pi f(t^{\rm ret}))^{2/3} \left(\frac{1+\cos^2 i}{2}\right) \cos(2\Phi(t^{\rm ret}))$$

$$h_\times(t,i) = \frac{4}{d_L(z)} (G\mathcal{M}_z)^{5/3} (\pi f(t^{\rm ret}))^{2/3} \cos i \sin(2\Phi(t^{\rm ret}))$$

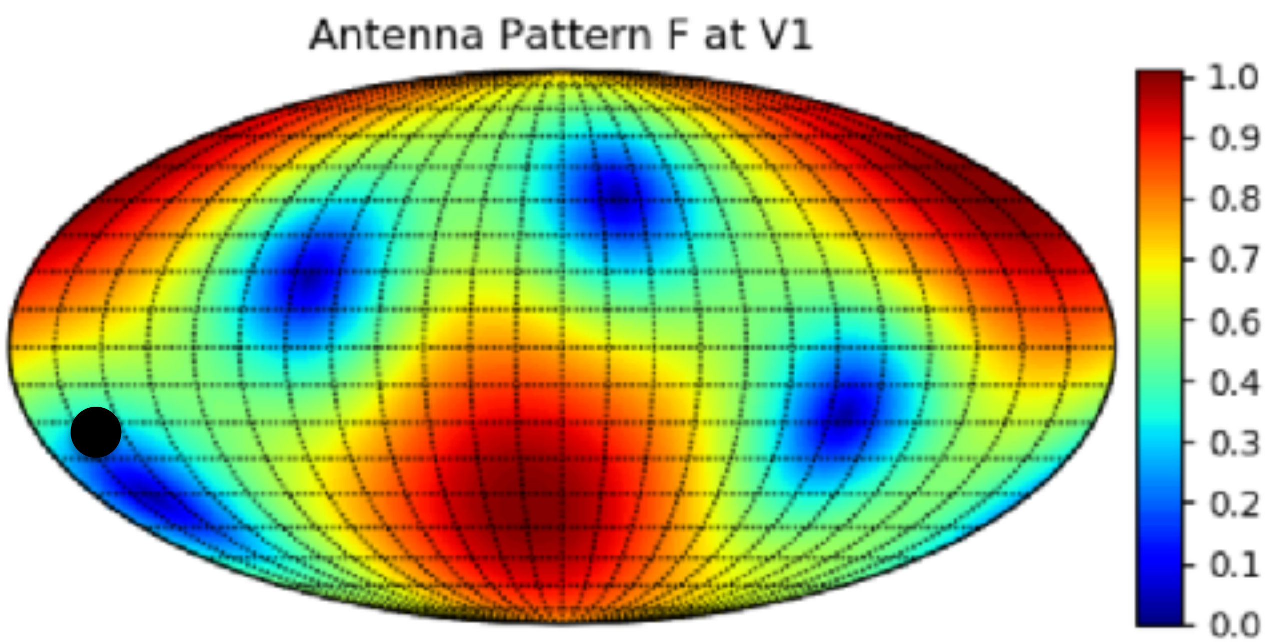
  – From their phase evolution determine redshifted chirp mass;
  – from the ratio of the two the inclination angle;
  – and hence from their amplitude the luminosity distance.

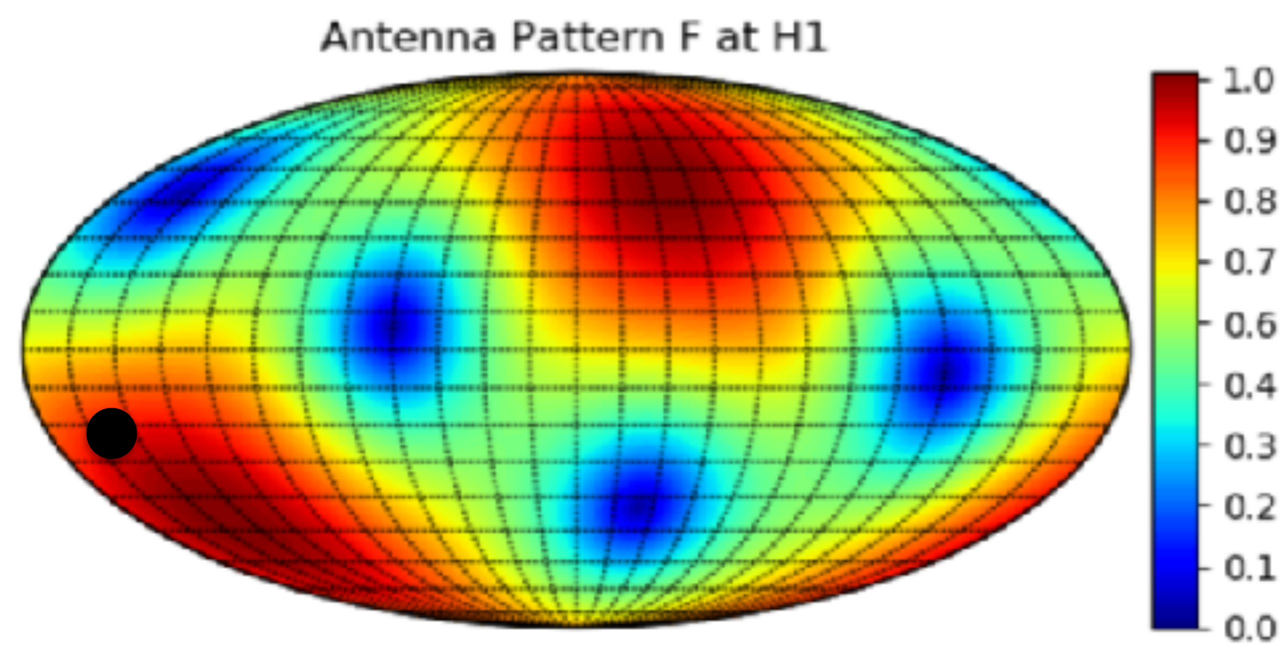
- But that's not the reality of GW interferometers
- Signal at detector *a*

2 polarisations of GW in GR

$$h_a(t,\alpha,\delta,\ldots) = F_a^+(t,\alpha,\delta,\ldots)h_+ + F_a^\times(t,\alpha,\delta,\ldots)h_\times$$

antenna pattern functions, depending on the geometry of the detector, position of the source in the sky defined by declination and right ascension, the polarisation angle etc (For Ligo-Virgo can ignore t dependence.)





● =GW170817

- What are typical distance errors? How do they depend on the position of the source in the sky? And the inclination?

Real space $$h_a(t,\alpha,\delta,\ldots) = F_a^+(t,\alpha,\delta,\ldots)h_+ + F_a^\times(t,\alpha,\delta,\ldots)h_\times$$

Fourier space

$$\boxed{\begin{aligned}\tilde{h}(f) &= \frac{\mathscr{A}}{f^{7/6}}e^{i\Psi(f,\mathscr{M}_z)}\\ \mathscr{A} &= \mathscr{A}(\alpha,\delta,\psi,\iota,d_L;\mathscr{M}_z)\end{aligned}}$$

$$\Psi(f,\mathscr{M}_z) = 2\pi f t_{c,d} - \frac{\pi}{4} - \phi_c + \frac{3}{128}\left(\frac{\pi G\mathscr{M}_z}{c^3}\right)^{-5/3}\frac{1}{f^{5/3}}$$

A quick and efficient method for forecasting errors is to use the “Gaussian” Fisher matrix approach

(…but distance posterior probability density is pretty non gaussian in general!)

[Finn, Cutler+Flanagan,…]

- Detector data

$$s = \text{noise} + \text{signal}(\vec{\theta})$$

- Assuming the noise in a detector to be stationary and gaussian with zero mean

$$(h_1|h_2) = 2\int_0^\infty df\,\frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_2^*(f)\tilde{h}_1(f)}{S_n(f)}$$

- SNR (signal to noise ratio):

$$\rho = (h|h)^{1/2} \equiv 2\left[\int_0^\infty df\,\frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)}\right]^{1/2},$$

- Fisher information matrix defined by

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta^i}\Big|\frac{\partial h}{\partial \theta^j}\right)$$

$$\theta^i = \{\mathscr{A}, \mathscr{M}_z\}$$

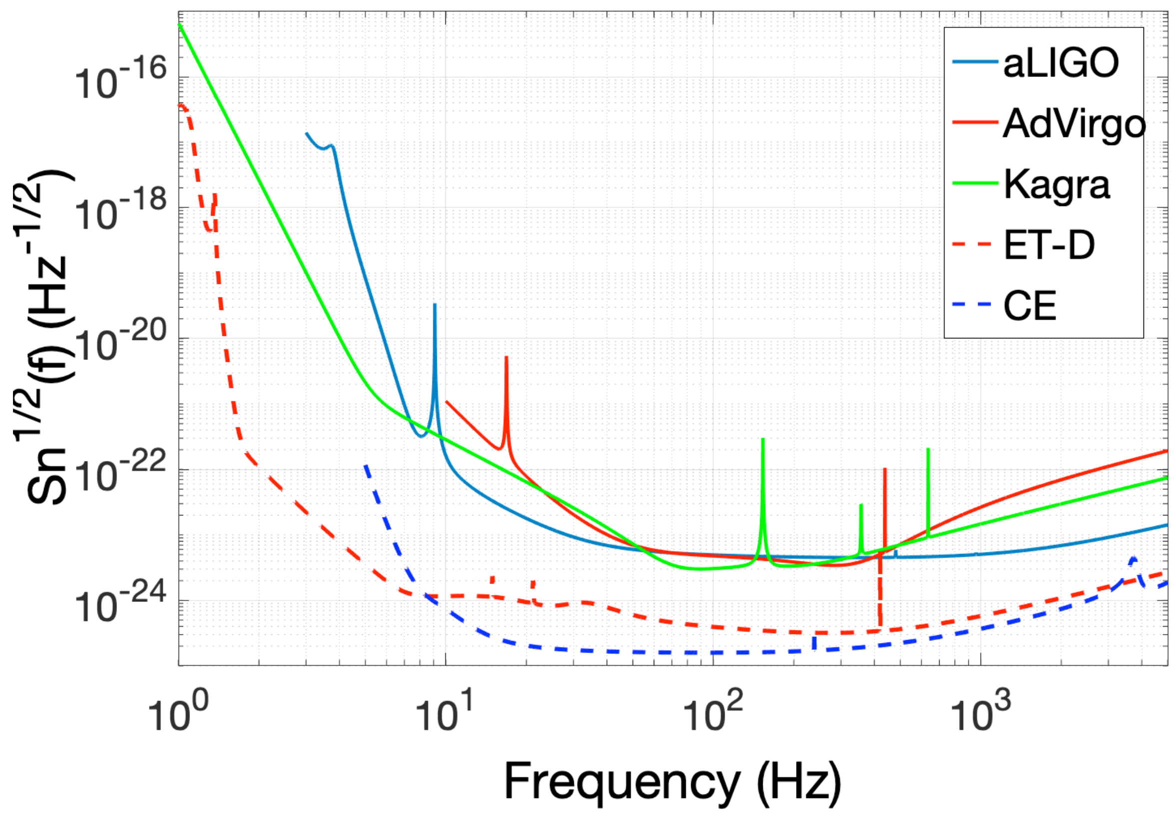
- Statistical errors are the estimated from the square root of the diagonal elements of the inverse matrix,

$$\Delta\theta^i \sim \sqrt{\Sigma^{ii}} \qquad \Sigma = \Gamma^{-1}$$

- Here:

$$\tilde{h}(f) = \frac{\mathscr{A}}{f^{7/6}} e^{i\Psi(f,\mathscr{M}_z)}$$

$$\frac{\partial \tilde{h}}{\partial \ln \mathcal{A}} = \tilde{h}. \qquad \frac{\partial \tilde{h}}{\partial \ln \mathcal{M}} = -\frac{5i}{4}(8\pi\mathcal{M}f)^{-5/3}\,\tilde{h}$$



$$\frac{\Delta \mathscr{A}}{\mathscr{A}} \sim 0.1 \left(\frac{10}{\rho}\right)$$

$$\frac{\Delta \mathscr{M}_z}{\mathscr{M}_z} \sim 10^{-5} \left(\frac{10}{\rho}\right) \left(\frac{\mathscr{M}_z}{M_\odot}\right)^{5/3}.$$

For a given an SNR, ρ , the error on the chirp mass is generally much smaller than the error on the amplitude, and thus smaller than the error on the amplitude

$$\mathscr{A} = \mathscr{A}(\alpha, \delta, \psi, \iota, d_L; \mathscr{M}_z)$$

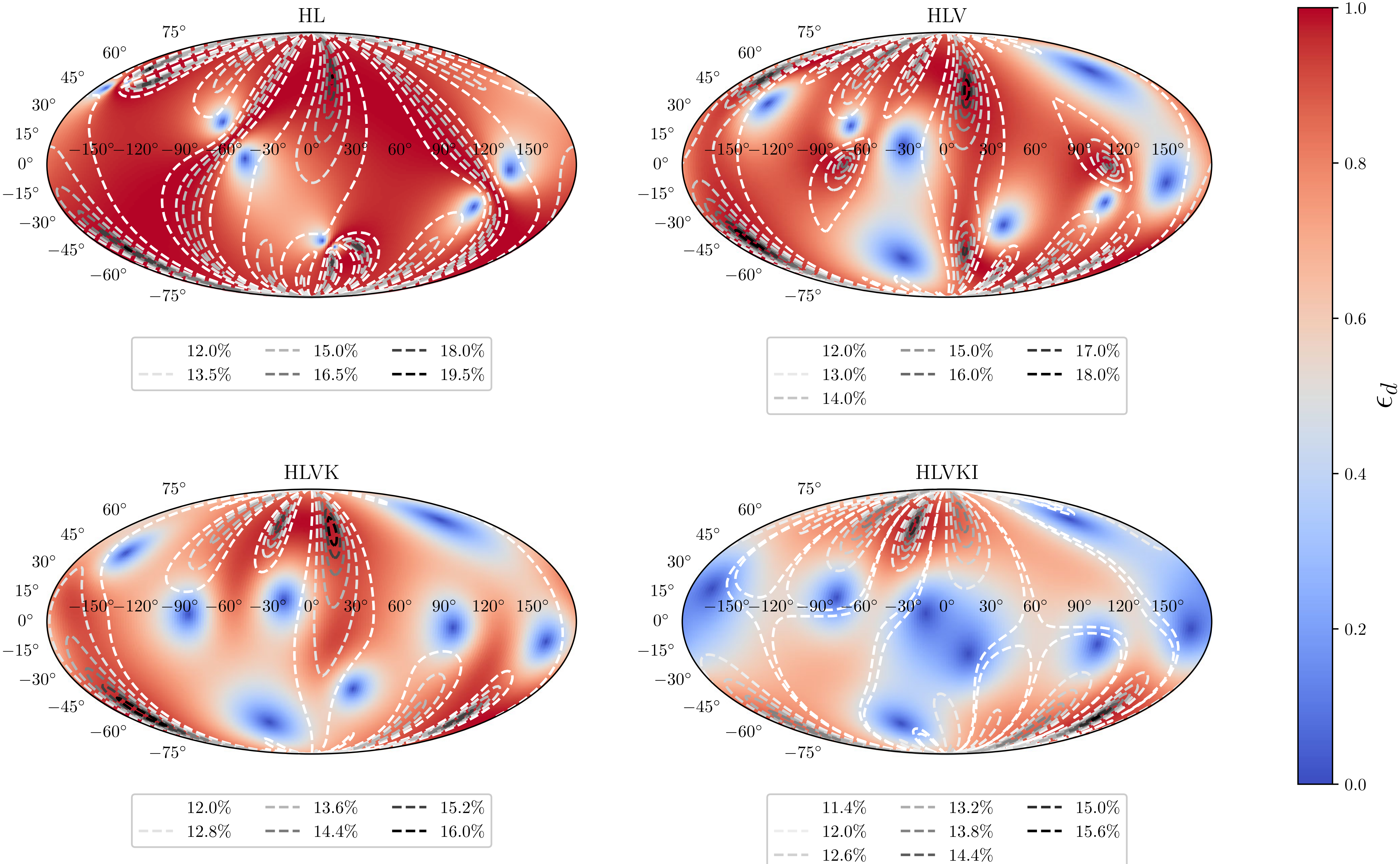
- Sky position? Are their any "golden spots" in the sky for which the error on dL would be much smaller? After all, GW170817 in a "blind spot" of Virgo. Would a similar event in a different part of the sky significantly change the dL estimation? If so by how much?

- Can one go beyond the gaussian approx, and develop an analytical estimator for dL, valid for source at any sky position and valid for any detector network?

- With more detectors, possibly more sensitive ones also, how does the accuracy increase, and vary as a function of sky position?

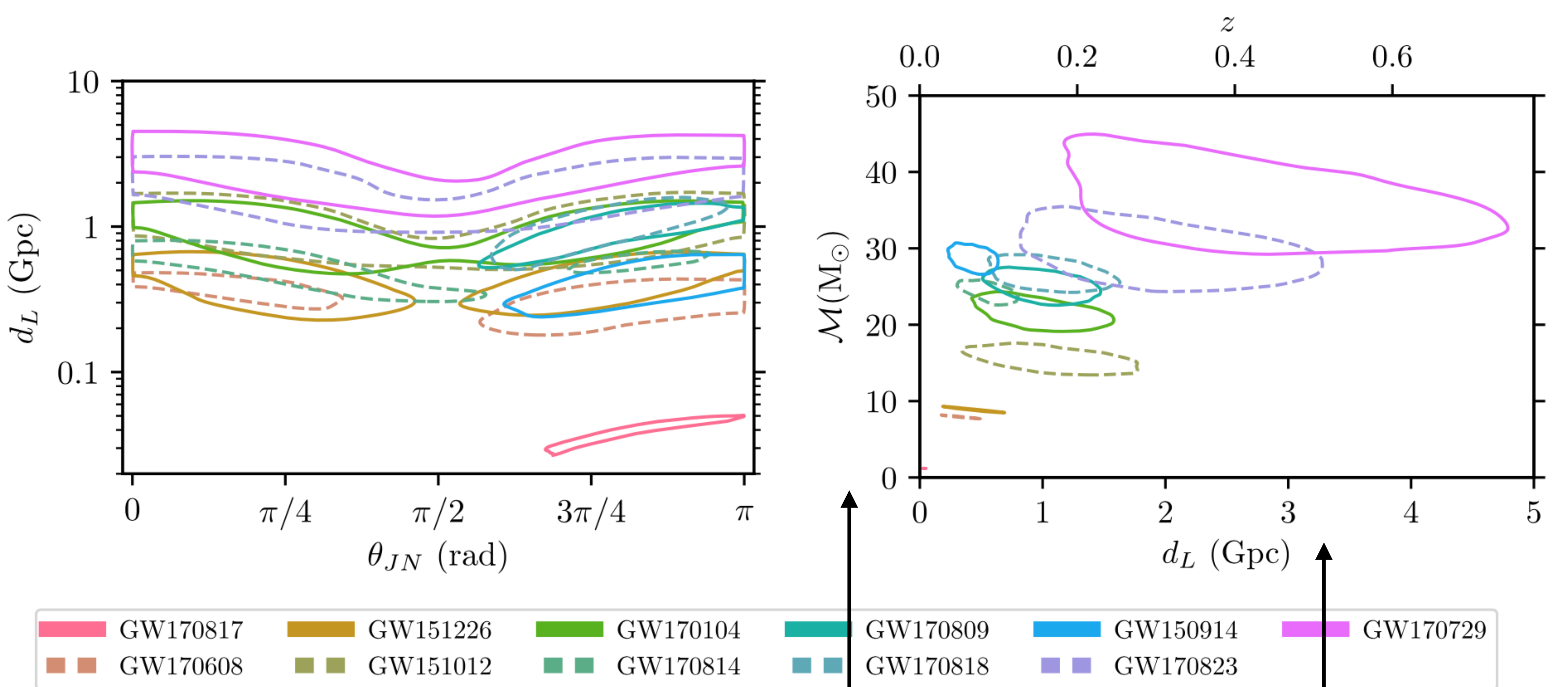
[K.Leyde et al, 1906.02670]

• Here Delta D/D, having marginalized over the other parameters, for SNR=33



– No “golden spots”….

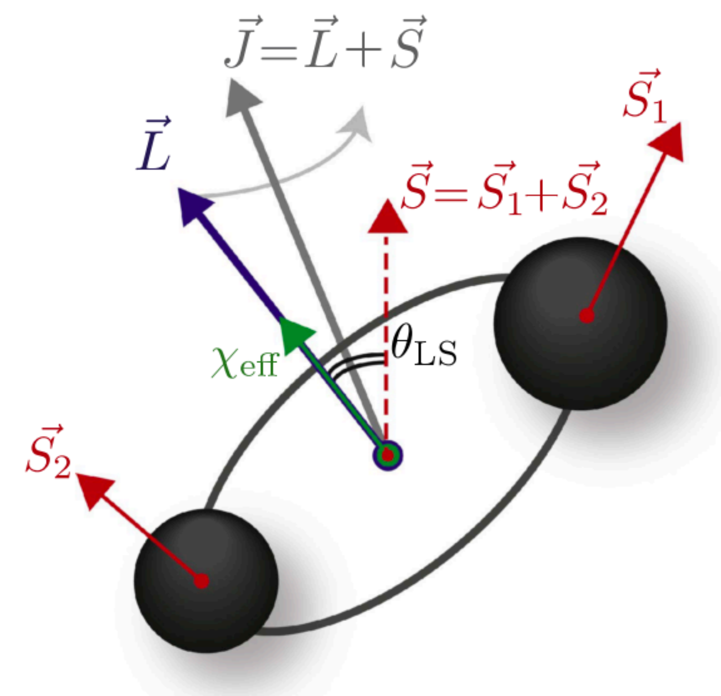
10 BBH events in GWTC-1 catalogue of LIGO-Virgo
(90% confidence level intervals)



error on chirp mass
increases with chirp mass

large errors due to distance inclination
degeneracy

Is it possible to reduce the error on dL?



In some cases, yes.

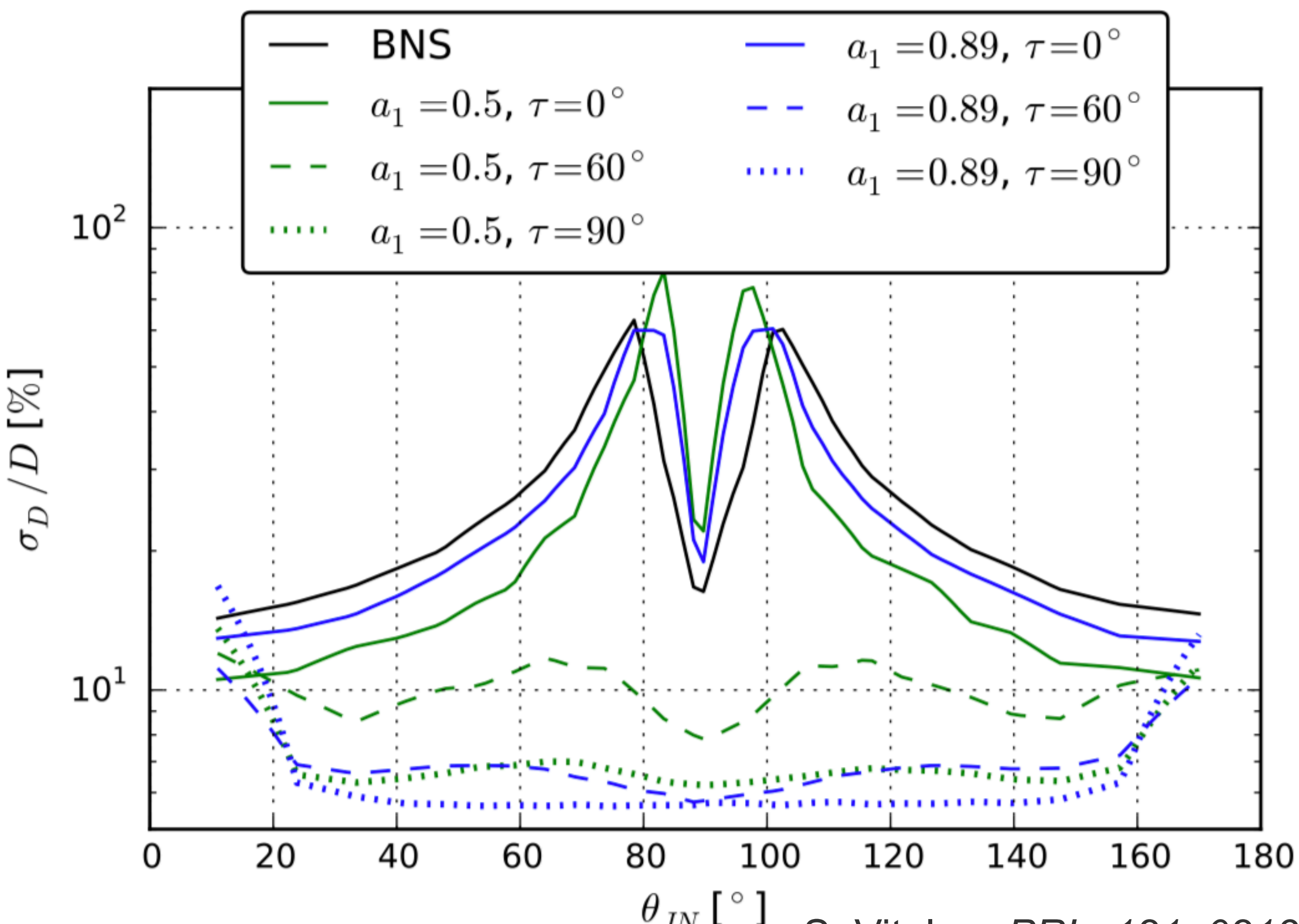
1) Binaries with spins:

if total spin is *misaligned* with the total orbital momentum L,
then these vectors change orientation due to spin-orbit interaction (1.5PN effect)
=> *detected radiation has a time-varying polarisation*.
=> As a result the *orbital plane precesses*, and inclination angle will
depend on time, thus providing an additional phase modulation to the GW signal

In case of NS-BH binaries, for same SNR, precision on luminosity distance can be increased by a factor of 10.

Here: NS has no spin.
BH has a spin tilted at angle τ
relative to orbital angular momentum

$\tau \neq 0$ have precession.



S. Vitale + *PRL, 121, 021303, 2018*

2) Higher order modes

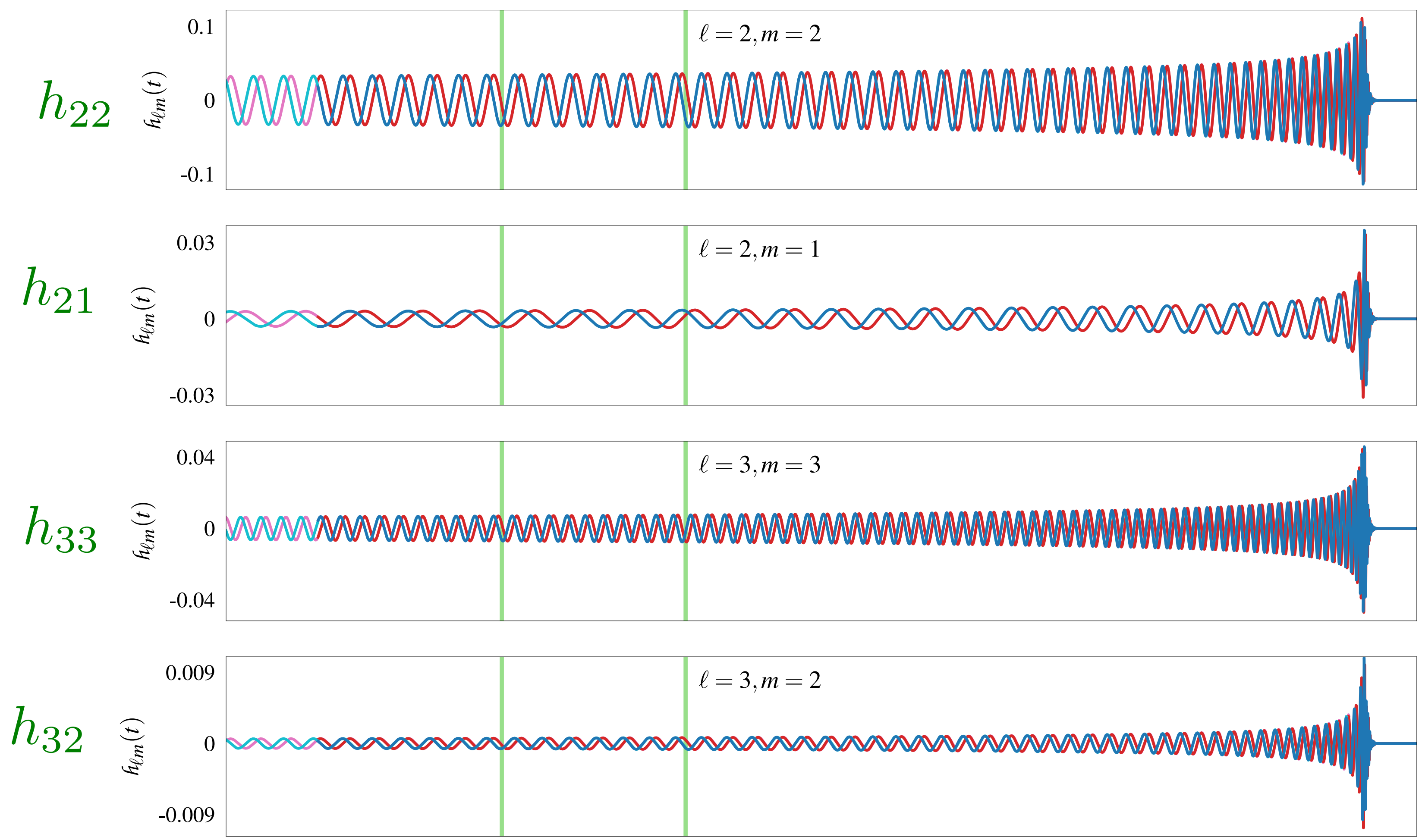
- Two polarisation modes generally decomposed into spin -2 weighted spherical harmonics:

$$h_+ - ih_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} Y_{\ell m}(i, \varphi)\, h_{\ell m},$$

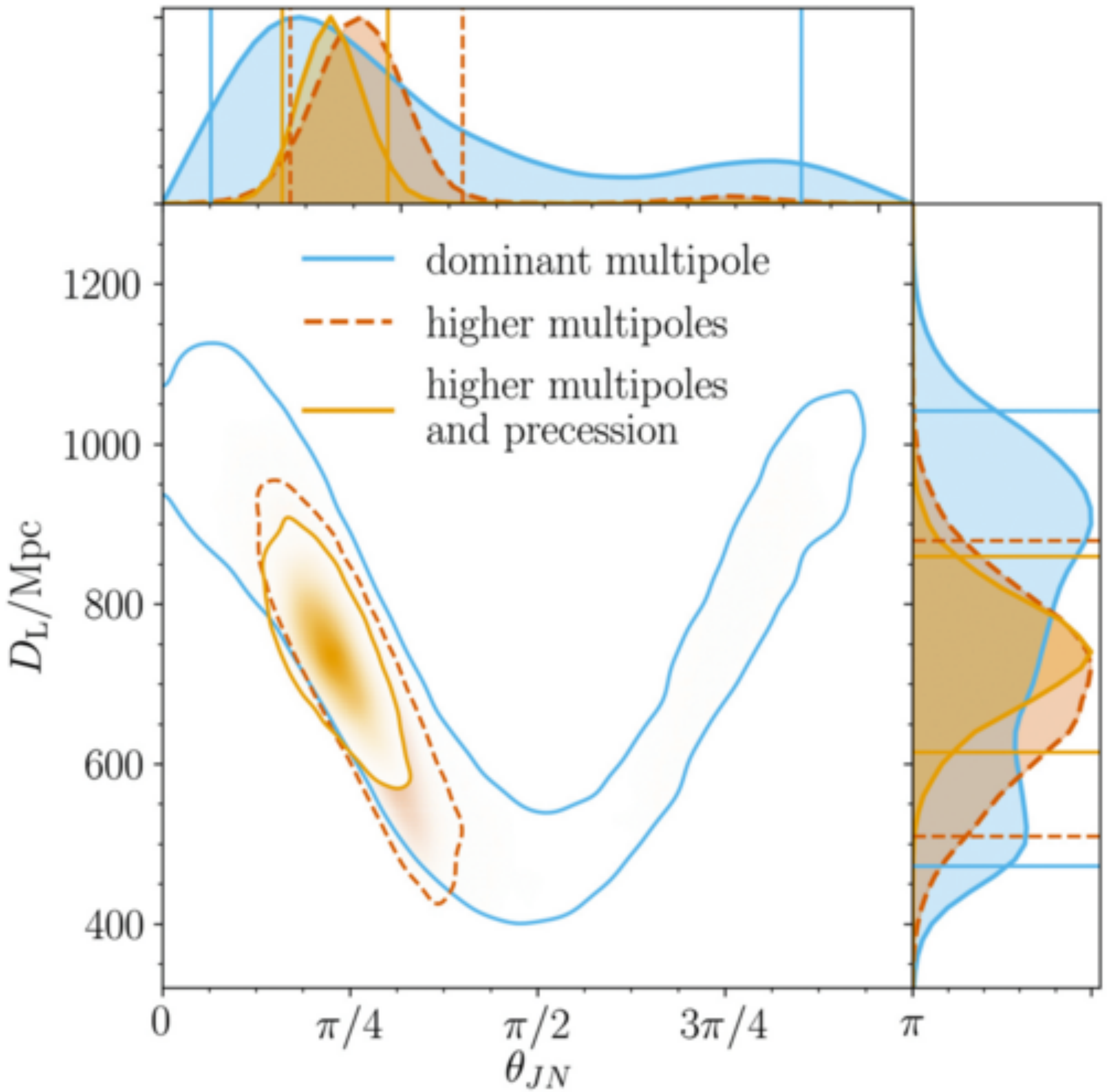
- So far discussed the dominant quadrupolar mode.

- Higher order modes generally depend on the mass difference, and scale differently with inclination.

$$\Delta = \frac{(m_1 - m_2)}{(m_1 + m_2)},$$



[1409.2349] mass ratio = 1/8



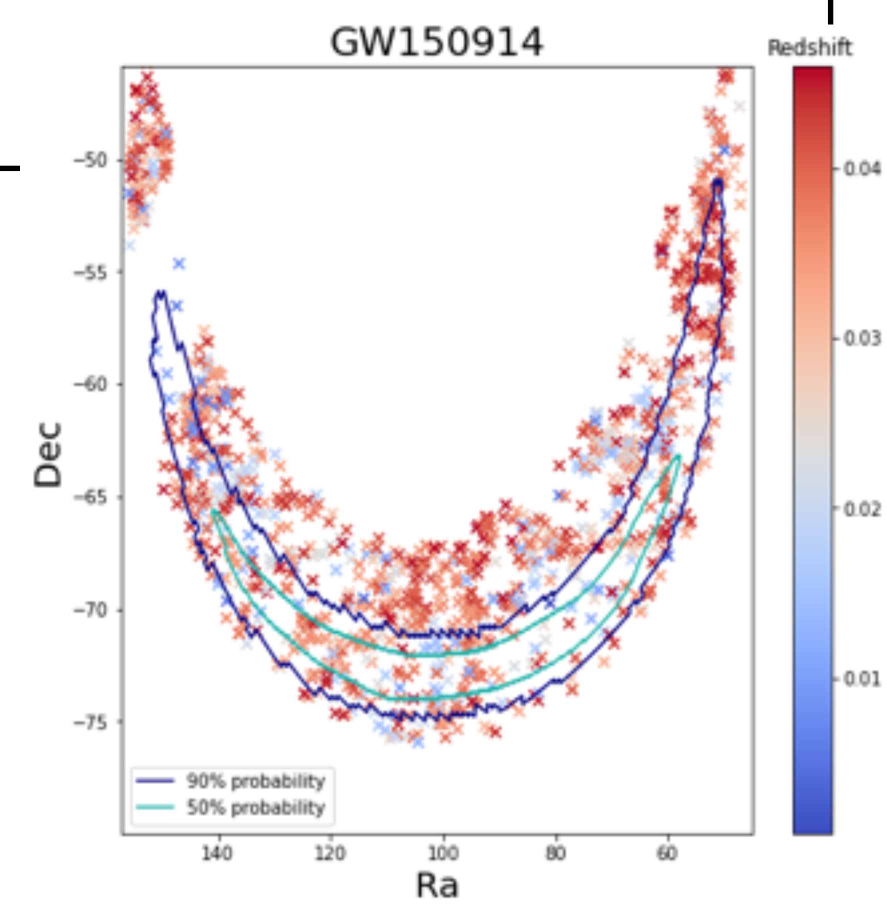
$m_1 \sim 30 M_\odot, m_2 \sim 8 M_\odot$

Fig. 4 GW190412: Posterior distribution for the luminosity distance and inclination. The central plot shows the 90% confidence level for different waveform approximants namely: the dominant multipole (and no precession), higher multipoles and no precession; and higher multipoles and precession. The impact of higher multipoles on constraining the inclination and distance is clear. The top and side plots show the marginal posteriors of ι and d_L respectively. Figure from [37]

5) Determining the redshift

- Crux of doing late-time cosmology with GWs is to determine r*edshift* of the sources.

1. A direct EM counterpart with an associated redshift measurement *[B.Schutz, '86]*
2. A collection of galaxies localized in the GW localization volume (i.e. using galaxy catalogues) *[B.Schutz, '86]*
3. Knowledge of the *source frame mass distribution*
4. *for NS, a* measure of the tidal deformability + equation of state
5. ….



1. direct EM counterpart (NS-NS or NS-BH mergers).

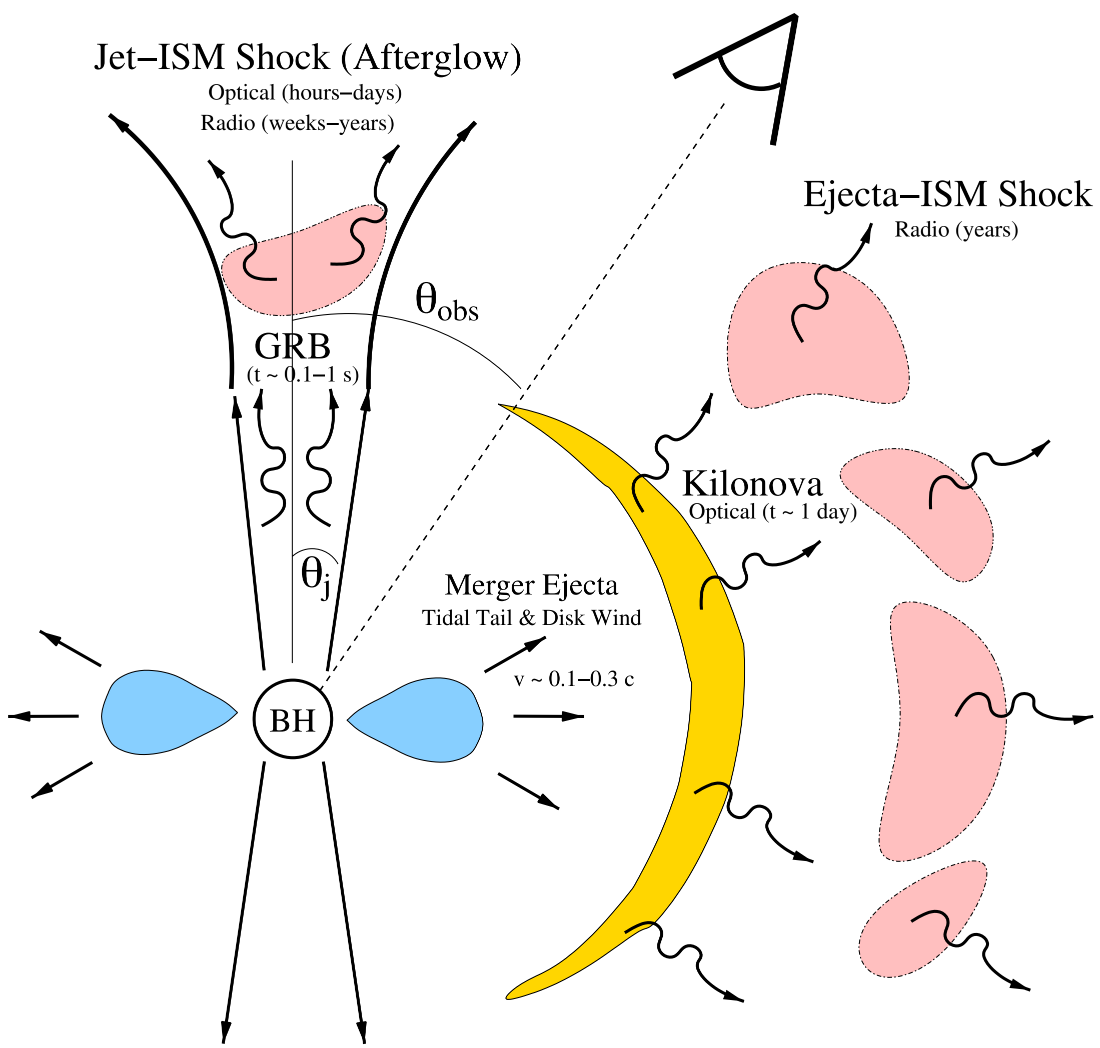
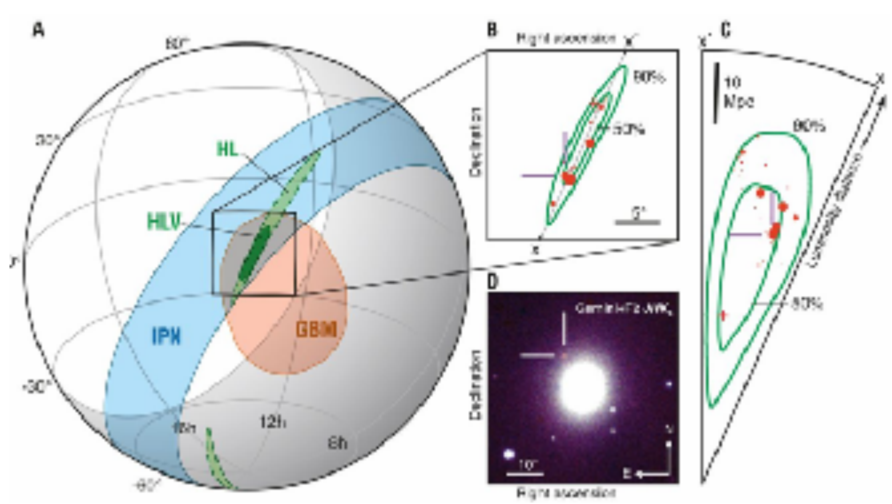


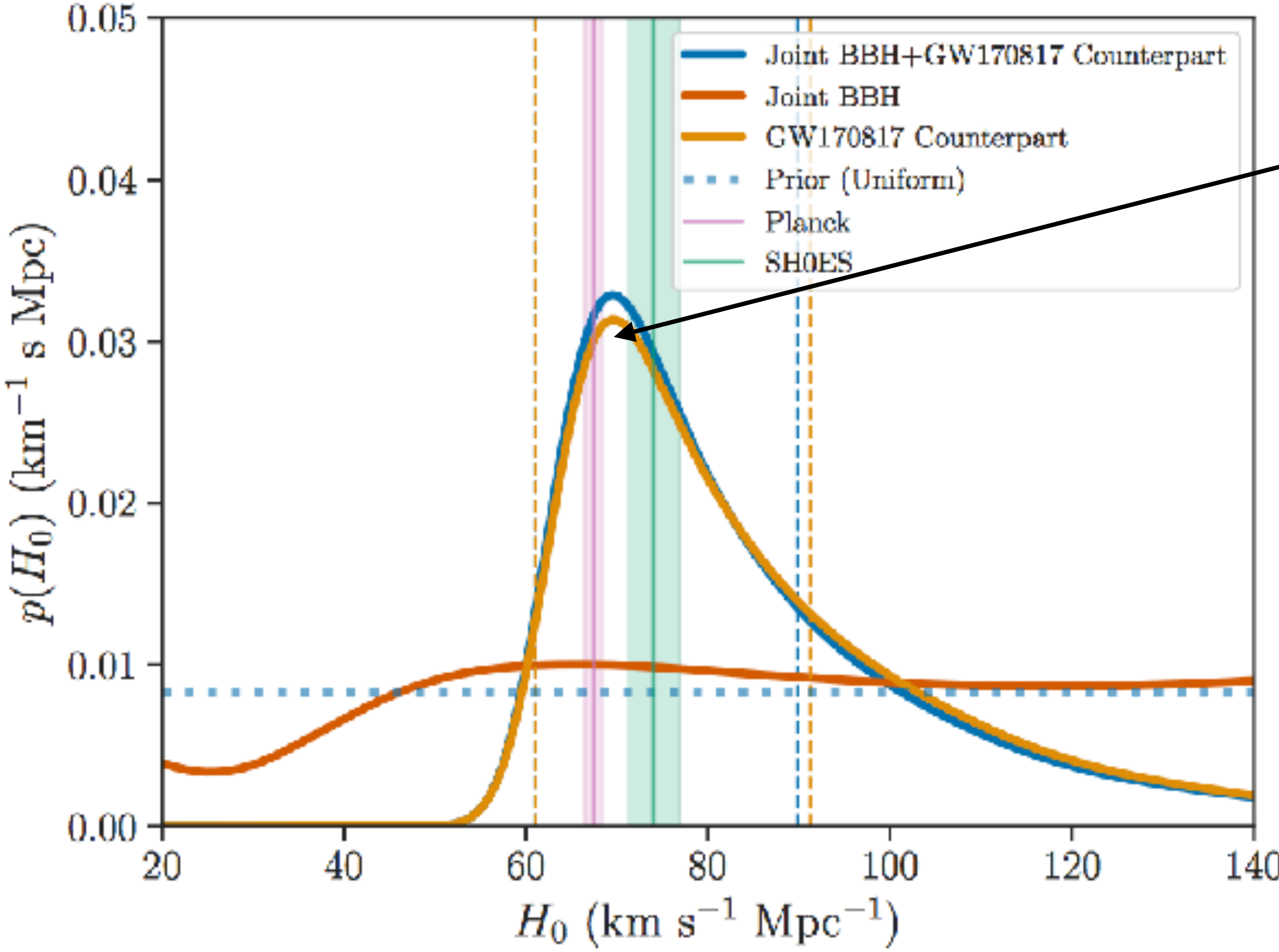
Figure 1. Summary of potential electromagnetic counterparts of NS–NS/NS–BH mergers discussed in this paper, as a function of the observer angle, θ_{obs}. Following the merger a centrifugally supported disk (blue) remains around the central compact object (usually a BH). Rapid accretion lasting $\lesssim 1$ s powers a collimated relativistic jet, which produces a short-duration gamma-ray burst (Section 2). Due to relativistic beaming, the gamma-ray emission is restricted to observers with $\theta_{obs} \lesssim \theta_j$, the half-opening angle of the jet. Non-thermal afterglow emission results from the interaction of the jet with the surrounding circumburst medium (pink). Optical afterglow emission is observable on timescales up to ~ days–weeks by observers with viewing angles of $\theta_{obs} \lesssim 2\theta_j$ (Section 3.1). Radio afterglow emission is observable from all viewing angles (isotropic) once the jet decelerates to mildly relativistic speeds on a timescale of weeks–months, and can also be produced on timescales of years from sub-relativistic ejecta (Section 3.2). Short-lived isotropic optical emission lasting ~few days (kilonova; yellow) can also accompany the merger, powered by the radioactive decay of heavy elements synthesized in the ejecta (Section 4).

For GW170817,
measure redshift from optical identification of the host galaxy (NGC4993)



binary neutron star merger GW170817

Marginalized posterior density for H_0



$$H_0 = 69^{+17}_{-8} \text{ km/Mpc/s}$$

Errors:

1) Hubble flow velocity: need to account for peculiar velocities, caused by local grav field, attraction to nearby galaxies

$$v_H = 3017 \pm 166 \,\mathrm{km\,s^{-1}}.$$

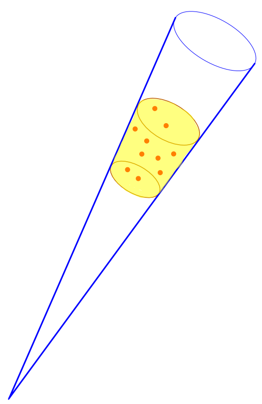
2) Distance error large,

$$d = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$$

3) statistical measurement error from noise in detectors + instrumentation calibration uncertainties:

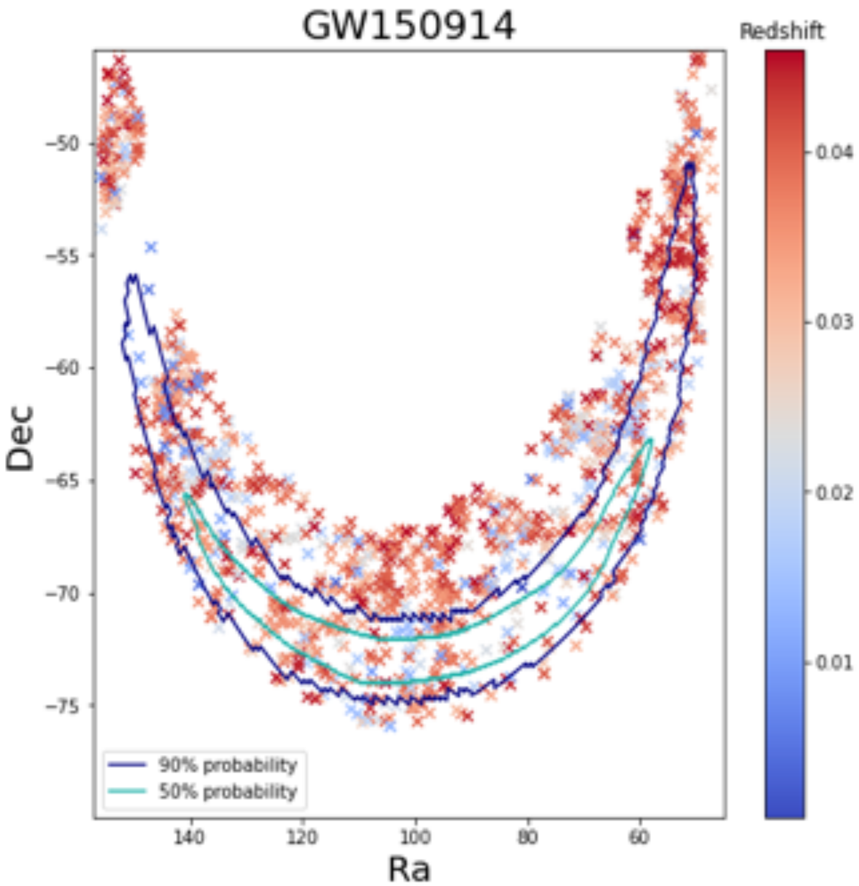
2. Most events are BBH with no EM counterpart: z from collection of galaxies localized in the GW localization volume (i.e. using galaxy catalogues)

[B.Schutz, '86]



with *good localization* of the GW event

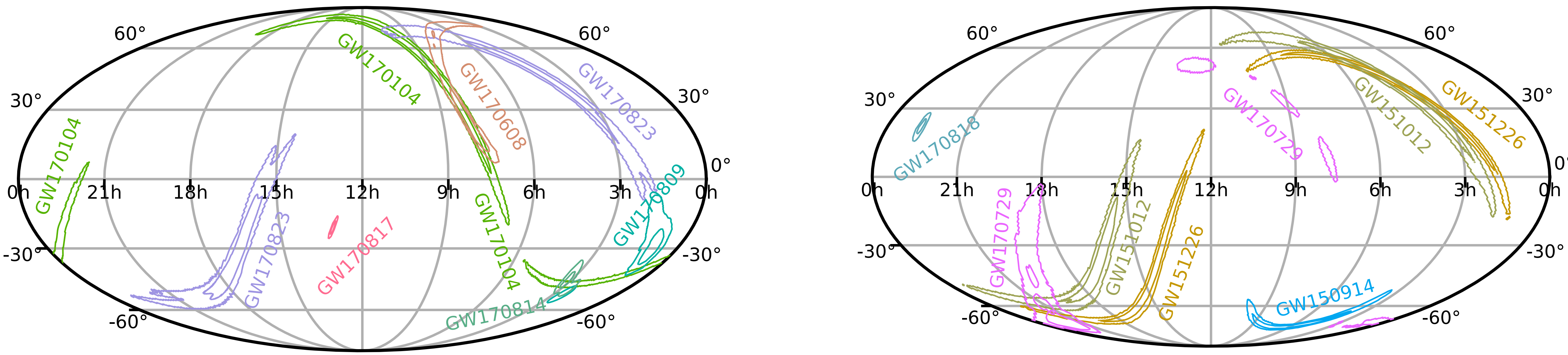
Infer z using directional information :
– using galaxy catalogues, identify all potential hosts in localization region
– incorporate redshift of each
– any confusion between host galaxies is averaged out by many sources



Method applied to O2 events.

– But not all events well localized (some with 2 detectors only)
– Furthermore, the catalogues may not cover completely the GW 3D localisation volume
– and also the catalogues may be incomplete.

[B.P.Abbot et al. arXiv:1908.06060v2]



B.P. Abbot et al Phys. Rev. X **9**, 031040

Event	SNR	$\Delta\Omega/\mathrm{deg}^2$	d_L/Mpc	z_{event}	V/Mpc^3	Galaxy catalog	Number of galaxies	m_{th}	$p(G\|z_{\mathrm{event}}, D_{\mathrm{GW}})$
GW150914	24.4	182	440^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	3.5×10^6	GLADE	3910	17.92	0.42
GW151012	10.0	1523	1080^{+550}_{-490}	$0.21^{+0.09}_{-0.09}$	5.8×10^8	GLADE	78195	17.97	0.01
GW151226	13.1	1033	450^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	2.4×10^7	GLADE	27677	17.93	0.41
GW170104	13.0	921	990^{+440}_{-430}	$0.20^{+0.08}_{-0.08}$	2.4×10^8	GLADE	42221	17.76	0.01
GW170608	15.4	392	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	3.4×10^6	GLADE	6267	17.84	0.60
GW170729	10.8	1041	2840^{+1400}_{-1360}	$0.49^{+0.19}_{-0.21}$	8.7×10^9	GLADE	77727	17.82	< 0.01
GW170809	12.4	308	1030^{+320}_{-390}	$0.20^{+0.05}_{-0.07}$	9.1×10^7	GLADE	18749	17.62	< 0.01
GW170814	16.3	87	600^{+150}_{-220}	$0.12^{+0.03}_{-0.04}$	4.0×10^6	DES-Y1	31554	23.84	> 0.99
GW170817	33.0	16	40^{+7}_{-15}	$0.01^{+0.00}_{-0.00}$	227	–	–	–	–
GW170818	11.3	39	1060^{+420}_{-380}	$0.21^{+0.07}_{-0.07}$	1.5×10^7	GLADE	1059	17.51	< 0.01
GW170823	11.5	1666	1940^{+970}_{-900}	$0.35^{+0.15}_{-0.15}$	3.5×10^9	GLADE	117680	17.98	< 0.01

probability that the galaxy is in the catalogue.

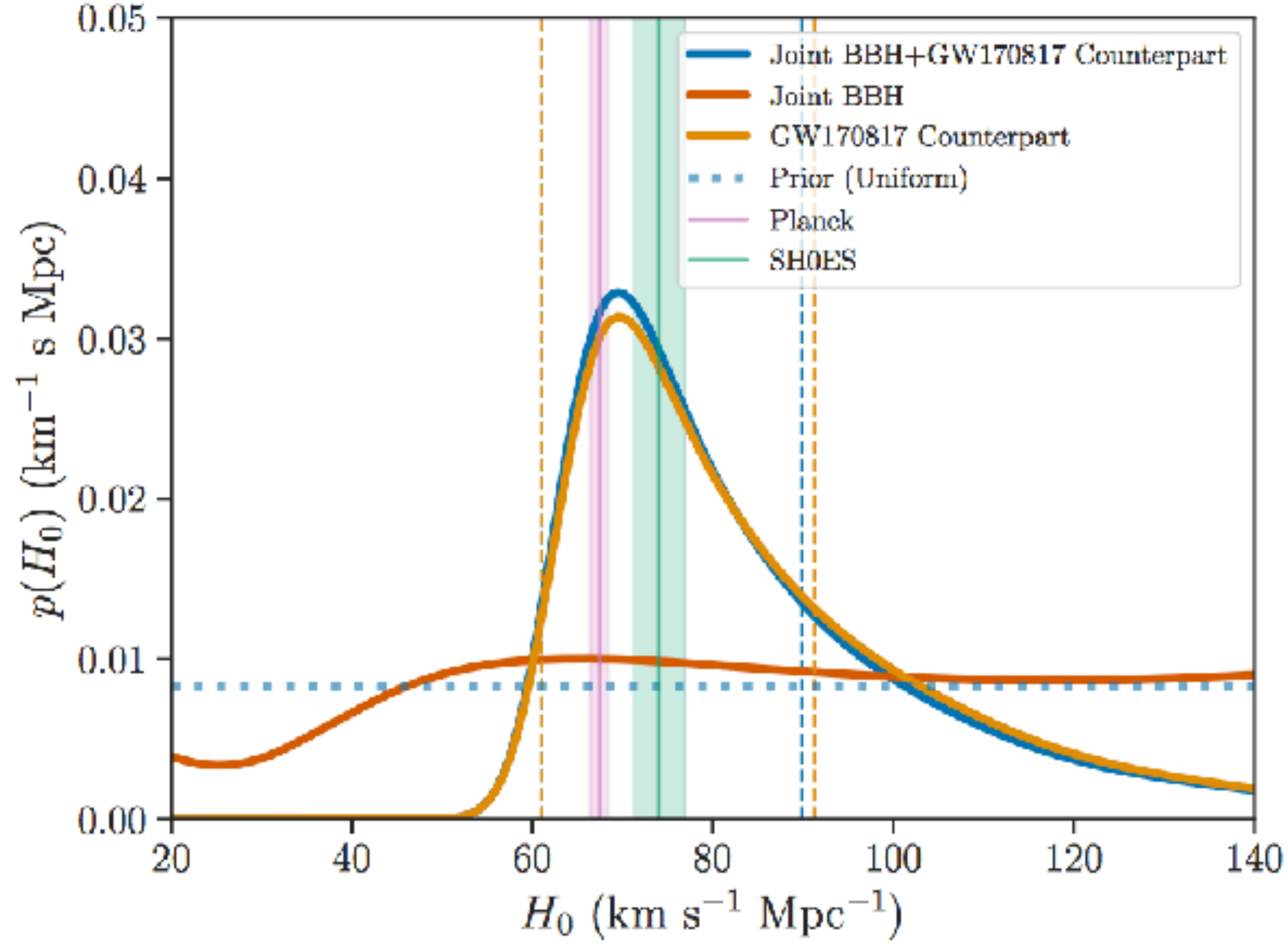
Table 1. Relevant parameters of the O1 and O2 detections: network signal-to-noise ratio (SNR) for the search pipeline (PyCBC/GstLAL) in which the signal is the loudest, 90% sky localization region $\Delta\Omega$ (deg^2), luminosity distance d_L (Mpc, median with 90% credible intervals), and estimated redshift z_{event} (median with 90% range assuming Planck 2015 cosmology) from Abbott et al. (2019b). In the remaining columns we report the corresponding 90% 3D localization comoving volumes, the number of galaxies within each volume for public catalogs which we find to be the most complete, and the apparent magnitude threshold, m_{th}, of the galaxy catalog associated with the corresponding sky region (as described in Section 3.3). The final column gives the probability that the host galaxy is inside the galaxy catalog for each event, $p(G|z_{\mathrm{event}}, D_{\mathrm{GW}})$, also evaluated at the median redshift for each event.

[B.P.Abbot et al. arXiv:1908.06060v2]

For O1 and O2 events from LIGO-Virgo

Only GW170817, with z from NGC4993

$$H_0 = 69^{+17}_{-8}\ \mathrm{km/Mpc/s}$$

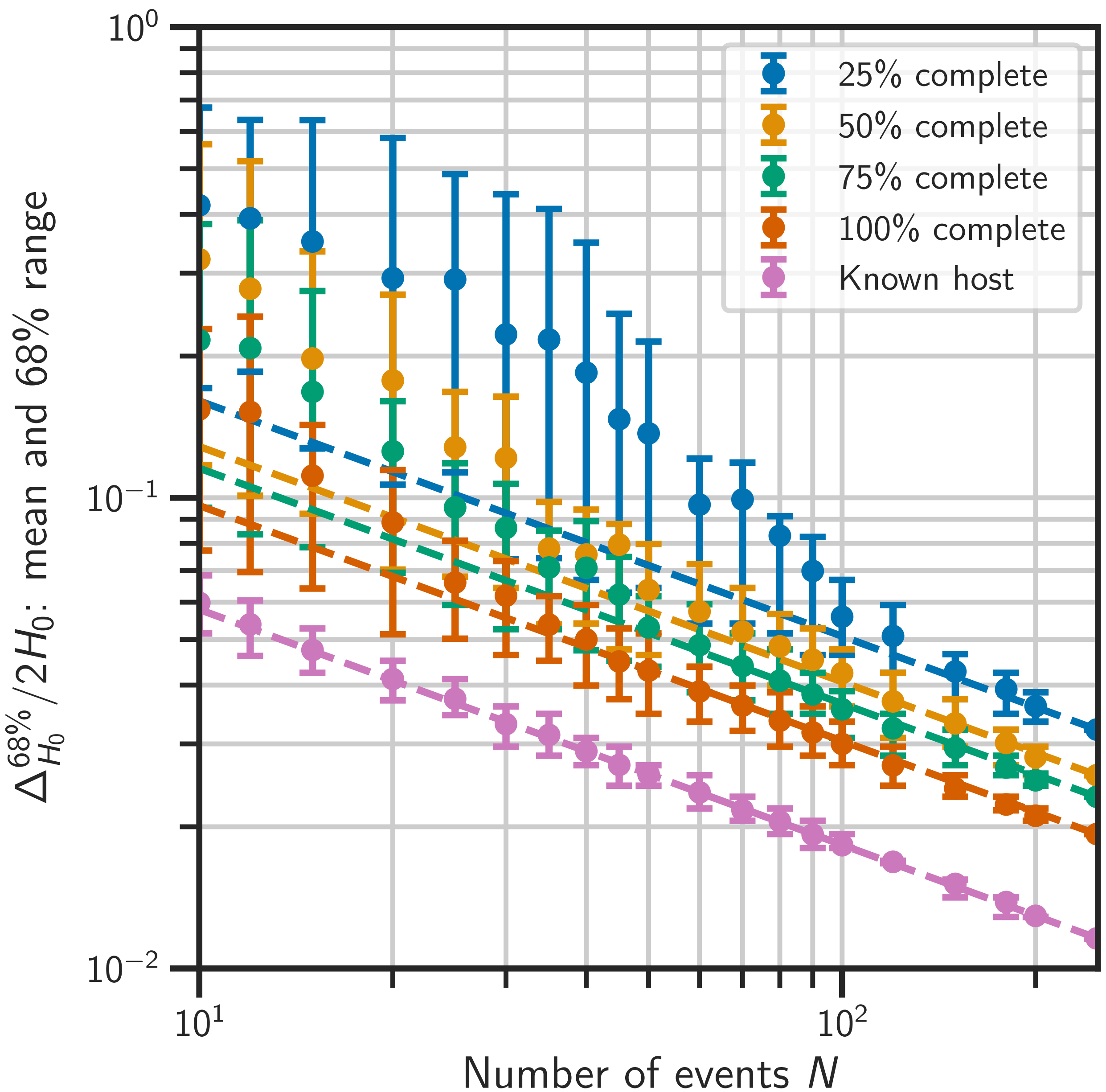


including 6 BBHs with SNR > 12 leads to ~4% improvement

$$H_0 = 69^{+16}_{-8}\ \mathrm{km/Mpc/s}$$

[B.P.Abbot et al. arXiv:1908.06060v2]

In future will have many more direct GW detections with LIGO-Virgo: **what are the prospects for solving H0 tension with LIGO-Virgo with methods 1 and 2 ?**



catalogue of simulated signals in 2015-2016 observing run.

H0 accurate to ~3% with:

~30 events with identified host galaxy,
~100 events with a galaxy catalogue that is 100% complete

dashed lines: $1/\sqrt{N}$

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Determining the redshift

- Crux of doing late-time cosmology with GWs is to determine *redshift* of the sources.

1. A direct EM counterpart with an associated redshift measurement *[B.Schutz, '86]*
2. A collection of galaxies localized in the GW localization volume (i.e. using galaxy catalogues) *[B.Schutz, '86]*
3. Knowledge of the *source frame mass distribution*
4. *for NS, a* measure of the tidal deformability + equation of state
5. ….

- **For ET and LISA,** galaxy catalogues will probably will be incomplete up to redshifts observed

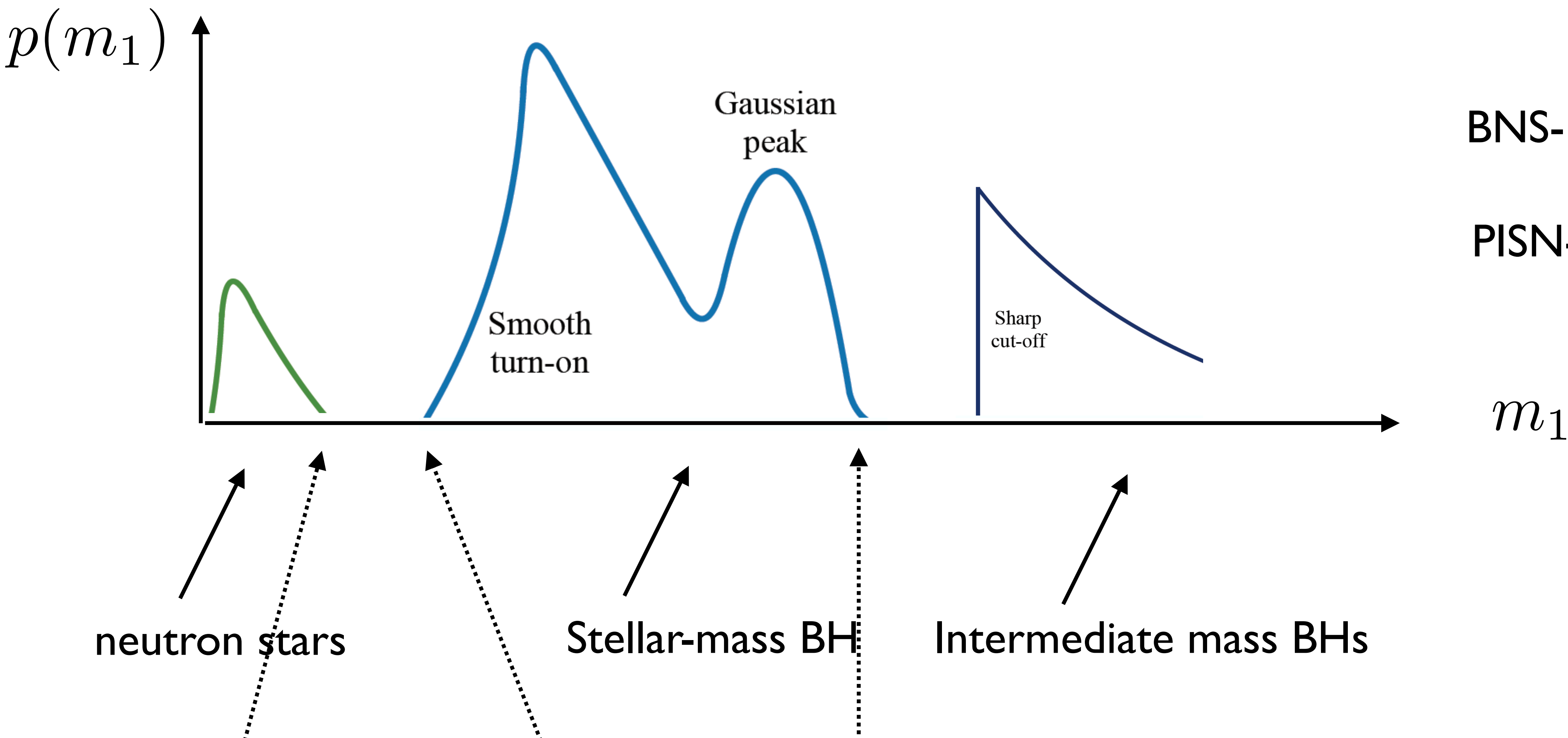
- Approaches 3. and 4. **use no EM data**, and hence work also for BBH (more numerous, heavier and observable to larger z). Basic idea:

$$m_{1,2}^{\rm det} = [1 + z(d_L, H_0, \ldots)] m_{1,2}^{\rm source}$$

from knowledge of source mass (for a population or individual source), together with given observed mass can infer z-distribution.

Very roughly expect errors to scale as $\sim 1/\sqrt{N}$

3) Knowledge of the *source frame mass distribution*



BNS-BH mass-gap

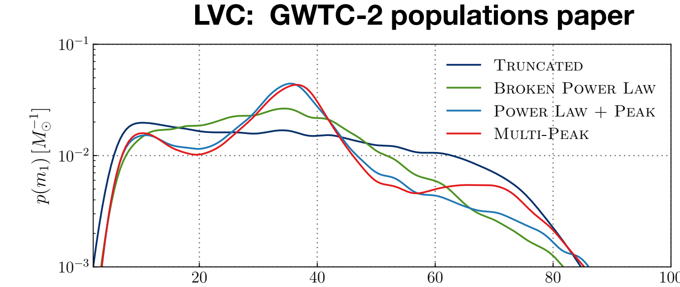
PISN-mass-gap?

maximum mass allowed by nuclear physics

Minimum mass allowed by stellar evolution

PISN mass-gap?
Pair instability prevents formation of remnant BH?

LVC: GWTC-2 populations paper



O3a
Observed mass spectrum
- Peak 35-40 Msun
- Decrease >60 Msun
- Cutoff <8 Msun

Knowledge of the *source frame mass distribution*

Forecasts.

Event rates :

ET larger detection horizon **->** larger detection rates:

BBH $10^5 - 10^6/\mathrm{year}$

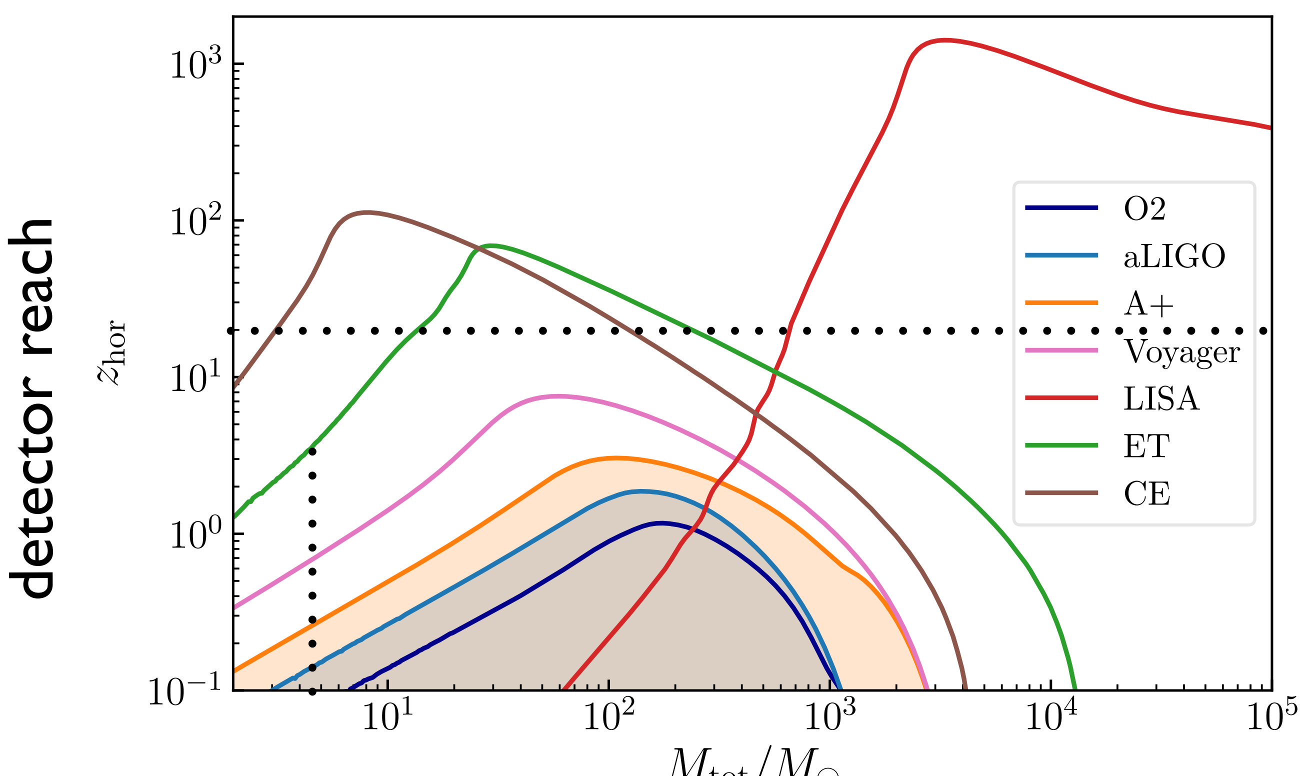
BNS $\sim 10^4/\mathrm{year}$ of which

$\sim \mathcal{O}(10^2 - 10^3)/\mathrm{year}$

with counterparts,
depending on EM facilities
operating at the time

and also NS-BH

[figure from 2006.02211]



Horizon redshift as a function of total *source frame* mass for an SNR detection threshold of rho=8. For LISA assumes 4 yrs obsv.

Must consider weak lensing of signals

$\rightarrow \sigma_{d_L}^{\mathrm{inst}} \sim \dfrac{2d_L}{\rho}$

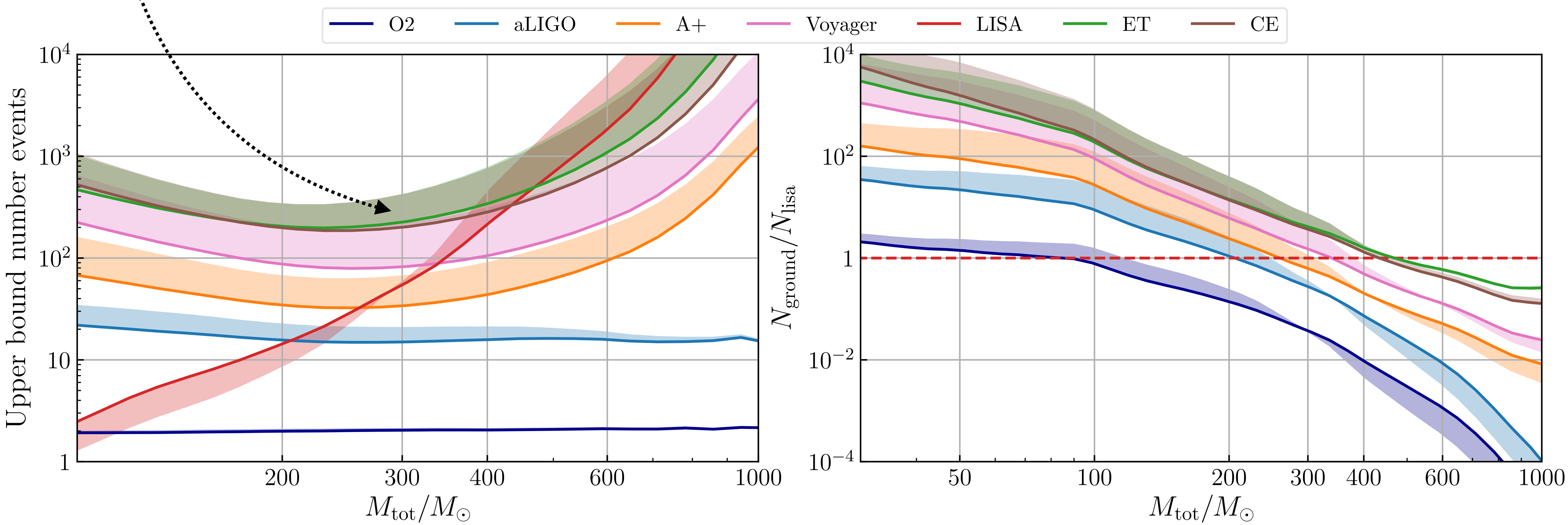
$\sigma_{d_L}^{\mathrm{lens}} \sim 0.05 d_L z$

similar order of magnitude
for z ~ few

Distribution of events as a function of their total source frame mass, for an observing time of 1year (except LISA=4years)

[figure from 2006.02211]

ET



maximum number of events detected given the upper bounds from the O1 and O2 runs [bands indicate difference resulting from a redshift evolution tracking the star formation rate]

Ratio between the number of non-spinning, equal mass binaries detected by different ground-based detectors, with respect to LISA

In range Mtot < 400 Msun, ET will detect more BBH in 1 year than LISA in 4 years, and several hundred events.

Forecasts for cosmological parameters

[Cai et al, 1608.08008]
[Belgacem et al,1907.01487]
[Ganz et al, in preparation]
[Ezquiaga et al, 2006.02211]
[You et al 2004.00036]
…

Generally proceed through **construction of a mock GW source catalogues**

– distribution of events in redshift

draw redshifts from a probability distribution $p(z) \propto \frac{dV_c}{dz}\frac{R(z)}{(1+z)}$
merger rate typically from O2 rates and populations estimates, or other models

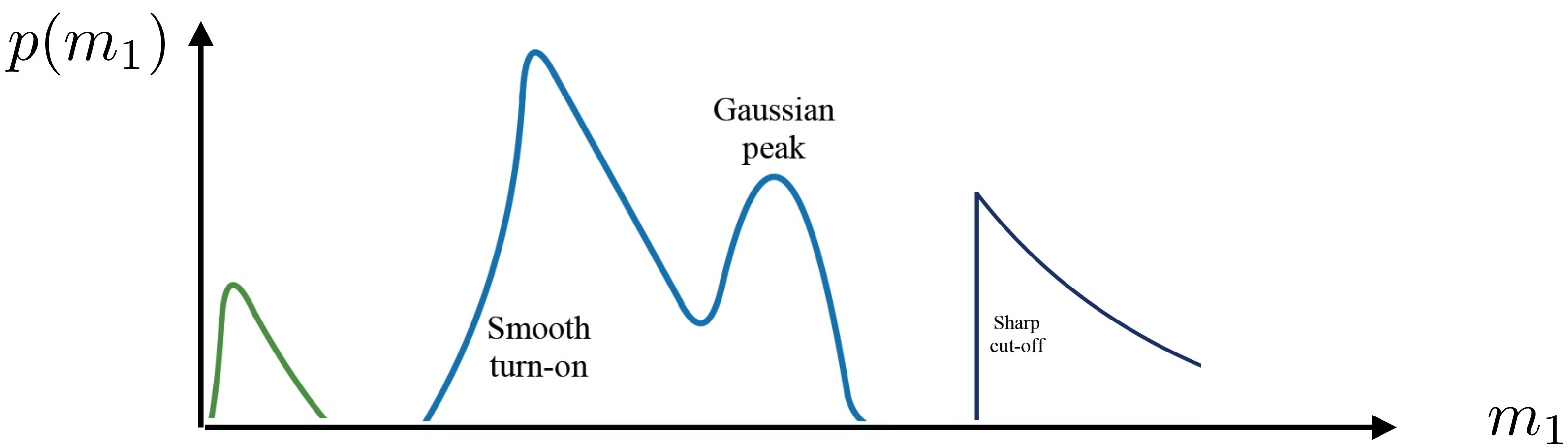
– Given fiducial values of $(H_0, \Omega_m, \ldots)$ determine $d_L(z)$

– Different assumptions made on error $\sigma_{d_L(z)}$

[assumed gaussian Fisher-Matrix approach, or analytic beyond gaussian, or using likelihoods from Bilby, or…
Include lensing error

– calculate SNR for each GW in the catalogue, if > threshold then event observable.

+ mass model, with masses drawn from a probability distribution $p(m_1, m_2)$

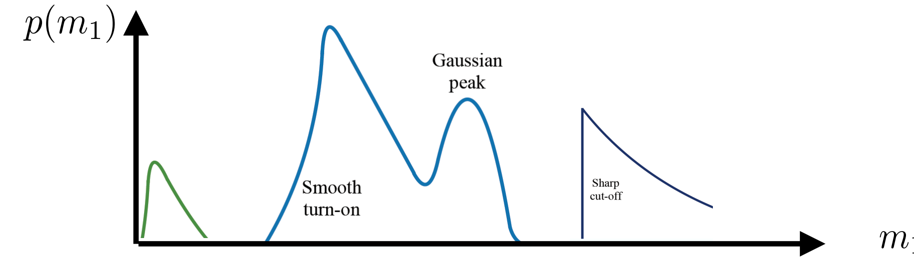


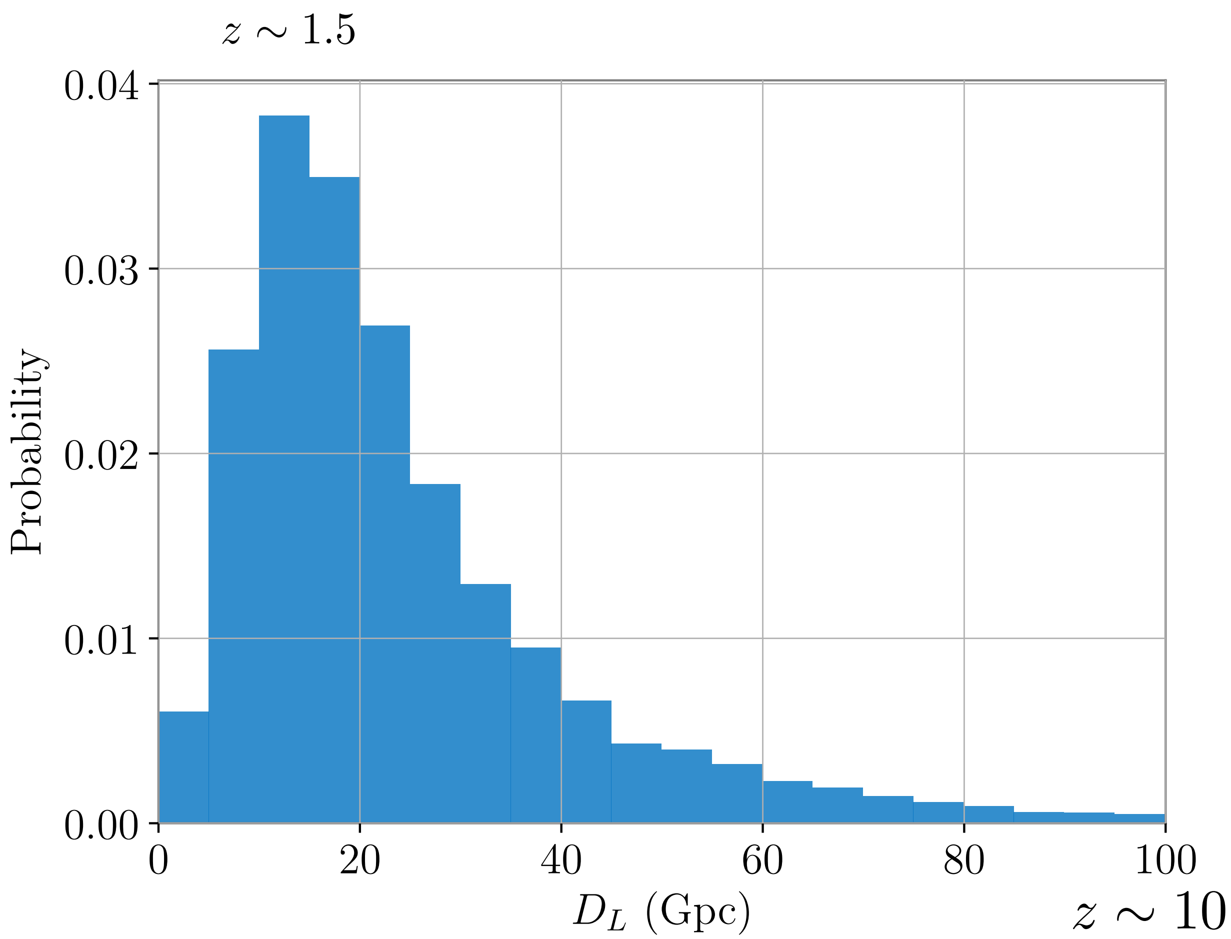
+ for BNS and/or NS-BH need criterion to determine those GW events which also observable in EM

Forecasts with ET; You et al

[You et al 2004.00036]

simulated population of BBHs in ET:
calibrated on *a power-law + gaussian peak model* from GWTC-2, and BBH merger rate from LVC 2020, *Mmax=65 and Mmin=5 fixed*





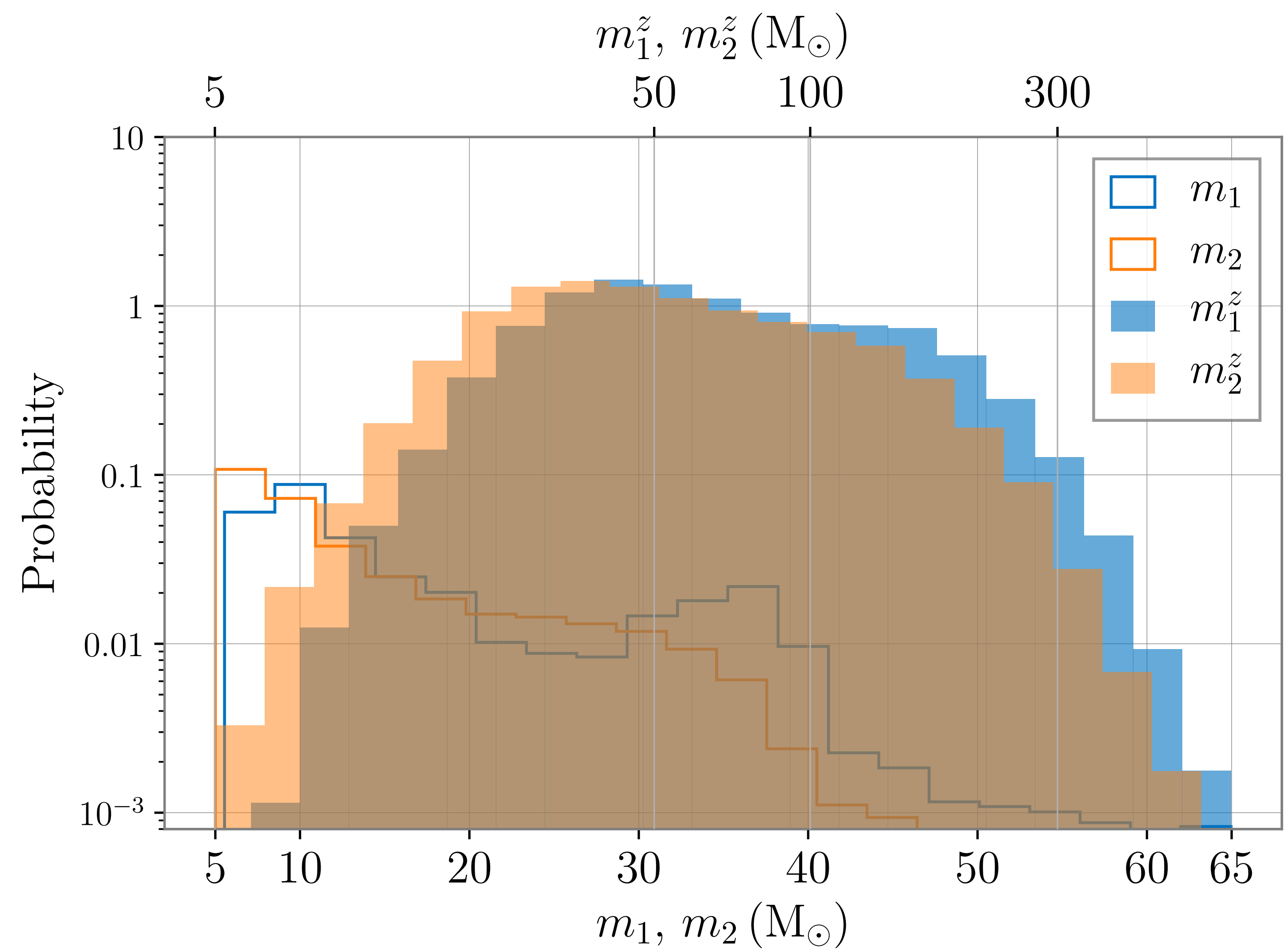


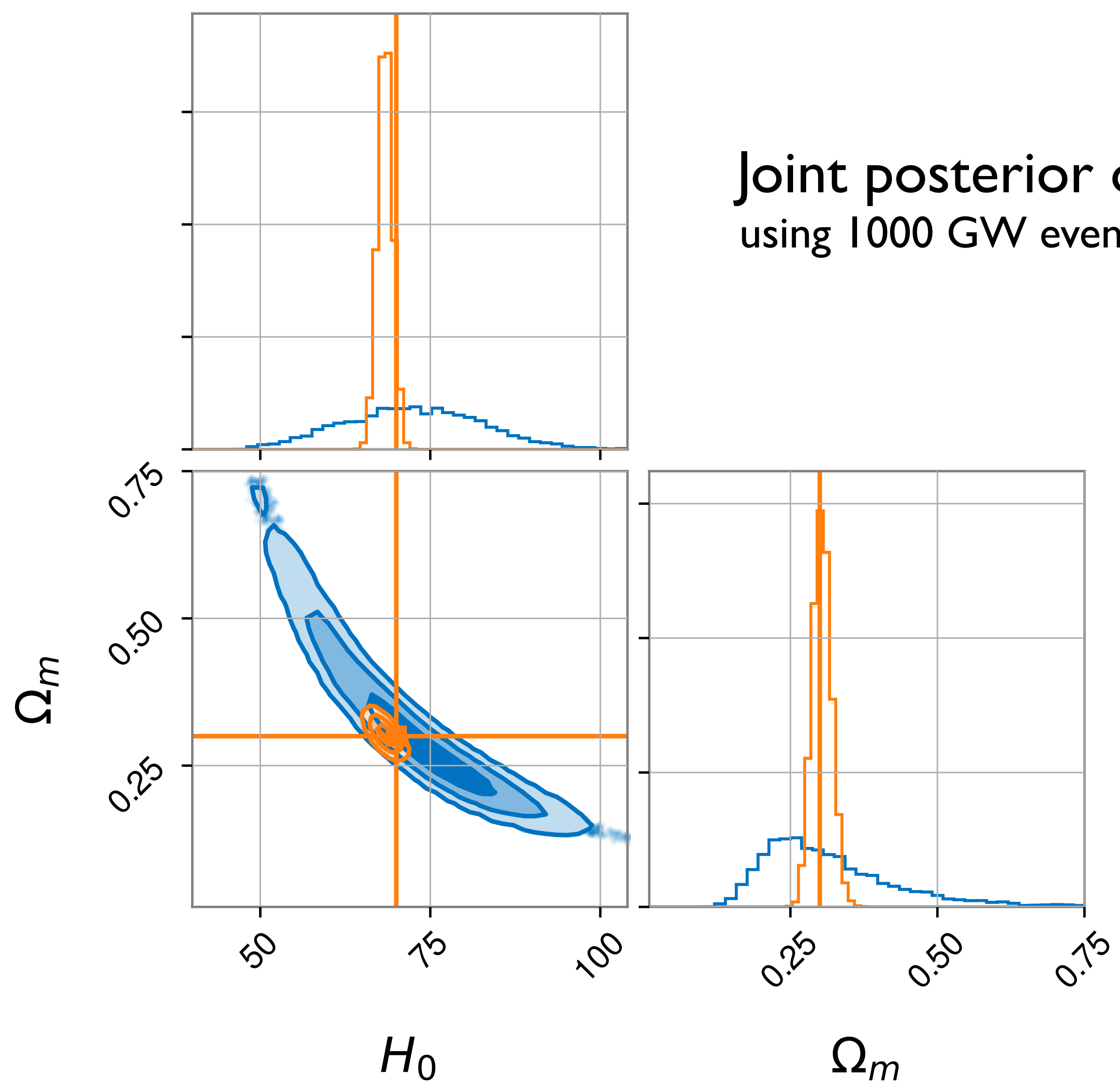
Figure 1. The distribution of luminosity distance (D_L) and black hole masses (m_1, m_2) for a simulated BBH population detectable by third-generation detectors, where the probability is normalized with the logarithm of mass.

$$m_{1,2}^z = m_{1,2}\left[1 + z_{1,2}(D_L)\right].$$

Forecasts with ET; You et al

[2004.00036]

- Carry out a full hierarchical Bayesian inference to compute posterior distributions on parameters describing the population, including (H_0, Ω_m)



Joint posterior on (H_0, Ω_m)
using 1000 GW events detected by ET

orange: assuming all population parameters known a priori

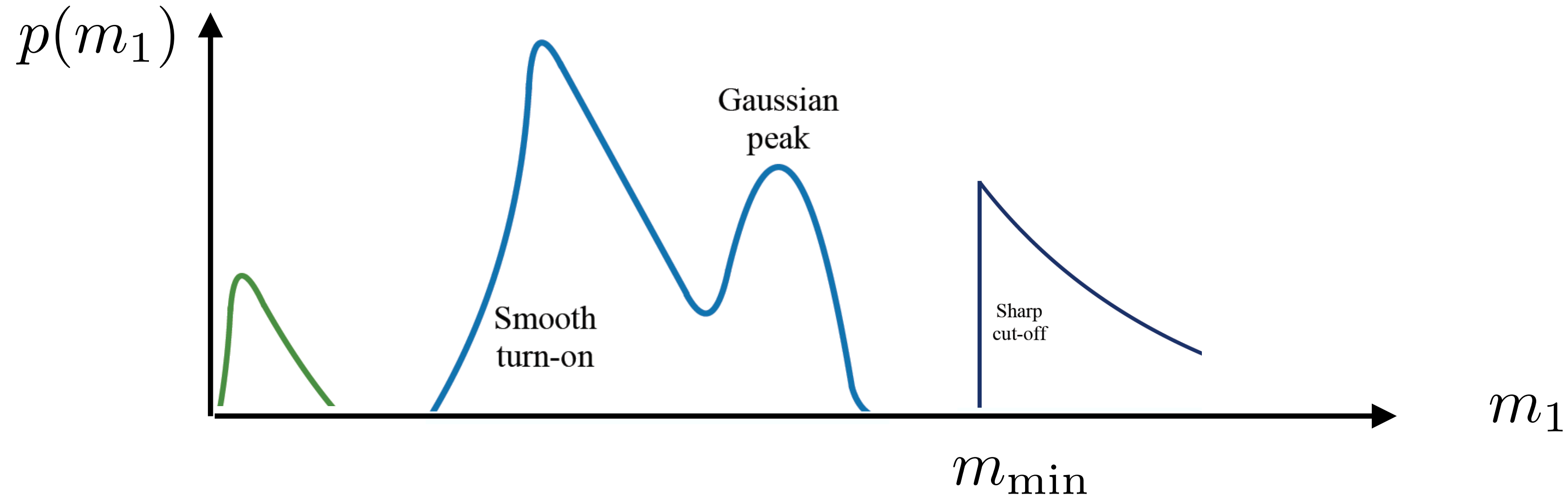
blue: marginalizing over population parameters

"1 year observation of ET will constrain the Hubble constant to a few % given our current knowledge of the black hole mass distribution, the cosmic star formation rate, and the binary merger delay time distribution. If/when our understanding of the above quantities is improved, which is plausible in the ET era, a sub-percent measurement precision is likely."

Forecasts with ET [Ezquiaga and Holz] *[2006.02211]*:

simulated population of intermediate mass BBHs in ET:

– assume a uniform distribution of BH masses above $m_{\min}$



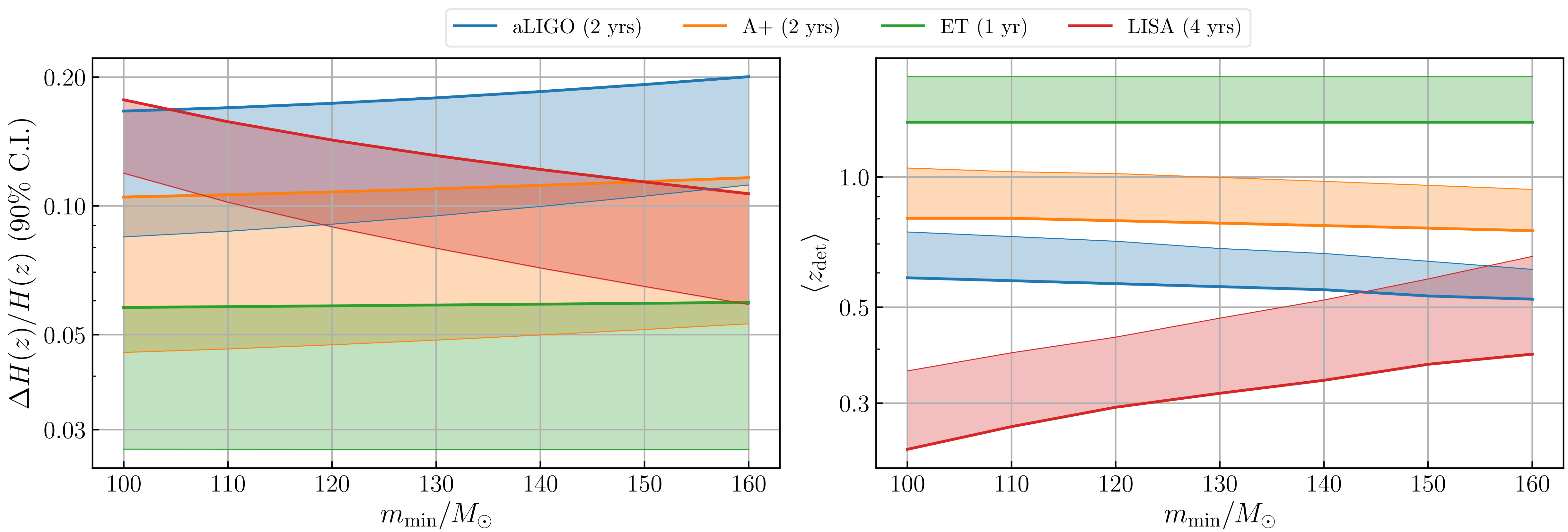


FIG. 5. (Left panel) Estimated fractional error on the Hubble parameter $\Delta H(z)/H(z)$ at 90% confidence interval (C.I.) obtained from standardizable GW sirens above the PISN gap, and (right panel) their most probable detected redshift. For both plots we assumed a uniform distribution of BBHs masses from $m_{\min}$ to $m_{\min} + 60 M_\odot$ with comoving merger rate $\mathcal{R}_c = 0.1\,\text{Gpc}^{-3}\text{yr}^{-1}$. The shaded regions represent the uncertainty in the redshift evolution of the merger rate between a constant rate (thick line) and a rate following the star formation rate (thin line)

Forecasts with ET [Belgacem et al]

[Belgacem et al, 1907.01487]

simulated population of BNS with EM counterparts:

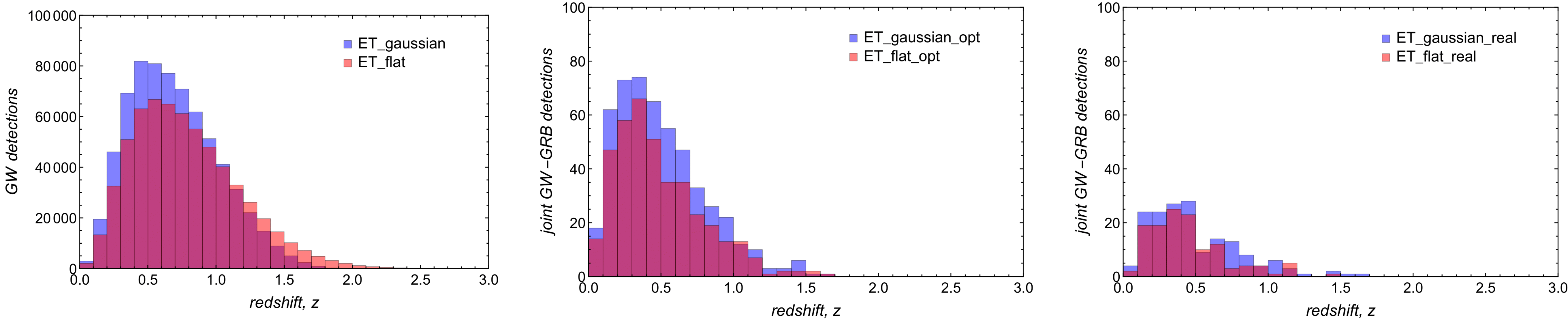


Figure 3. Left panel: the redshift distributions of 10-years of BNS detections by a ET detector. Middle panel: the coincident detections made by THESEUS in the 'optimistic' scenario for the FOV. Right panel: the coincident detections in the 'realistic' scenario. Notice that the vertical scale for the left panel is very different from that in the middle and right panels.

$\sim 6 \times 10^5$ events with SNR>12 over 10 years, of which optimistically ~400-500

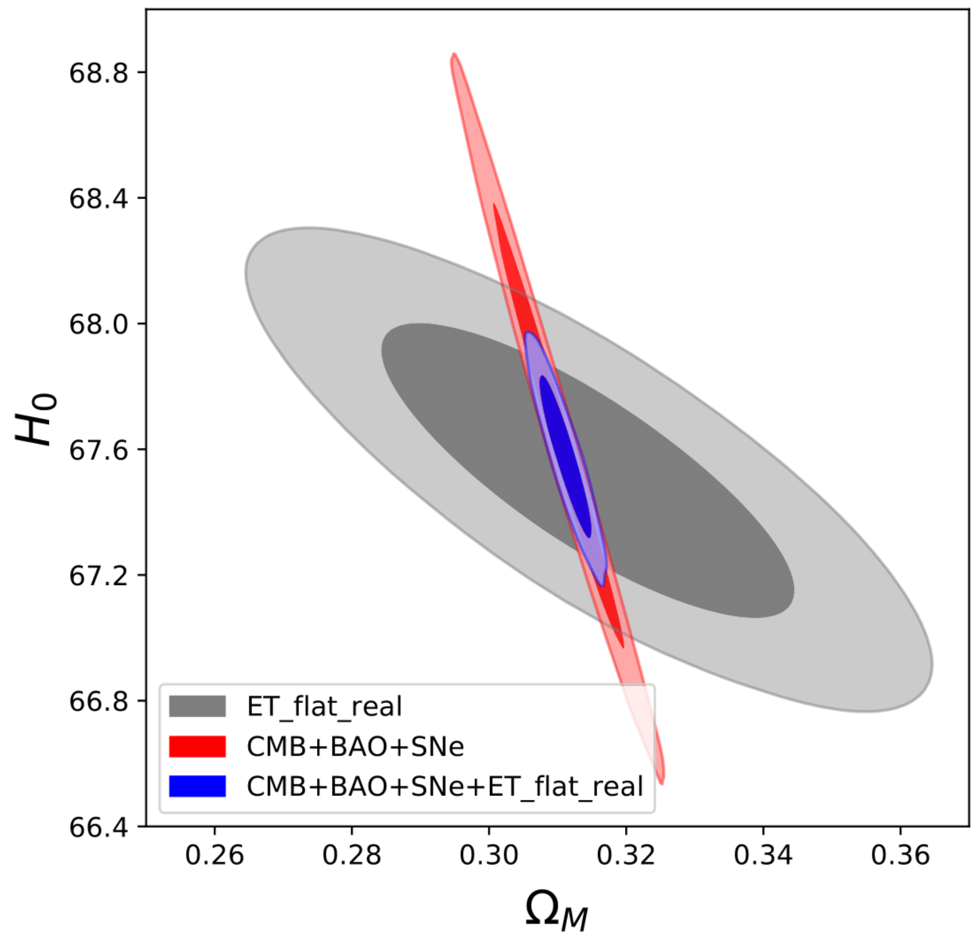
realistically ~100-200 have EM counterpart

CONSTRAINTS: simulated population of BNS with EM counterparts in ET:

[Belgacem et al, 1907.01487]

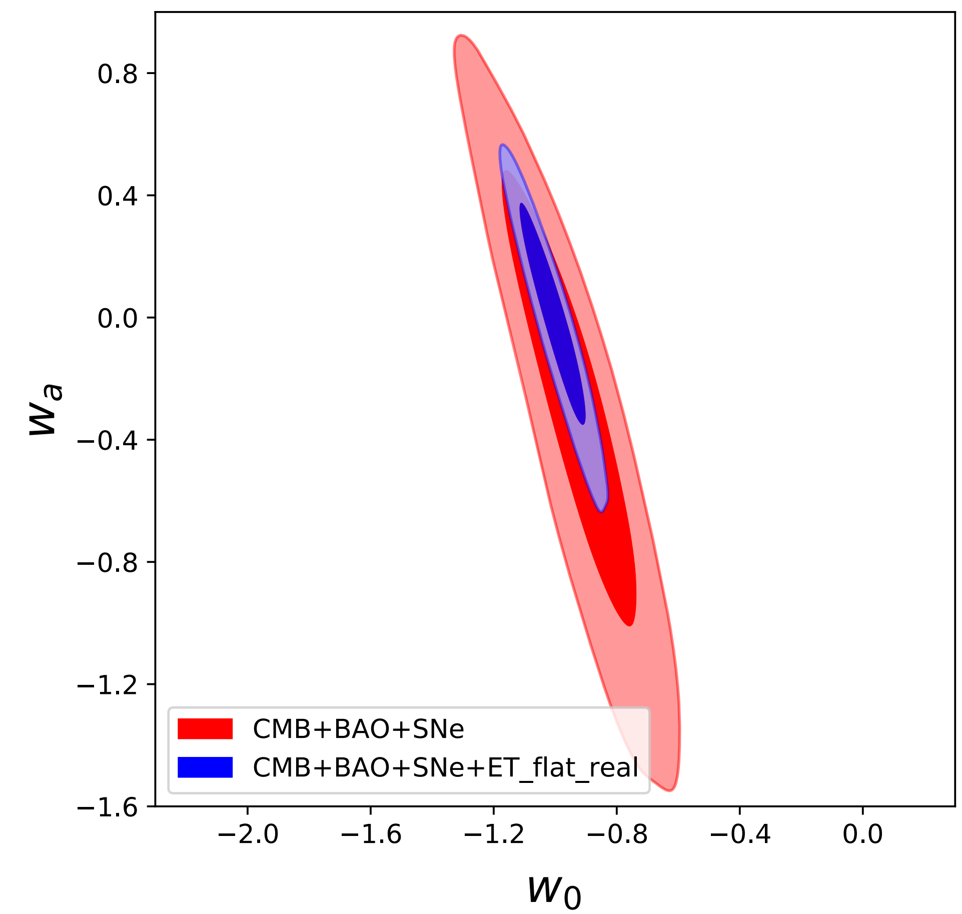
For LambdaCDM

	$\Delta H_0/H_0$	$\Delta\Omega_M/\Omega_M$
ET_flat_real	0.42 %	6.17 %
CMB+BAO+SNe	0.72 %	2.11 %
CMB+BAO+SNe+ET_flat_real	0.26 %	0.82 %



Allowing evolving Dark Energy $w = w_0 + w_a \frac{z}{1+z}$

(w_0, w_a)extension	Δw_0	Δw_a
CMB+BAO+SNe	0.140	0.483
CMB+BAO+SNe+ET	0.058	0.224



Assuming all population parameters are known a priori

Forecasts with ET [Cai et al]

[Cai et al, 1608.08008]

simulated population of BNS and BH-NS with EM counterparts in ET:

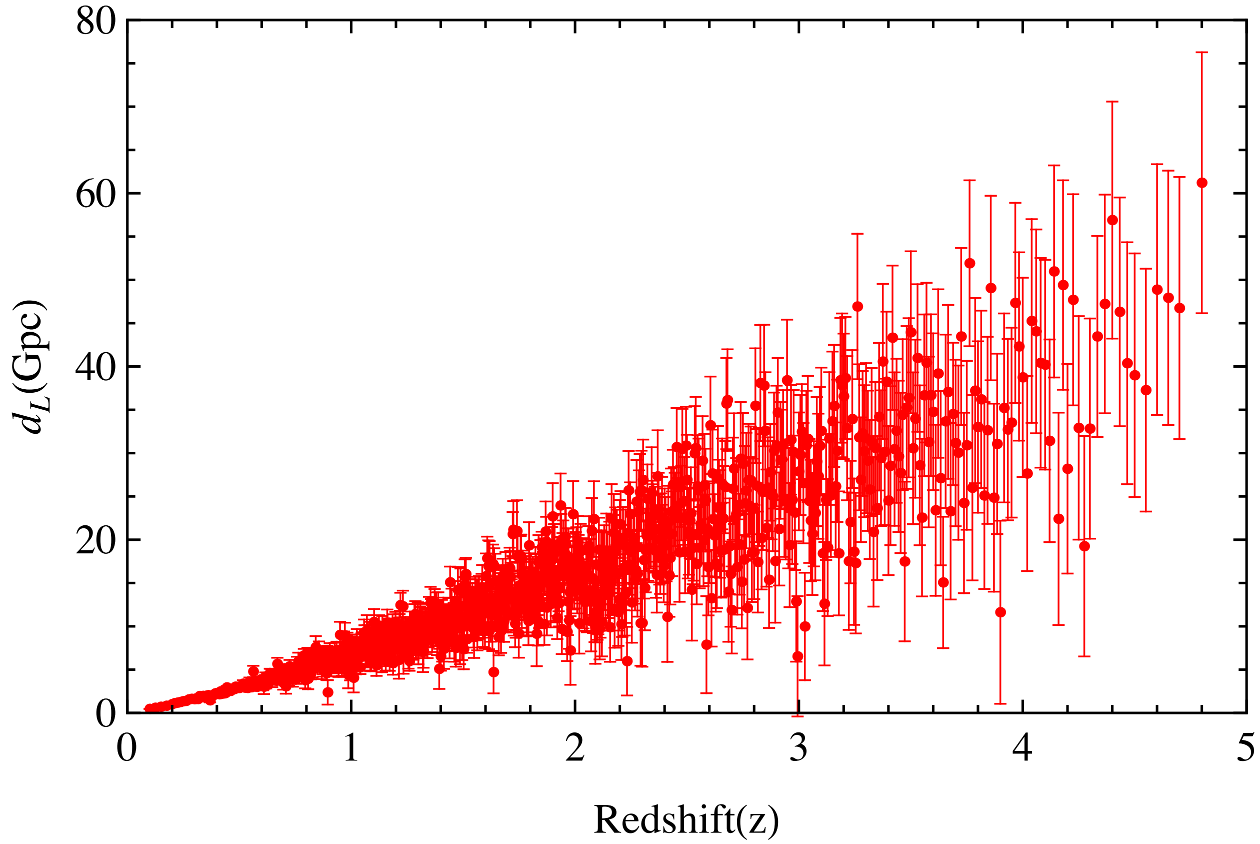


FIG. 1: An example catalogue with 1000 observed events of redshift, luminosity distance, and the error of the luminosity distances from the fiducial model.

- NS mass distribution uniform in interval

$$[1,2]\ M_\odot$$

- BH mass distribution uniform in interval

$$[3,10]\ M_\odot$$

Predictions more optimistic, as include also NS-BH population
(i) these can also emit a counterpart => more events
(ii) louder signals

CONSTRAINTS: simulated population of BNS and BH-NS with EM counterparts in ET:

[Cai et al, 1608.08008]

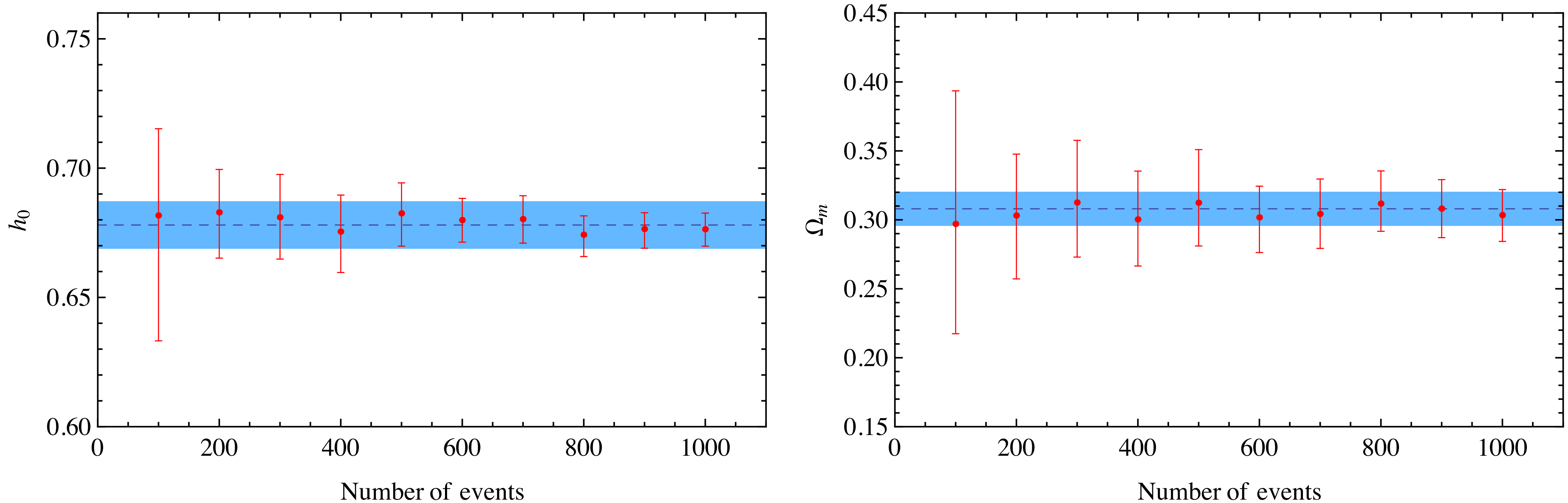


FIG. 2: Sixty-eight percent confidence level (C.L.) (red line) and the best fit (red dot) for H_0 (left) and Ω_m (right) for a variable number of GW events with EM counterpart. The fiducial model is shown as the dashed line. For a comparison, the blue shaded area is the 68% C.L. constrained by the *Planck* temperature data combined with *Planck* lensing in the current *Planck* 2015 results.

with ~600 events get an accuracy on H0 comparable to Planck

Conclusion and outlook

• Different ways to extract information on H0 and dark energy using GW measurements. Direct EM counterparts, galaxy catalogues, and/or through BBH, BNS populations.
Cosmology hand in hand with astrophysics

• In near future: expect important impact on measurements of cosmological parameters, certainly resolving the Hubble tension

• Number of effects to consider: overlapping sources and parameter estimation; higher order modes; precessing spins; waveform accuracy ? etc

• Other methods: redshift from tidal deformation and post-merger signal *[Messenger et al]*

– if a NS mergers with a compact object and is tidally deformed, extra phase in waveform which depends on the source mass through the tidal deformation parameter. Provided an equation of state is know, and if the tidal deformation parameters are accurately measured -> source mass and hence z. (Prediction, in Lambda CDM of inference on H0 of ~7% with O(10000) events)

• GWs can also constrain *modified gravity theories*, particularly those in which the *propagation* of GWs is affected.

$$h_A \propto \frac{1}{d_L^{\mathrm{GW}}}$$

$$d^{\mathrm{GW}}(z) = d_{\mathrm{EM}}(z) \exp\left[\int_0^z \frac{\alpha_M(z)}{1+z} dz\right]$$