A compressible two-layer mixed-flow model.

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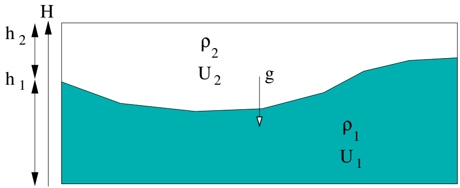
The Compressible Two-Layer (CTL) model [1,2] has been proposed in order to model liquid/gas flows in pipes with the following features:

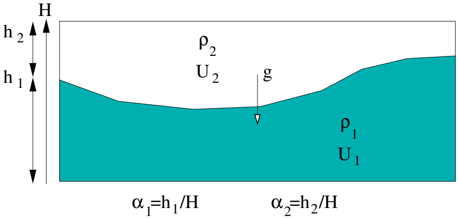
- uniform diameter of the pipes;
- one velocity for each phase (possibility of counter-current flows);
- compressibility of the phases;
- thermal aspects where first out of the scope of the targetted applications (barotropic EOS, no equation for energies);
- stratification due to gravity (liquid is below the gas);
- initially without mass transfer.



$\alpha_1 = h_1/H$ $\alpha_2 = h_2/H$

[1] *A compressible two-layer model for transient gas–liquid flows in pipes*, C. Demay, J.-M. Hérard, CMAT, 2017.

[2] *Modélisation et simulation d'écoulements transitoires diphasiques eau-air dans les circuits hydrauliques*, C. Demay, PhD Thesis, 2017.



Barotropic Baer-Nunziato like model:

$$\begin{cases} \dfrac{\partial}{\partial t}(\alpha_2) + Q_2/m_2 \dfrac{\partial}{\partial x}(\alpha_2) = \Phi_2 = -\Phi_1, \\ \dfrac{\partial}{\partial t}(m_1) + \dfrac{\partial}{\partial x}(Q_1) = M_1, \\ \dfrac{\partial}{\partial t}(Q_1) + \dfrac{\partial}{\partial x}\left(Q_1^2/m_1 + \alpha_1 P_1(\alpha_1/m_1)\right) + \mathscr{P}_I(\alpha_1, m_1) \dfrac{\partial}{\partial x}(\alpha_2) = \overline{V} M_1 - \lambda_u(U_1 - U_2), \\ \dfrac{\partial}{\partial t}(m_2) + \dfrac{\partial}{\partial x}(Q_2) = -M_1, \\ \dfrac{\partial}{\partial t}(Q_2) + \dfrac{\partial}{\partial x}\left(Q_2^2/m_2 + \alpha_2 P_2(\alpha_2/m_2)\right) - \mathscr{P}_I(\alpha_1, m_1) \dfrac{\partial}{\partial x}(\alpha_2) = -\overline{V} M_1 + \lambda_u(U_1 - U_2), \end{cases}$$

with: $\alpha_1 + \alpha_2 = 1$, $m_k = \alpha_k \rho_k$, $Q_k = m_k U_k$, $\overline{V} = (U_1 + U_2)/2$ and where $\tau_k \to P_k(\tau_k)$ are EOS specified by the user. Interfacial pressure reads:

$$\mathscr{P}_I = P_1(\tau_1) - \frac{g \alpha_1 H \rho_1}{2}.$$

Closure of the source terms thanks to a convex energy (counterpart of entropic criterion for BN model with energy):

$$E = E_1 + E_1^p + E_2, \quad \text{with } E_k = m_k(e_k + U_k^2/2) \quad \text{and } E_1^p = \alpha_1 m_1 gH/2.$$

This leads to:

$$\frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x}\left(U_1(E_1 + E_1^p + \alpha_1 P_1) + U_2(E_2 + \alpha_2 P_2)\right) = \left(h1 + \frac{\alpha_1 gH}{2} - h_2\right) M_1 + (P_2 - \mathscr{P}_I)\Phi_1 - \lambda_u(U_1 - U_2)^2,$$

with $h_k = e_k + P_k \tau_k$ and $\lambda_u \geq 0$. Classical choices are possible for M_1 and Φ_1, i.e.:

$$M_1 = -\lambda_m \left(h1 + \frac{\alpha_1 gH}{2} - h_2\right), \quad \text{and } \Phi_1 = -\lambda_p(P_2 - \mathscr{P}_I), \quad \text{with } \lambda_m, \lambda_p \geq 0.$$

But, an other choice is proposed on the basis of [3]. The following energy is defined:

$$(\alpha_1, m_1) \mapsto \mathscr{E}_0^{\#}(\alpha_1, m_1) = m_1 e_1\left(\frac{\alpha_1}{m_1}\right) + \alpha_1 m_1 \frac{gH}{2} + (m^0 - m_1)e_2\left(\frac{1-\alpha_1}{m^0 - m_1}\right),$$

with $m_0 = m_1 + m_2$.

It can be proved that it is strictly convex provided that the following sufficient condition is fulfilled:

$$\frac{1}{2}\sqrt{gH} < \min(c_1, c_2),$$

i.e. speed of gravity waves < speed of sound in pure phases. We assume that this condition holds in the following.

[3] *Various choices of source terms for a class of two-fluid two-velocity models*, O. Hurisse, ESAIM: M2AN, 2021.

We have:

$$\frac{\partial}{\partial \alpha_1}\left(\mathscr{E}_0^{\#}\right) = P_2 - \mathscr{P}_I, \quad \text{and} \quad \frac{\partial}{\partial m_1}\left(\mathscr{E}_0^{\#}\right) = h_1 - h_2 + \frac{\alpha_1 g H}{2},$$

so that:

$$\begin{aligned}
&\frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x}\left(U_1(E_1 + E_1^p + \alpha_1 P_1) + U_2(E_2 + \alpha_2 P_2)\right) \\
&= \frac{\partial}{\partial m_1}\left(\mathscr{E}_0^{\#}\right) M_1 + \frac{\partial}{\partial \alpha_1}\left(\mathscr{E}_0^{\#}\right) \Phi_1 - \lambda_u (U_1 - U_2)^2 \\
&= \nabla_{(\alpha_1, m_1)} \mathscr{E}_0^{\#}(\alpha_1, m_1) . \begin{pmatrix} \Phi_1 \\ M_1 \end{pmatrix} - \lambda_u (U_1 - U_2)^2 .
\end{aligned}$$

Since $\mathscr{E}_0^{\#}$ is strictly convex wrt. (α_1, m_1), there exists a unique minimizer $(\overline{\alpha_1}, \overline{m_1})$ so that:

$$0 \geq \mathscr{E}_0^{\#}(\overline{\alpha_1}, \overline{m_1}) - \mathscr{E}_0^{\#}(\alpha_1, m_1) \geq \nabla_{(\alpha_1, m_1)} \mathscr{E}_0^{\#}(\alpha_1, m_1) . \begin{pmatrix} \overline{\alpha_1} - \alpha_1 \\ \overline{m_1} - m_1 \end{pmatrix} .$$

Hence, by choosing:

$$M_1 = \lambda(\overline{m_1} - m_1) \quad \text{and} \quad \phi_1 = \lambda(\overline{\alpha_1} - \alpha_1),$$

with the same λ for both source terms and $\lambda \geq 0$, we get:

$$\frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x}\left(U_1(E_1 + E_1^p + \alpha_1 P_1) + U_2(E_2 + \alpha_2 P_2)\right) \leq 0$$

- Extension of the CTL model [1,2] to flows with mass transfer.
- Homogeneous-like "BGK" source terms for two-fluid models (extension to BN model with energy, see [3]).
- Thermodynamical disequilibrium source terms are written in a more linear form wrt. primal variables.
- In a numerical point of view, treatment of these terms is more easy.
- Coupling of convection terms and source terms is more easy, see [4] for an example while using a MAC/SIMPLE fractional step sheme.

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[2] *Modélisation et simulation d'écoulements transitoires diphasiques eau-air dans les circuits hydrauliques*, C. Demay, PhD Thesis, 2017.

[3] *Various choices of source terms for a class of two-fluid two-velocity models*, O. Hurisse, ESAIM: M2AN, 2021.

[4] *A semi-implicit fractional step algorithm on staggered meshes for simulating a compressible two-layer mixed-flows model*, O. Hurisse, submitted.