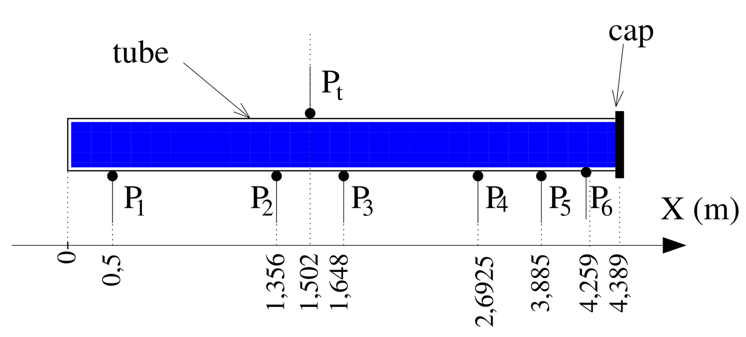
OUT-OF-EQUILIBRIUM TWO-PHASE FLOW MODEL

Coupling with complex equations of state.

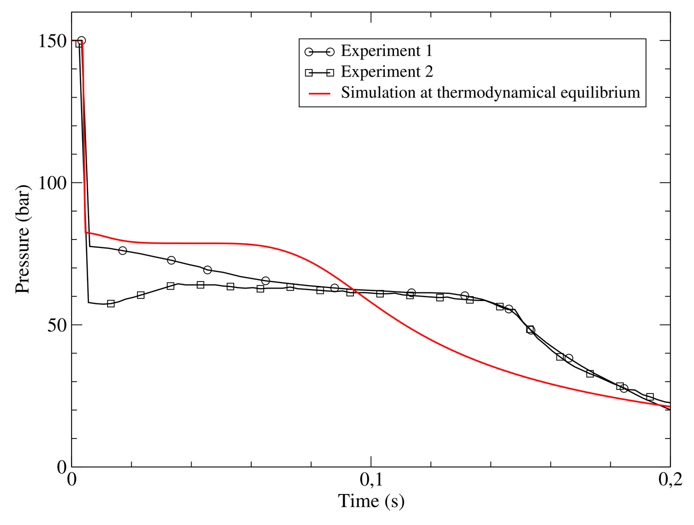
CONTEXT

Safety issue demands reliable numerical simulations of two-phase flow phenomena which may occur within a pressurized water nuclear reactor.

Some accidental scenarios, like vapor explosion or LOCA, involve rapid transients with important mass transfer. One may then need a model able to account for thermodynamical disequilibria.



SUPERCANON Experiment (B. Riegel PhD, 1978)



Time evolution of the pressure at the point P1 for two experimental runs and a numerical simulation at thermodynamical equilibrium.

THE HOMOGENEOUS MODEL

Code Saturne embedded a module based on the homogeneous two-phase flow model proposed in [1], assuming the full thermodynamical disequilibrium - i.e. in terms of the pressures, temperatures and chemical potentials.

System of Equations :

$$\begin{cases} \dfrac{\partial}{\partial t}(\rho Y) + \dfrac{\partial}{\partial x}(\rho U Y) = \rho\Gamma_Y \\ \dfrac{\partial}{\partial t}(\rho) + \dfrac{\partial}{\partial x}(\rho U) = 0 \\ \dfrac{\partial}{\partial t}(\rho U) + \dfrac{\partial}{\partial x}(\rho U^2 + P) = 0 \\ \dfrac{\partial}{\partial t}(\rho E) + \dfrac{\partial}{\partial x}(\rho U E + U P) = 0, \end{cases} \qquad (1)$$

where $Y = (\alpha_v, y_v, z_v) =$ (volume fraction, mass fraction, energy fraction).

Source terms Γ_Y **:** $\quad \Gamma_Y = \frac{Y_{eq}-Y}{\lambda}$, $\quad$ as in [1, 2]

λ : characteristic time-scale ; $Y_{eq} = (\alpha_{v,eq}, y_{v,eq}, z_{v,eq})$: equilibrium fractions which maximize s for a given e and τ.

Thermodynamical closure, for P :

- Specific entropy : $s = y_v s_v(\tau_v, e_v) + y_l s_l(\tau_l, e_l)$, with $s_k(\tau_k, e_k)$ (phasic entropies) specified by the user.
- Definition of P and T for the mixture through the Gibbs relation :

$$T ds = de + P d\tau + \partial_\alpha s|_{e,\tau,y,z} d\alpha + \partial_y s|_{e,\tau,y,z} dy + \partial_z s|_{e,\tau,y,z} dz.$$

$$P = \frac{\frac{\alpha_l}{T_l}P_l + \frac{\alpha_v}{T_v}P_v}{\frac{z_l}{T_l} + \frac{z_v}{T_v}} \text{ and } \frac{1}{T} = \frac{z_l}{T_l} + \frac{z_v}{T_v}. \qquad (2)$$

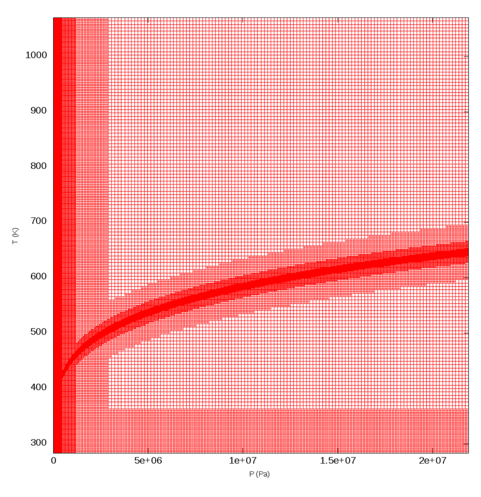
Hyperbolicity of (1) closed by (2) if :
(i) For $k \in \{l, v\}$, s_k strictly concave w.r.t. (τ_k, e_k) ; (ii) $T \geq 0$. [2].

VERIFICATION WITH A LOOK-UP TABLE

Classically, simple analytical Equations of State are used, like the Stiffened Gas EoS :

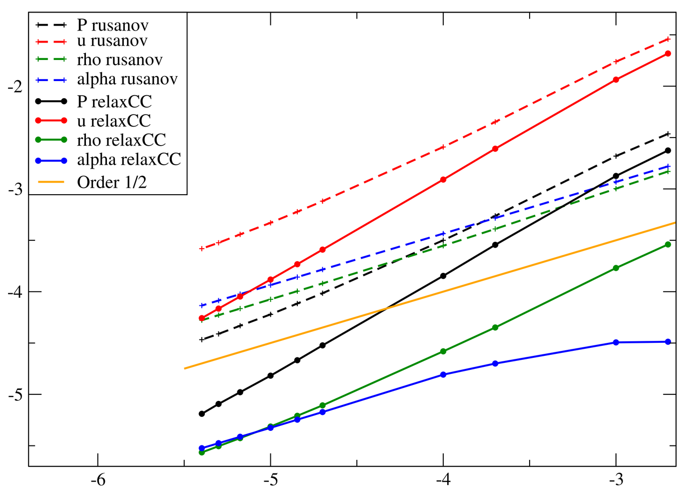
$$s_k(\tau_k, e_k) = C_{v,k} \ln\left((e_k - \Pi_k \tau_k)\tau_k^{\gamma_k - 1}\right) + s_k^0.$$

However, more complex laws may be needed (cf SUPERCANON experiment). A look-up table, based on the IAPWS-IF97 formulations, has been coupled with the homogeneous model. Robust and accurate numerical schemes are then required.

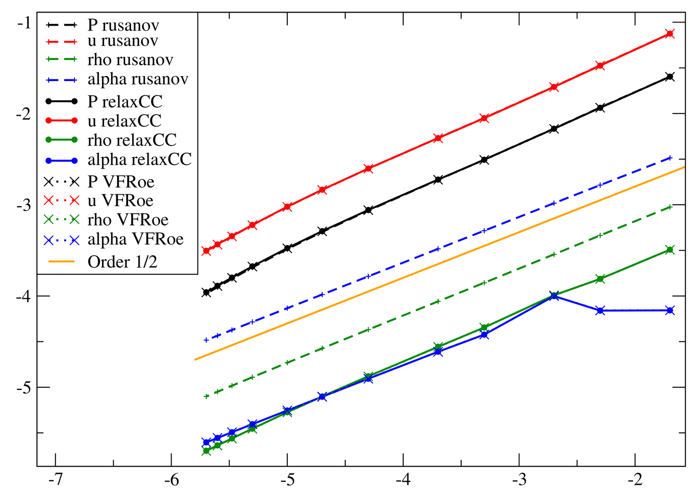


Mesh of the (P, T)-plane for the look-up tables.

Convergence curves obtained for Riemann problems with one intermediate state highlight that the relaxation scheme proposed in [4] is more accurate than the Rusanov scheme for both EoS. The VFRoe scheme is not robust enough when using the look-up tables :



Convergence curves with the look-up table; log(L1-error) VS log(mesh size) (from 100 to 150 000 cells).



Convergence curves with the Stiffened Gas EoS; log(L1-error) VS log(mesh size) (from 100 to 500 000 cells).

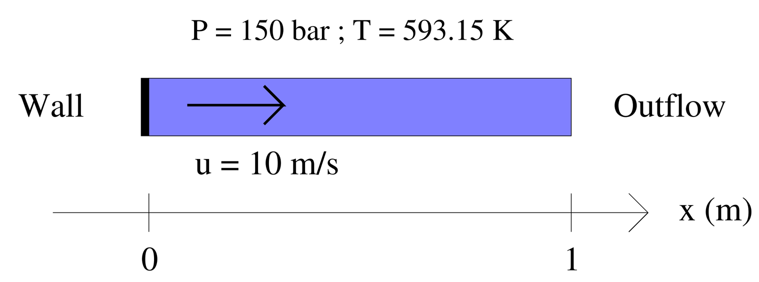
INFLUENCE OF THE RELAXATION TIME

Classical nucleation theory assumes that the bubble nucleation rate J (the number of bubbles created per unit time in unit volume) follows an Arrhenius law :

$$J = J_0 \exp\left(-\frac{E_a}{k_B T}\right), \quad \text{with } E_a = \frac{16\pi\gamma^3}{3(\Delta P)^2}. \qquad (3)$$

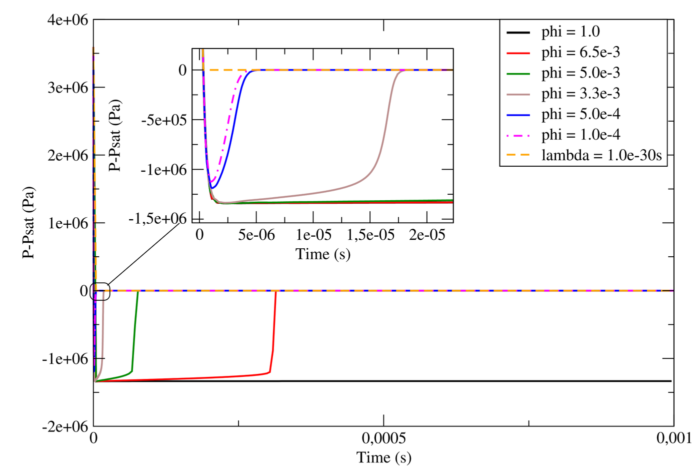
We proposed a simplified model for λ, based on assumptions made for instance in [3] :

$$\lambda = \frac{a_0}{\Delta P^3} \exp\left(-\frac{\varphi E_a}{k_B T}\right). \qquad (4)$$

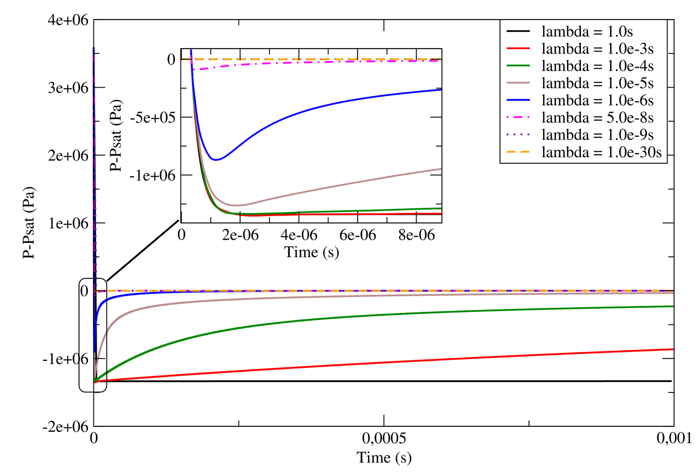


A very simple case has been studied with different closure laws for λ : vaporization near a wall due to a sudden pressure drop in the liquid.

Even if this simplified nucleation model is not completely physical at the moment, this study shows how strongly relaxation time law can modify mixture behavior throughout the simulation :



Time evolution of $P - P_{sat}$ near the wall with λ following (4) for several φ.



Time evolution of $P - P_{sat}$ near the wall for several constant λ.

REFERENCES

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[2] O. Hurisse Numerical simulations of steady and unsteady two-phase flows using a homogeneous model. *Computers & fluids*, 2017.

[3] F. Caupin, E. Herbert Cavitation in water: a review. *Comptes Rendus Physique*, 2006.

[4] C. Chalons, J.-F. Coulombel Relaxation approximation of the Euler equations. *Journal of Mathematical Analysis and Applications*, 2008.

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